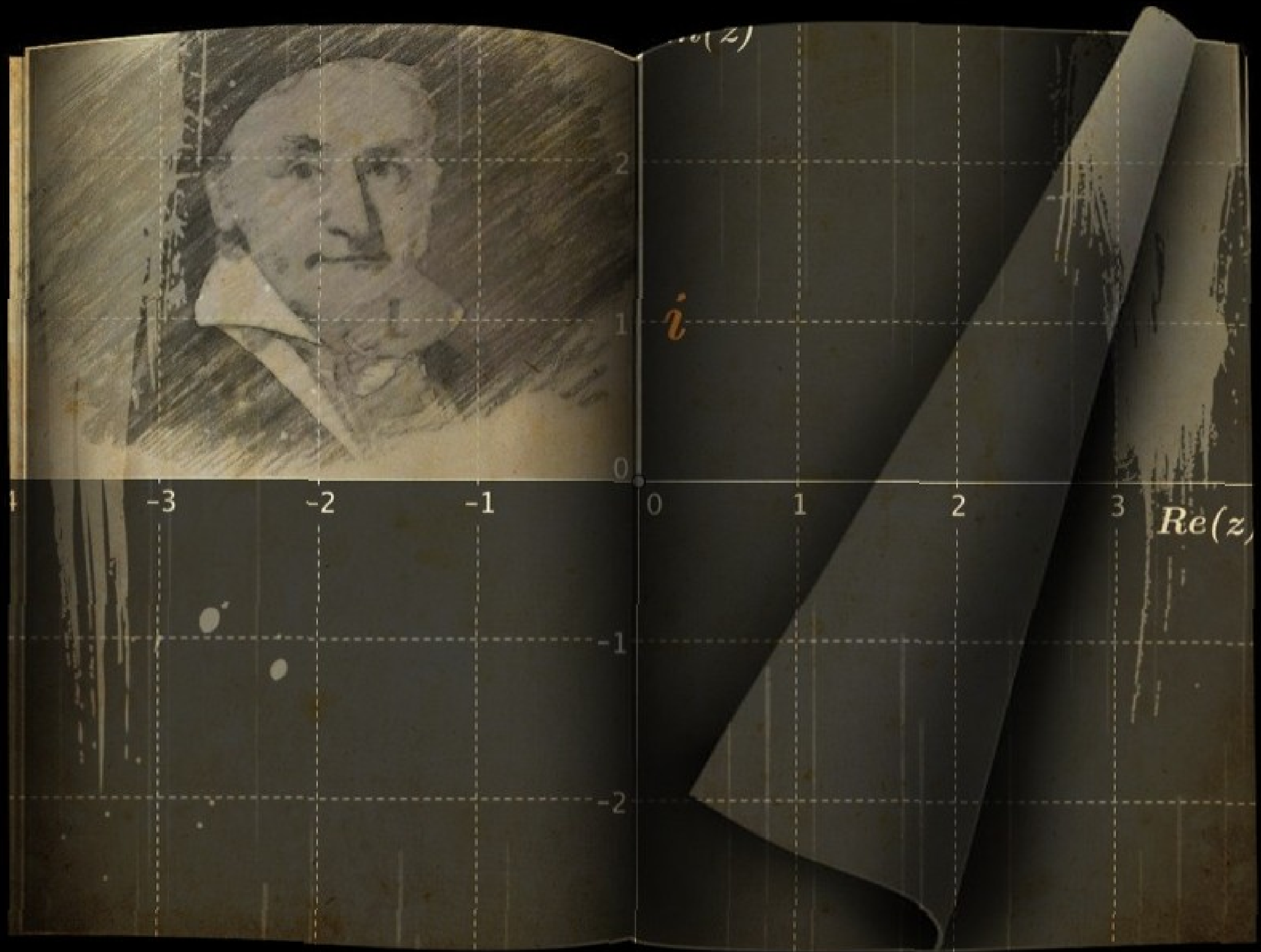


Mengen komplexer Zahlen in der Gaußschen Zahlenebene

Aufgaben, Teil 1

Stellen Sie die folgende Menge in der Gaußschen Zahlenebene dar:

$$M_a = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) \geqslant 1 \}$$

$$M_b = \{ z \in \mathbb{C} \mid \operatorname{Im}(z) \geqslant -3 \}$$

$$M_c = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) \geqslant -1, \quad \operatorname{Im}(z) \geqslant 1 \}$$

