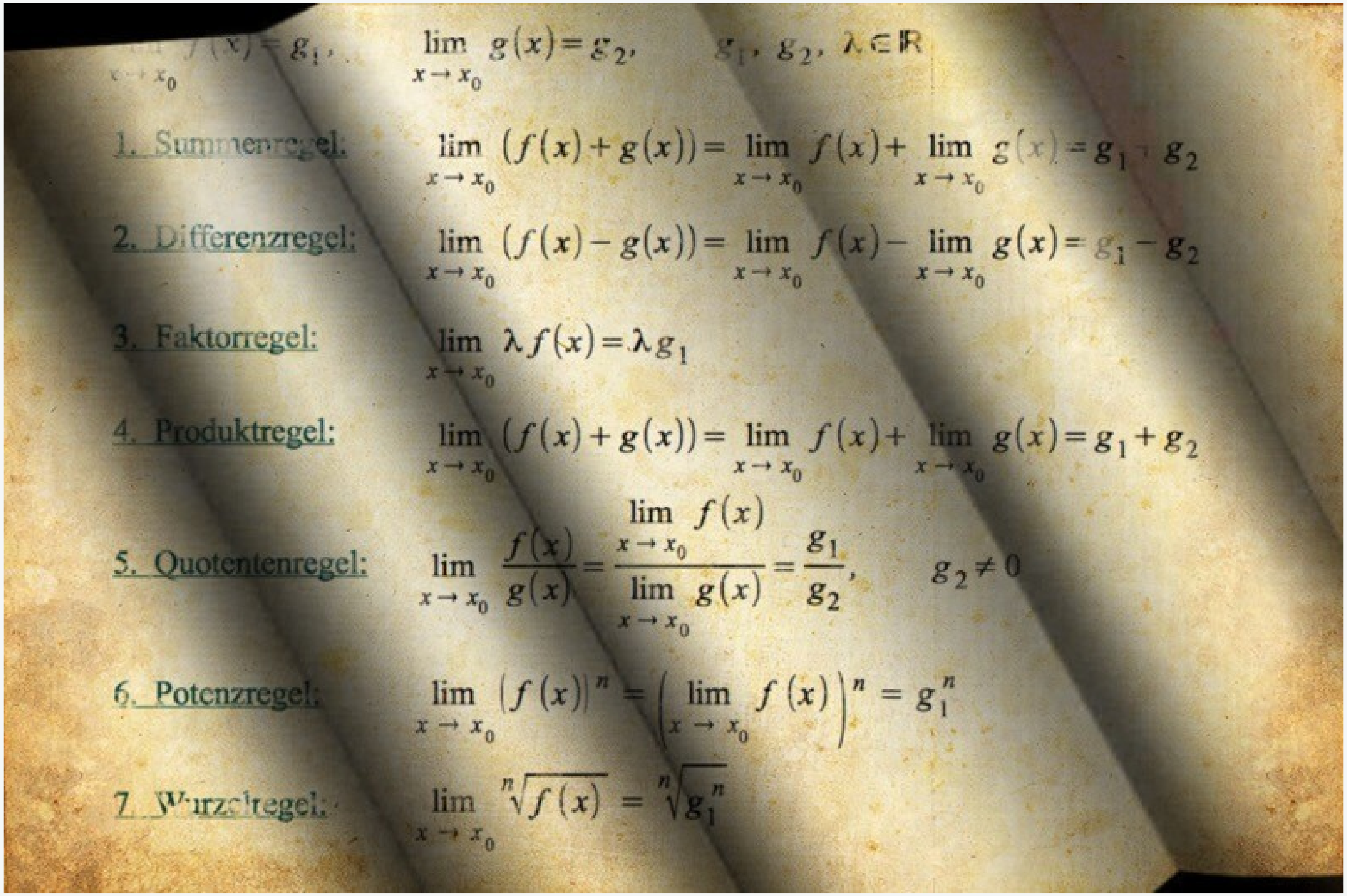

$$\lim_{x \to x_0} f(x) = g_1, \qquad \lim_{x \to x_0} g(x) = g_2, \qquad g_1,\ g_2,\ \lambda \in \mathbb{R}$$