$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant \operatorname{Re}(z) < 3 \}$$

$$M_e = \{ z \in \mathbb{C} \mid -3 \leqslant \operatorname{Re}(z) \leqslant 2, \quad -1 < \operatorname{Im}(z) < 2 \}$$

$$M_f = \{ z \in \mathbb{C} \mid \operatorname{Im}(z) - \operatorname{Re}(z) \leqslant 2 \}$$

$$M_g = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) + \operatorname{Im}(z) = 1 \}$$

$$M_h = \{ z \in \mathbb{C} \mid 1 \leqslant |\operatorname{Re}(z)| \leqslant 3, \quad 2 \leqslant \operatorname{Im}(z) \leqslant 4 \}$$

$$M_i = \{ z \in \mathbb{C} \mid A \cap B \cap C \}$$

$$A \quad : \mathrm{Re}(z) + \mathrm{Im}(z) \geqslant 1$$

$$B \quad : -\mathrm{Re}(z) + \mathrm{Im}(z) \geqslant -3$$

$$C \quad : \mathrm{Im}(z) \leqslant 4$$

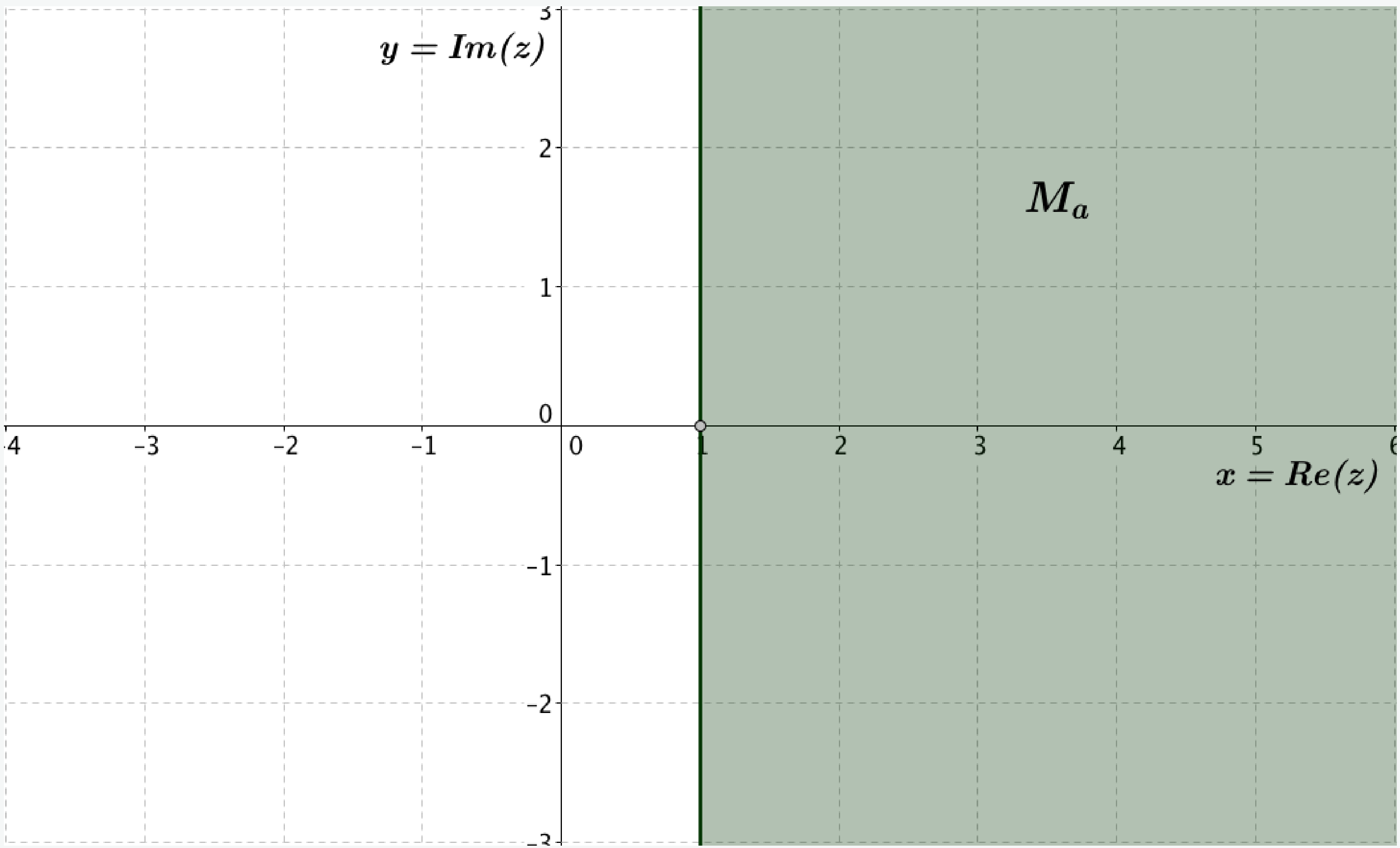


Abb. L-1a: Graphische Darstellung der Aufgabe

$$M_a = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) \geqslant 1 \} : \quad z = x + i\, y, \quad \operatorname{Re}(z) = x \geqslant 1$$