1. Summenregel: $\lim\limits_{x \to x_0} \left(f(x) + g(x)\right) = \lim\limits_{x \to x_0} f(x) + \lim\limits_{x \to x_0} g(x) = g_1 + g_2$

2. Differenzregel: $\lim\limits_{x \to x_0} \left(f(x) - g(x)\right) = \lim\limits_{x \to x_0} f(x) - \lim\limits_{x \to x_0} g(x) = g_1 - g_2$

3. Faktorregel: $\lim\limits_{x \to x_0} \lambda f(x) = \lambda g_1$

4. Produktregel: $\lim\limits_{x \to x_0} \left(f(x) + g(x)\right) = \lim\limits_{x \to x_0} f(x) + \lim\limits_{x \to x_0} g(x) = g_1 + g_2$

5. Quotentenregel: $\lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to x_0} f(x)}{\lim\limits_{x \to x_0} g(x)} = \dfrac{g_1}{g_2}, \qquad g_2 \neq 0$

6. Potenzregel: $\lim\limits_{x \to x_0} \left(f(x)\right)^n = \left(\lim\limits_{x \to x_0} f(x)\right)^n = g_1^n$

7. Wurzelregel: $\lim\limits_{x \to x_0} \sqrt[n]{f(x)} = \sqrt[n]{g_1^n}$

Rechenregeln für Grenzwerte

Rechenregeln für Grenzwerte

$$\lim_{x \to x_0} f(x) = g_1, \qquad \lim_{x \to x_0} g(x) = g_2, \qquad g_1,\ g_2,\ \lambda \in \mathbb{R}$$

1. Summenregel: $$\lim_{x \to x_0} \left(f(x) + g(x) \right) = \lim_{x \to x_0} f(x) + \lim_{x \to x_0} g(x) = g_1 + g_2$$

2. Differenzregel: $$\lim_{x \to x_0} \left(f(x) - g(x) \right) = \lim_{x \to x_0} f(x) - \lim_{x \to x_0} g(x) = g_1 - g_2$$

3. Faktorregel: $$\lim_{x \to x_0} \lambda f(x) = \lambda g_1$$

4. Produktregel: $$\lim_{x \to x_0} \left(f(x) \cdot g(x) \right) = \lim_{x \to x_0} f(x) \cdot \lim_{x \to x_0} g(x) = g_1 \cdot g_2$$

5. Quotentenregel: $$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to x_0} f(x)}{\lim\limits_{x \to x_0} g(x)} = \frac{g_1}{g_2}, \qquad g_2 \neq 0$$

6. Potenzregel: $$\lim_{x \to x_0} \left(f(x) \right)^n = \left(\lim_{x \to x_0} f(x) \right)^n = g_1^n$$

7. Wurzelregel: $$\lim_{x \to x_0} \sqrt[n]{f(x)} = \sqrt[n]{g_1^n}$$

$$x \to \infty$$

$$\lim_{x\to\infty} \frac{2x^3 - 4x^2 + 2}{x^3 + 7x - 12} = \lim_{x\to\infty} \frac{x^3\left(2 - \frac{4}{x} + \frac{2}{x^3}\right)}{x^3\left(1 + \frac{7}{x^2} - \frac{12}{x^3}\right)} = \lim_{x\to\infty} \frac{2 - \frac{4}{x} + \frac{2}{x^3}}{1 + \frac{7}{x^2} - \frac{12}{x^3}} =$$

$$\overset{5}{=} \frac{\lim\limits_{x\to\infty}\left(2 - \frac{4}{x} + \frac{2}{x^3}\right)}{\lim\limits_{x\to\infty}\left(1 + \frac{7}{x^2} - \frac{12}{x^3}\right)} \overset{1,\,2}{=} \frac{\lim\limits_{x\to\infty} 2 - \lim\limits_{x\to\infty} \frac{4}{x} + \lim\limits_{x\to\infty} \frac{2}{x^3}}{\lim\limits_{x\to\infty} 1 + \lim\limits_{x\to\infty} \frac{7}{x} - \lim\limits_{x\to\infty} \frac{12}{x^3}} = 2$$

Bestimmen Sie folgende Grenzwerte:

$a)\ \lim\limits_{x \to 0} \frac{x}{x^2 + 1}, \qquad b)\ \lim\limits_{x \to -2} \frac{x + 2}{x^2 + 3x + 2}$

$c)\ \lim\limits_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}, \qquad d)\ \lim\limits_{x \to 0} \frac{-x}{1 - \sqrt{x + 1}}$

$e)\ \lim\limits_{x \to \infty} \frac{3}{x^2 + x + 5}, \qquad f)\ \lim\limits_{x \to 0} \frac{2x^2 + 4x}{3x^2 - 5}$

$g)\ \lim\limits_{x \to \infty} \frac{x^4 + x^2 + 1}{2x^3 + 6}, \qquad h)\ \lim\limits_{x \to \infty} \frac{x^5 - 12x^3}{x^2 - 4x + 6}$

$$a)\quad \lim_{x \to 0} \frac{x}{x^2 + 1} = 0$$

$$b)\quad \lim_{x \to -2} \frac{x + 2}{x^2 + 3x + 2} = \lim_{x \to -2} \frac{x + 2}{(x + 2)(x + 1)} = \lim_{x \to -2} \frac{1}{(x + 1)} = -1$$

$$c)\quad \lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} = \lim_{x \to 9} (\sqrt{x} + 3) = 6$$

$$x - 9 = (\sqrt{x})^2 - 3^2 = (\sqrt{x} - 3)(\sqrt{x} + 3)$$

$$d)\quad \lim_{x \to 0} \frac{-x}{1 - \sqrt{x + 1}} = \lim_{x \to 0} (1 + \sqrt{x + 1}) = 2$$

$$-x = -x - 1 + 1 = 1 - (x + 1) = 1 - (\sqrt{x + 1})^2 =$$

$$= (1 - \sqrt{x + 1})(1 + \sqrt{x + 1})$$

$$e)\quad \lim_{x \to \infty} \frac{3}{x^2 + x + 5} = 0, \qquad f)\quad \lim_{x \to 0} \frac{2x^2 + 4x}{3x^2 - 5} = 0$$

$$g)\quad \lim_{x \to \infty} \frac{x^4 + x^2 + 1}{2x^3 + 6} = \infty, \qquad h)\quad \lim_{x \to \infty} \frac{x^5 - 12x^3}{x^2 - 4x + 6} = \infty$$