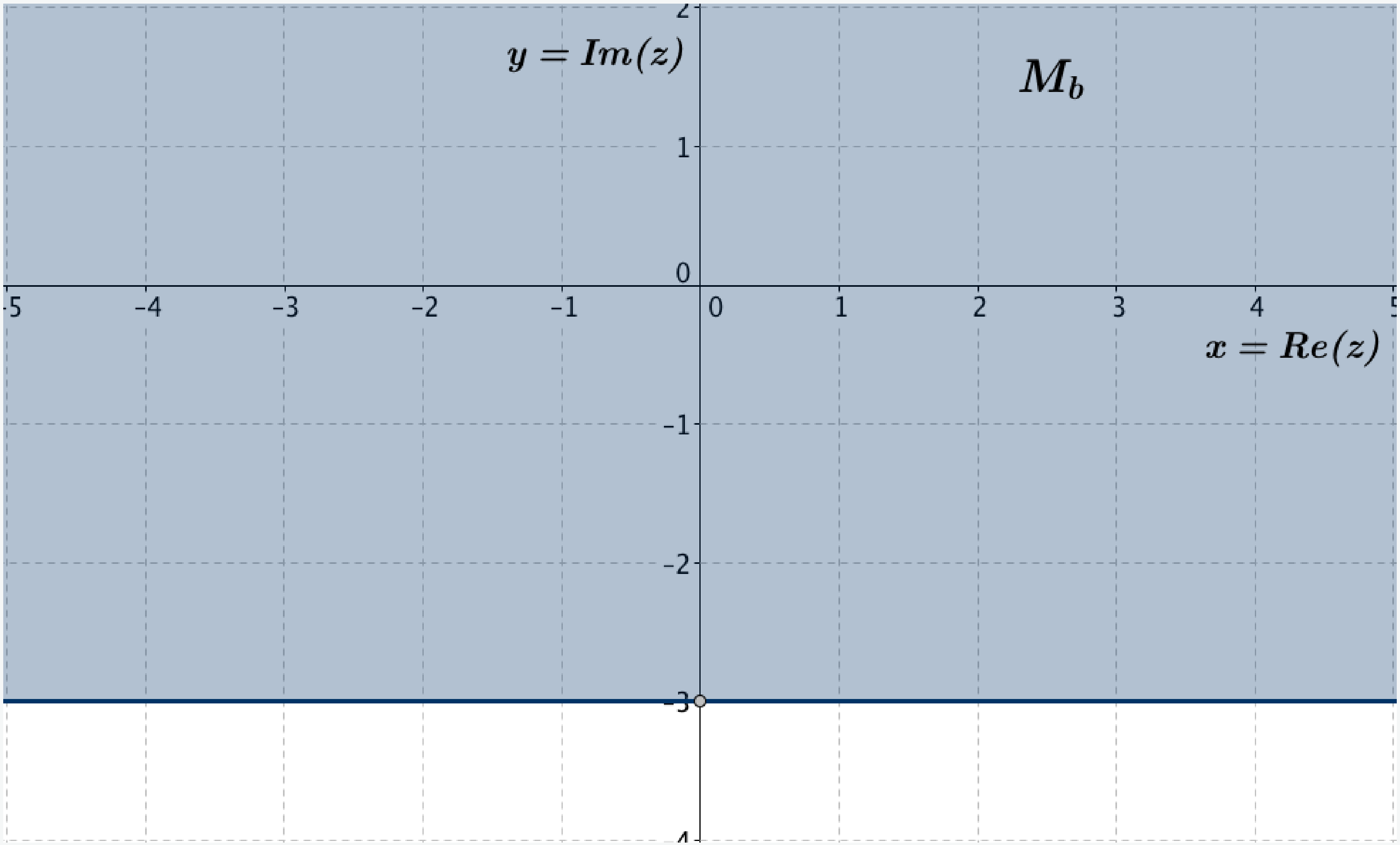


Abb. L-1b: Graphische Darstellung der Aufgabe

$$M_b = \{ z \in \mathbb{C} \mid \operatorname{Im}(z) \geqslant -3 \} : \quad z = x + i\,y, \quad \operatorname{Im}(z) = y \geqslant -3$$