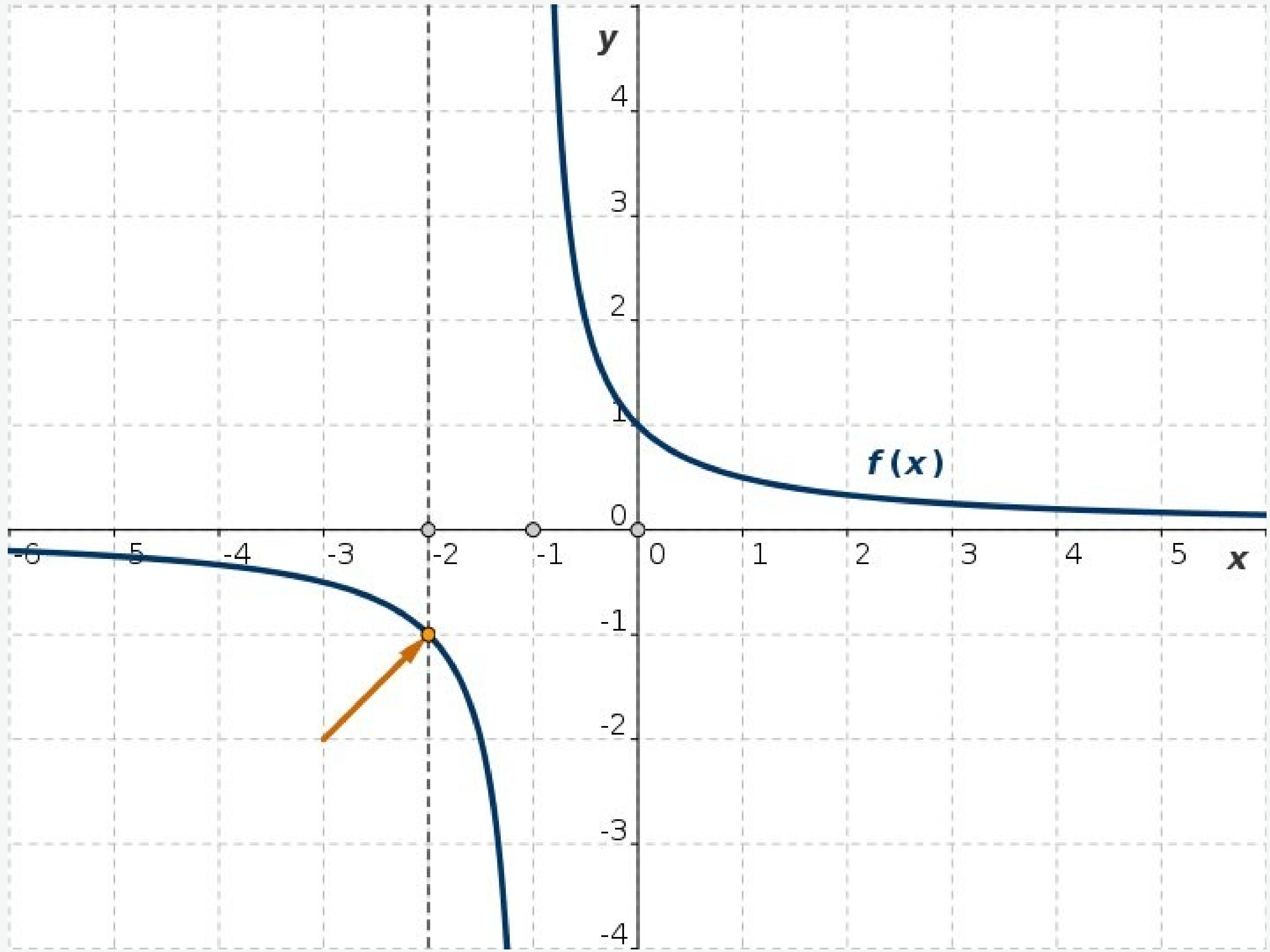

Abb. L1b: Graphische Darstellung der Funktion $y = f(x)$

$$f(x) = \frac{x + 2}{x^2 + 3x + 2}$$

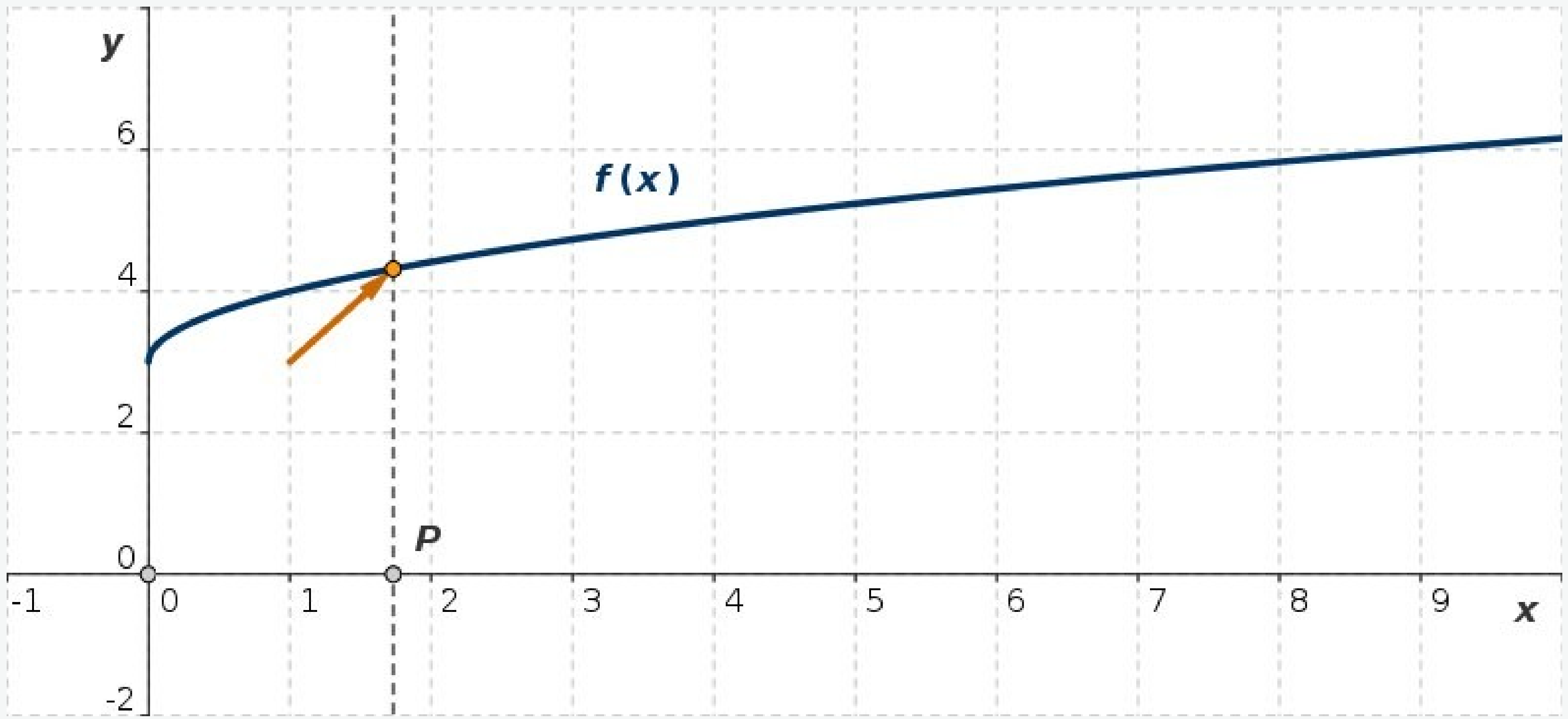

Abb. L1c: Graphische Darstellung der Funktion $y = f(x)$

$$f(x) = \frac{x - 9}{\sqrt{x} - 3}$$

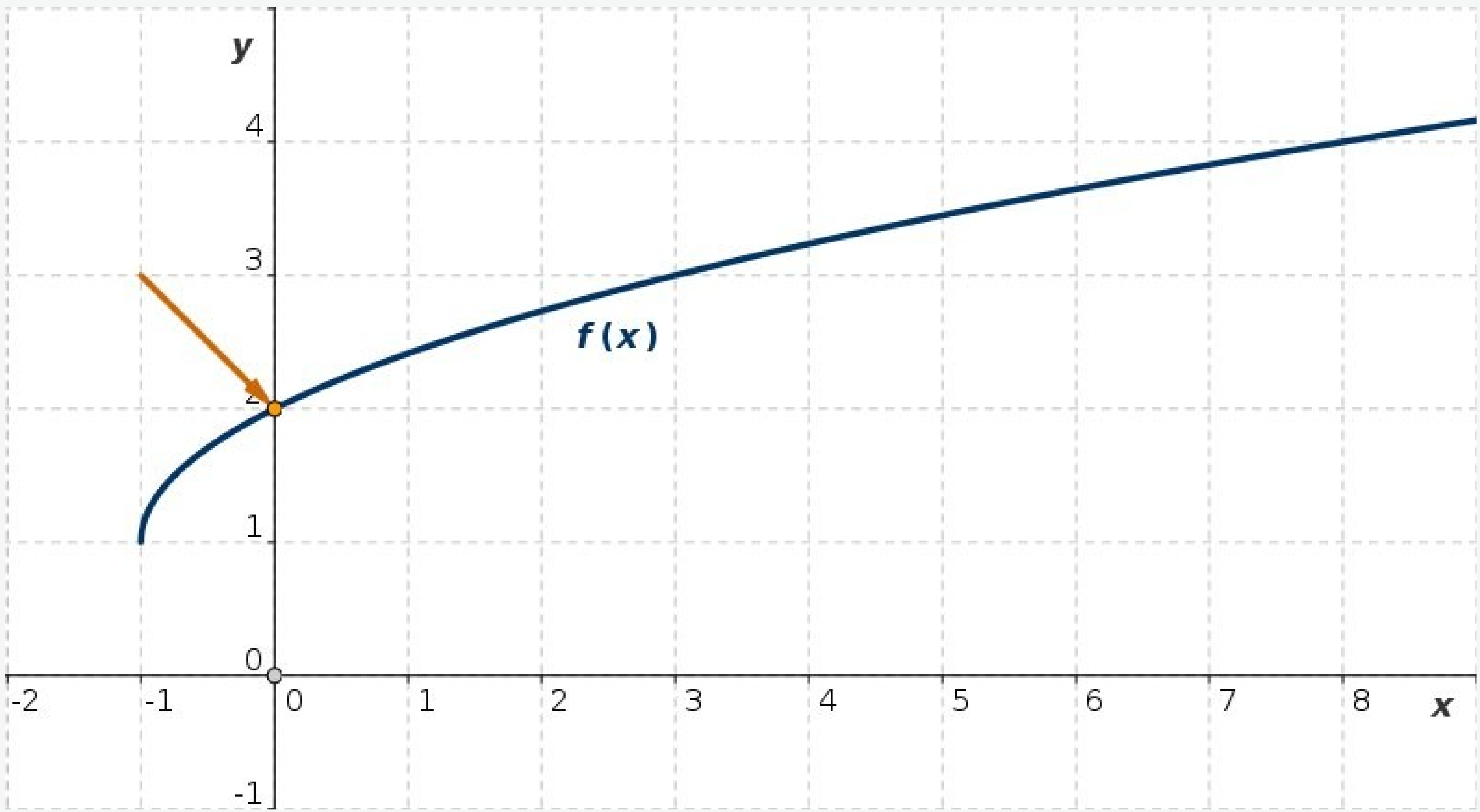

Abb. L1d: Graphische Darstellung der Funktion y = f (x)

$$f(x) = \frac{-x}{1 - \sqrt{x+1}}$$

Grenzwerte von Funktionen: Aufgabe 2

a) $$\lim_{x \to 0} \left(6 \sin^2 x + 4 \cos^2 x + \frac{x - 1}{x + 2} \right)$$

b) $$\lim_{x \to 1} \frac{x^2 - 1}{x^2 + 5x - 6},$$

c) $$\lim_{x \to \infty} \frac{4x - 1}{x - 2}$$

d) $$\lim_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}, \qquad \lim_{x \to 0} \frac{\sin (3x)}{\sin (2x)}$$

$$\alpha \beta \neq 0, \qquad \lim_{x \to 0} \frac{\sin x}{x} = 1$$

e) $$\lim_{x \to -\pi/2} \frac{1 + \cos (2x)}{\cos x}$$

$$a)\quad \lim_{x\to 0}\left(6\sin^2 x + 4\cos^2 x + \frac{x-1}{x+2}\right) = 6\lim_{x\to 0}\sin^2 x + 4\lim_{x\to 0}\cos^2 x$$

$$+\lim_{x\to 0}\frac{x-1}{x+2} = 4 - \frac{1}{2} = \frac{7}{2}$$

$$b)\quad \lim_{x\to 1}\frac{x^2-1}{x^2+5x-6} = \lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(x+6)} = \lim_{x\to 1}\frac{x+1}{x+6} = \frac{2}{7}$$

$$c)\quad \lim_{x\to\infty}\frac{4x-1}{x-2} = 4$$

$$d)\quad \lim_{x\to 0}\frac{\sin\alpha x}{\sin\beta x} = \lim_{x\to 0}\frac{\frac{\sin\alpha x}{\alpha x}\,\alpha x}{\frac{\sin\beta x}{\beta x}\,\beta x} = \frac{\alpha}{\beta}\,\frac{\lim\limits_{x\to 0}\frac{\sin\alpha x}{\alpha x}}{\lim\limits_{x\to 0}\frac{\sin\beta x}{\beta x}} = \frac{\alpha}{\beta}$$

$$\lim_{x\to 0}\frac{\sin(3x)}{\sin(2x)} = \frac{3}{2}$$

$$e)\quad \lim_{x\to -\pi/2}\frac{1+\cos(2x)}{\cos x} = \lim_{x\to -\pi/2}\frac{2\cos^2 x}{\cos x} = \lim_{x\to -\pi/2} 2\cos x = 0$$