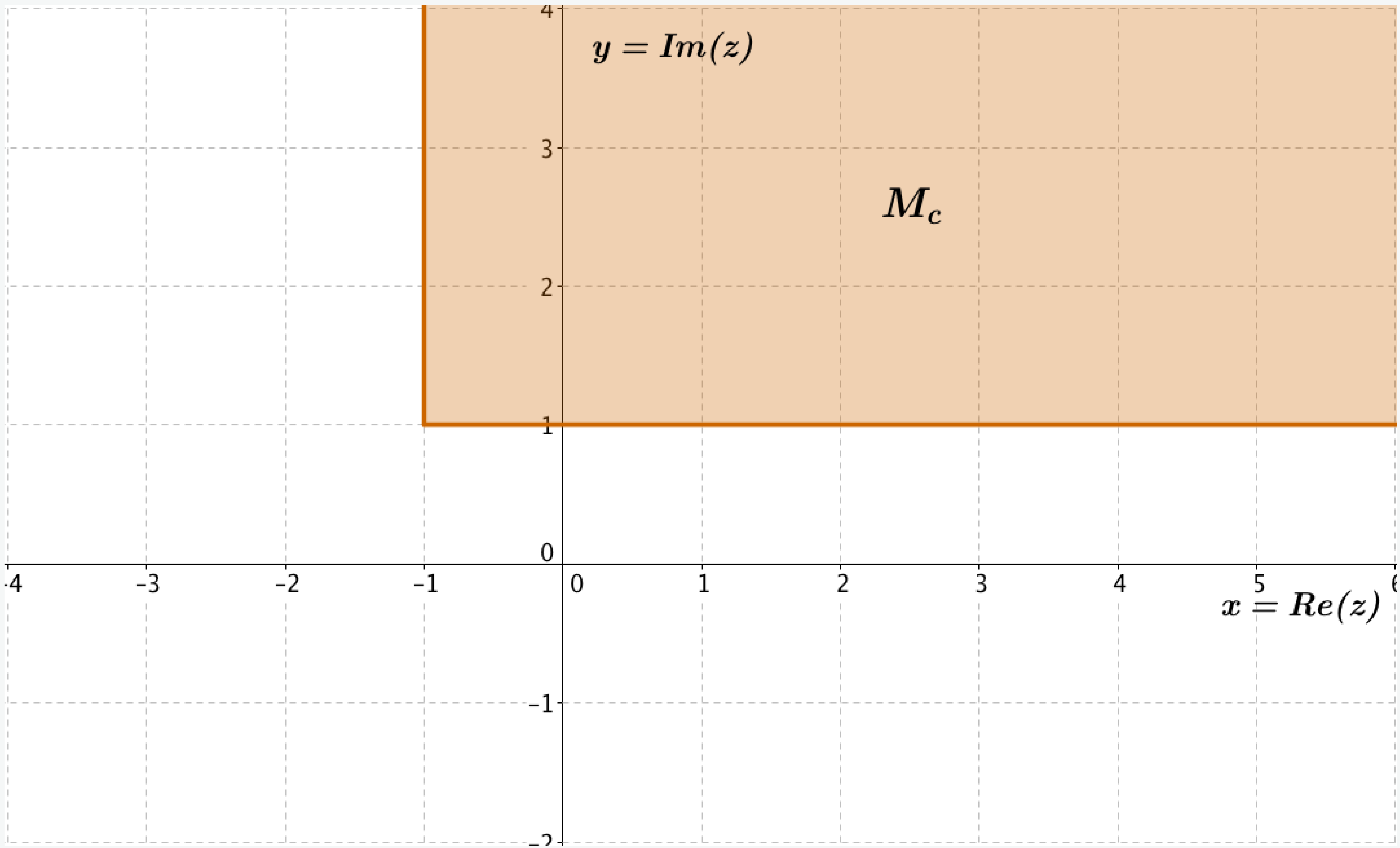


Abb. L-1c: Graphische Darstellung der Aufgabe

$$M_c = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) \geqslant -1 , \quad \operatorname{Im}(z) \geqslant 1 \}$$

$$z = x + i\, y , \quad \operatorname{Re}(z) = x \geqslant -1, \quad \operatorname{Im}(z) = y \geqslant 1$$