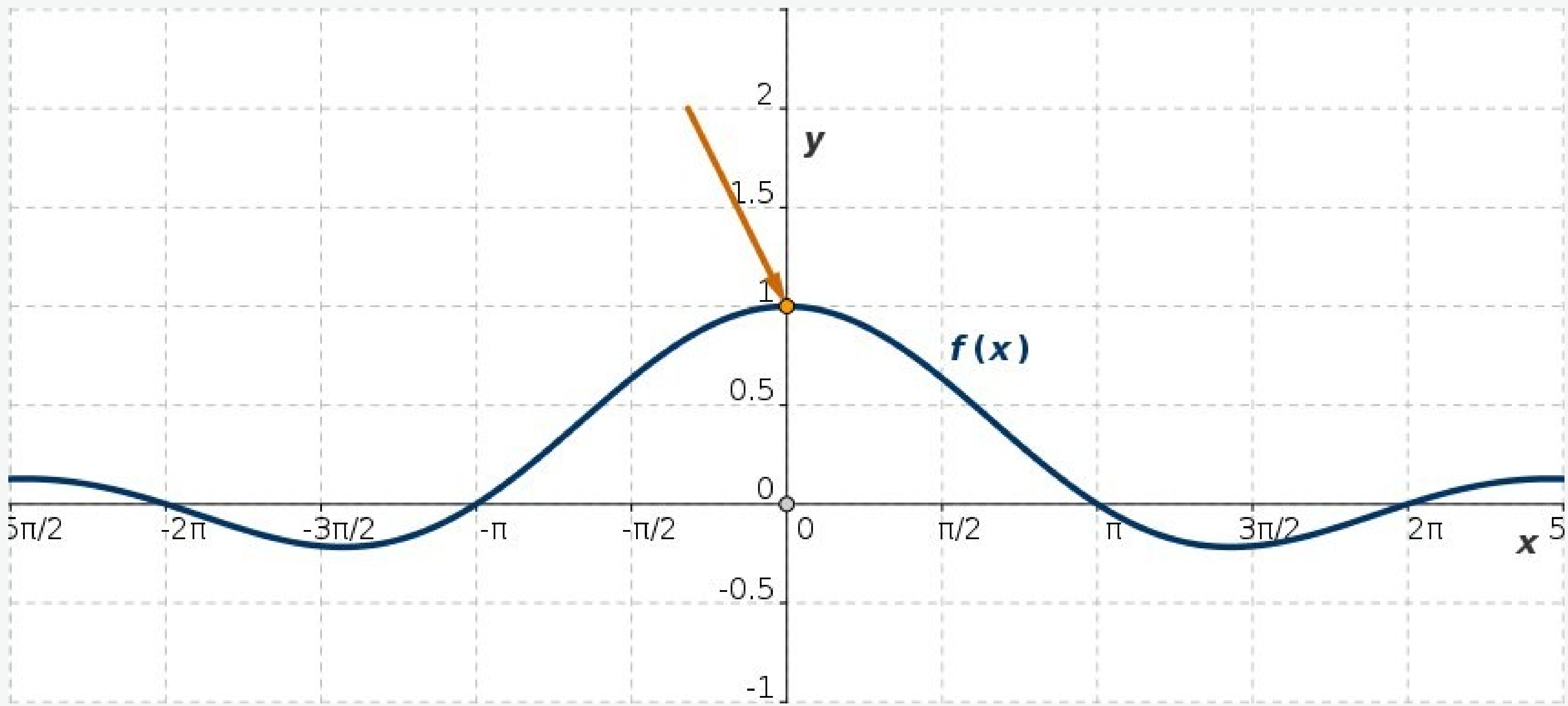

Abb. L2d-1: Graphische Darstellung der Funktion y = f (x)

$$f(x) = \frac{\sin x}{x}$$

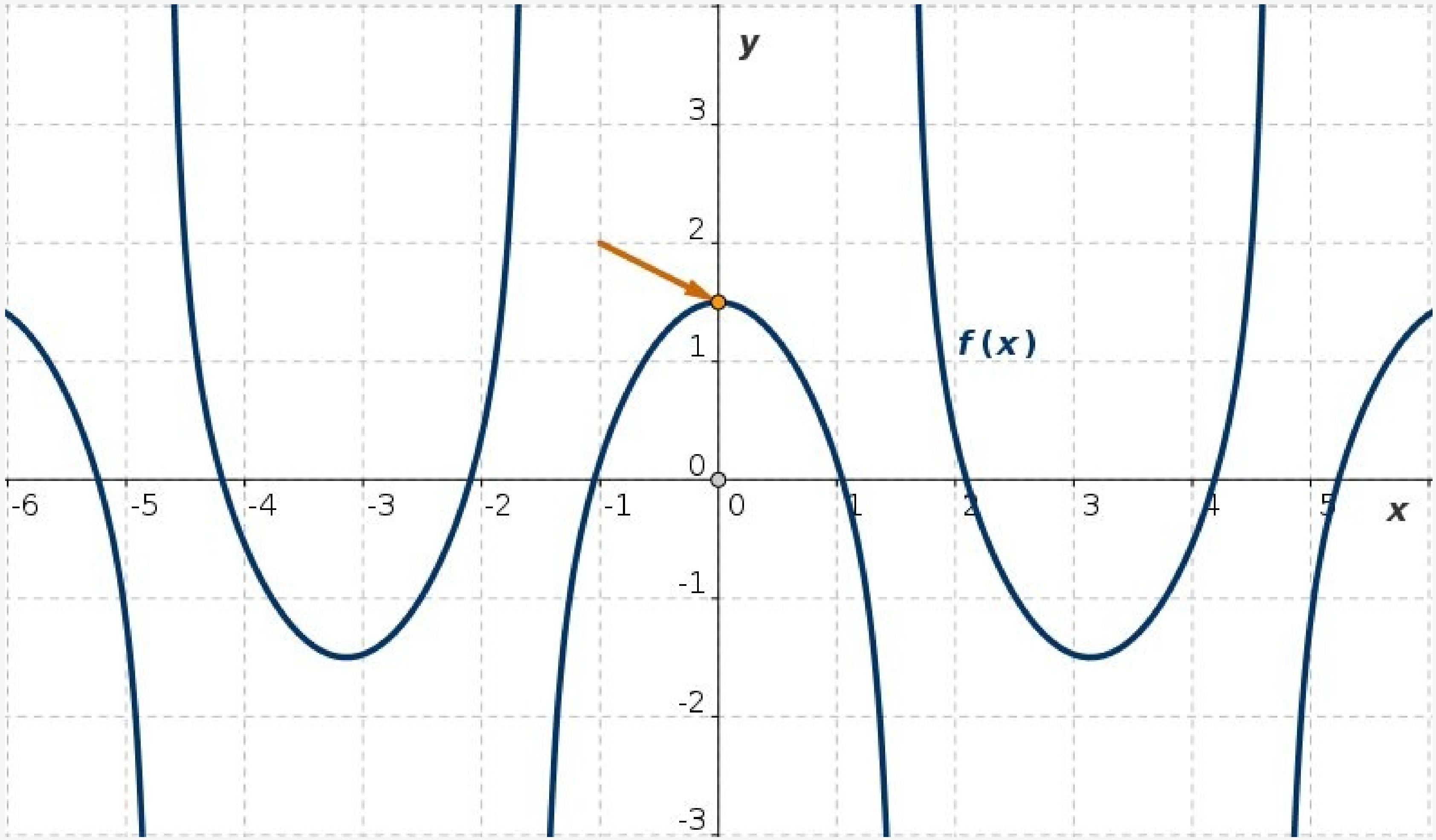

Abb. L2d-2: Graphische Darstellung der Funktion $y = f(x)$

$$f(x) = \frac{\sin(3x)}{\sin(2x)}$$