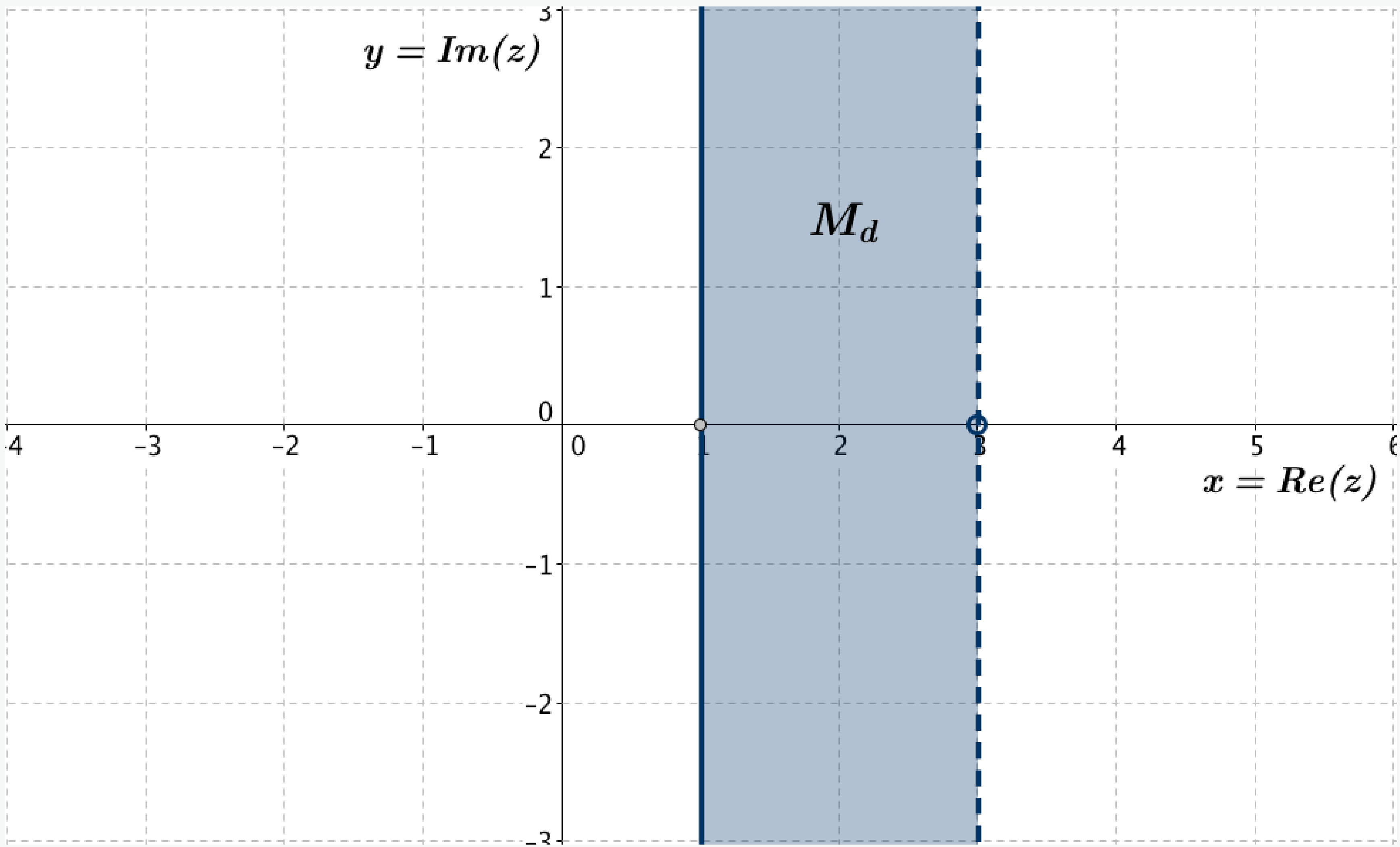


Abb. L-1d: Graphische Darstellung der Aufgabe

$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant \operatorname{Re}(z) < 3 \}, \qquad z = x + i\,y, \quad 1 \leqslant \operatorname{Re}(z) = x < 3$$

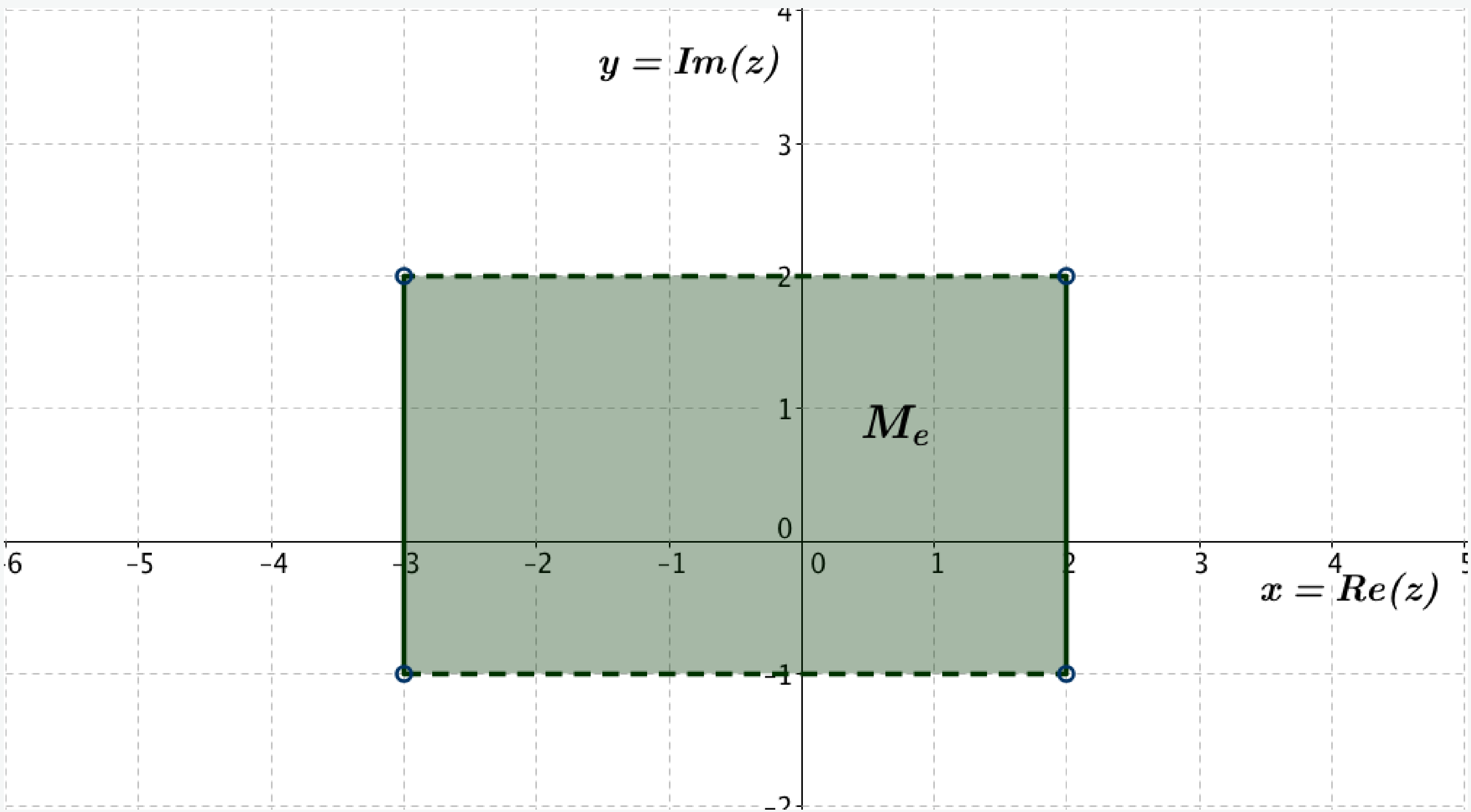


Abb. L-1e: Graphische Darstellung der Aufgabe

$$M_e = \{ z \in \mathbb{C} \mid -3 \leqslant \operatorname{Re}(z) = x \leqslant 2, \quad -1 < \operatorname{Im}(z) = y < 2 \}$$

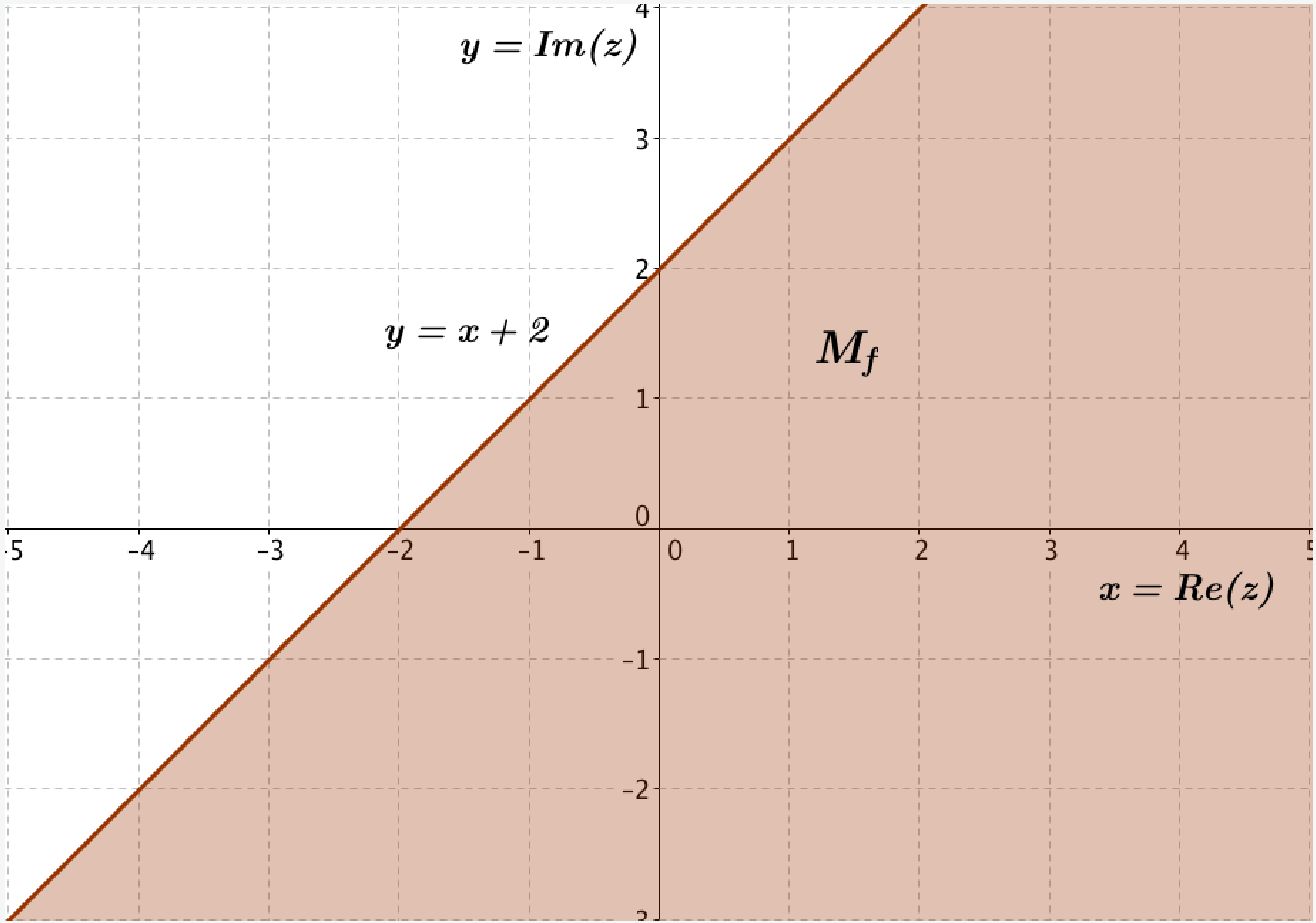


Abb. L-1f: Graphische Darstellung der Aufgabe

$$M_f = \{ z \in \mathbb{C} \mid \operatorname{Im}(z) - \operatorname{Re}(z) \leqslant 2 \}$$

$$\operatorname{Im}(z) - \operatorname{Re}(z) = y - x \leqslant 2, \qquad y \leqslant x + 2$$

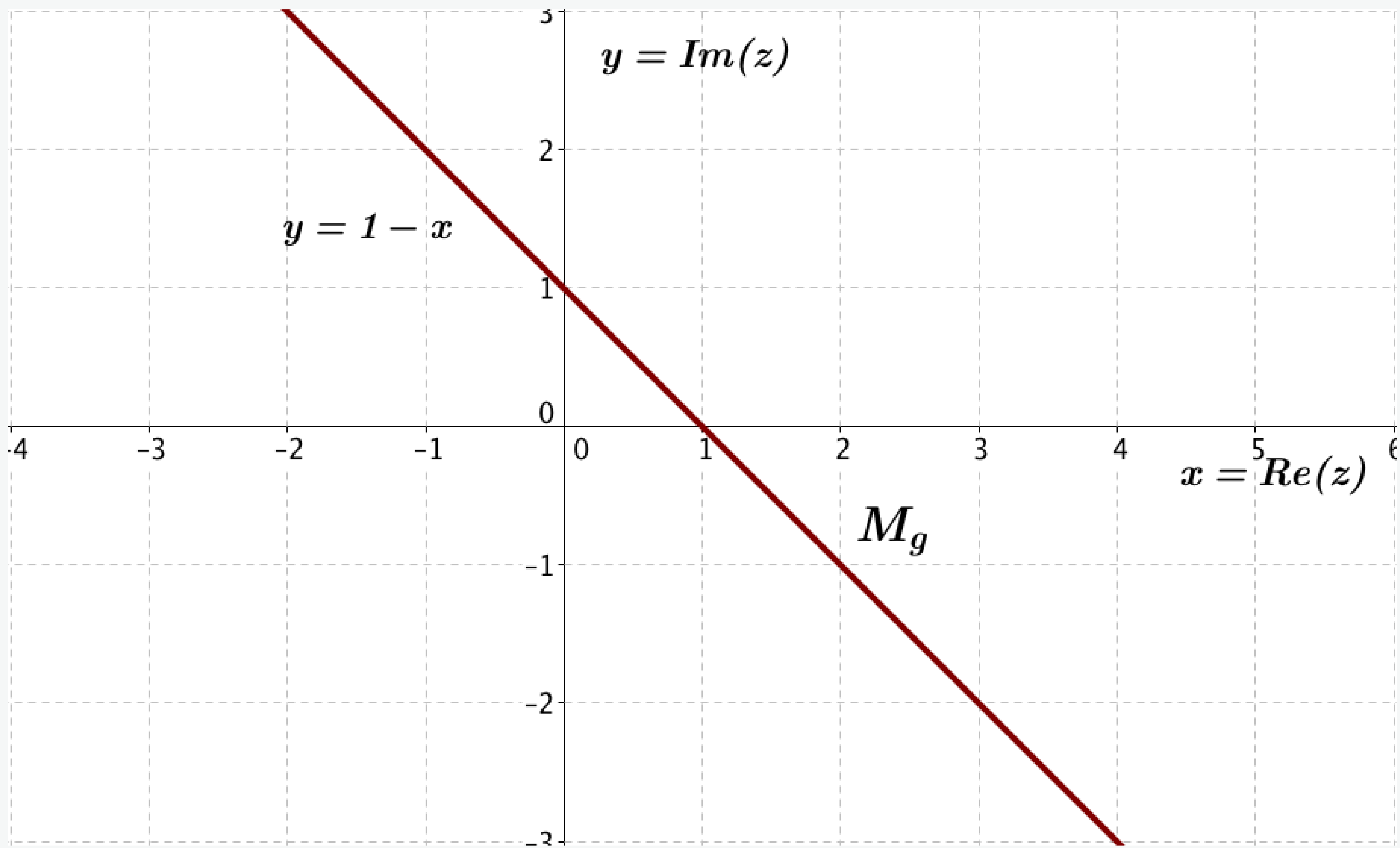


Abb. L-1g: Graphische Darstellung der Aufgabe

$$M_g = \{ \, z \in \mathbb{C} \mid \operatorname{Re}(z) + \operatorname{Im}(z) = 1 \, \}$$

$$\operatorname{Re}(z) + \operatorname{Im}(z) = x + y = 1, \qquad y = 1 - x$$

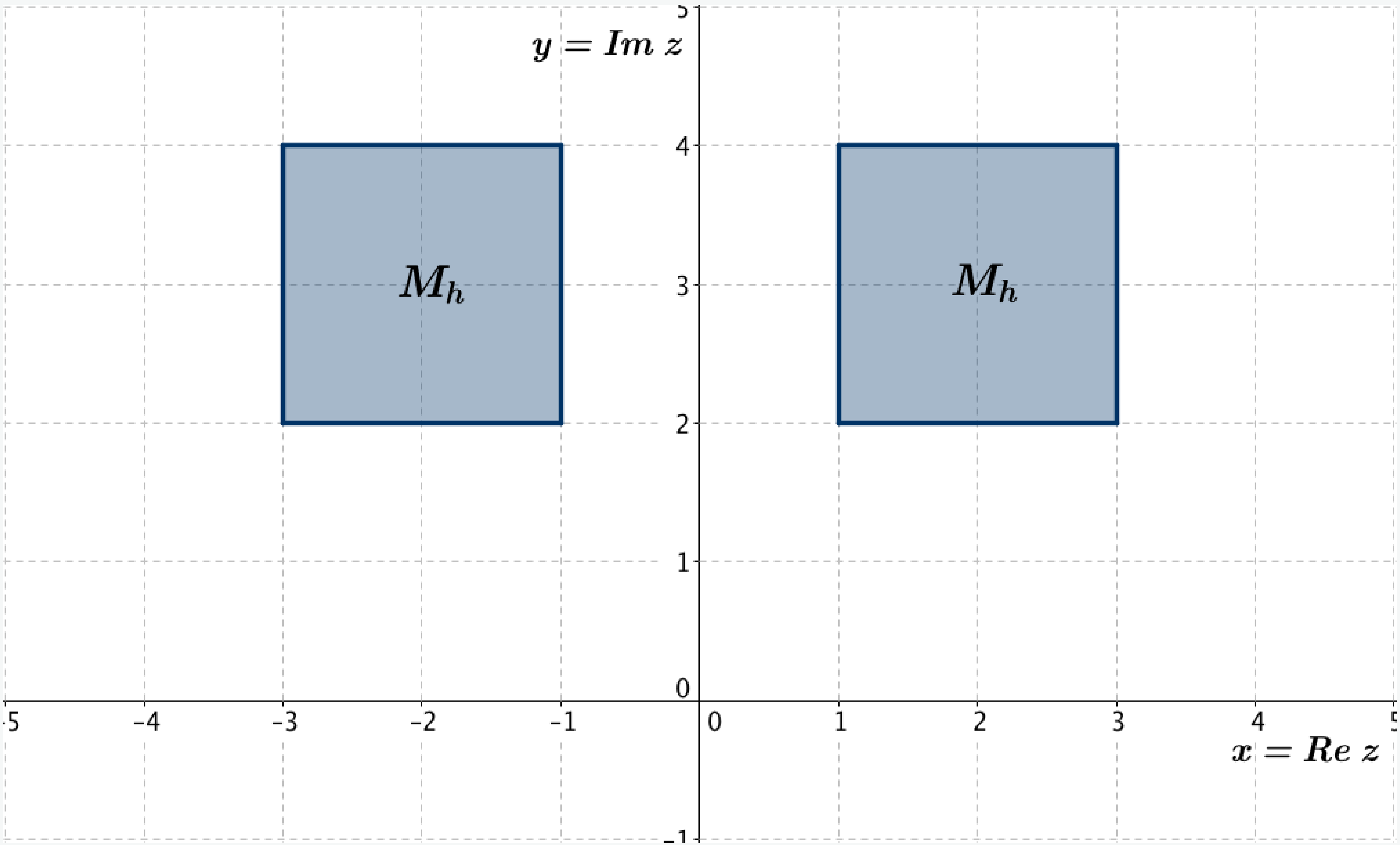


Abb. L-1h: Graphische Darstellung der Aufgabe

$$M_h = \{ z \in \mathbb{C} \mid 1 \leqslant |\operatorname{Re}(z)| \leqslant 3, \quad 2 \leqslant \operatorname{Im}(z) \leqslant 4 \}$$

$$x \in [-3, -1] \cup [1, 3], \qquad y \in [2, 4]$$

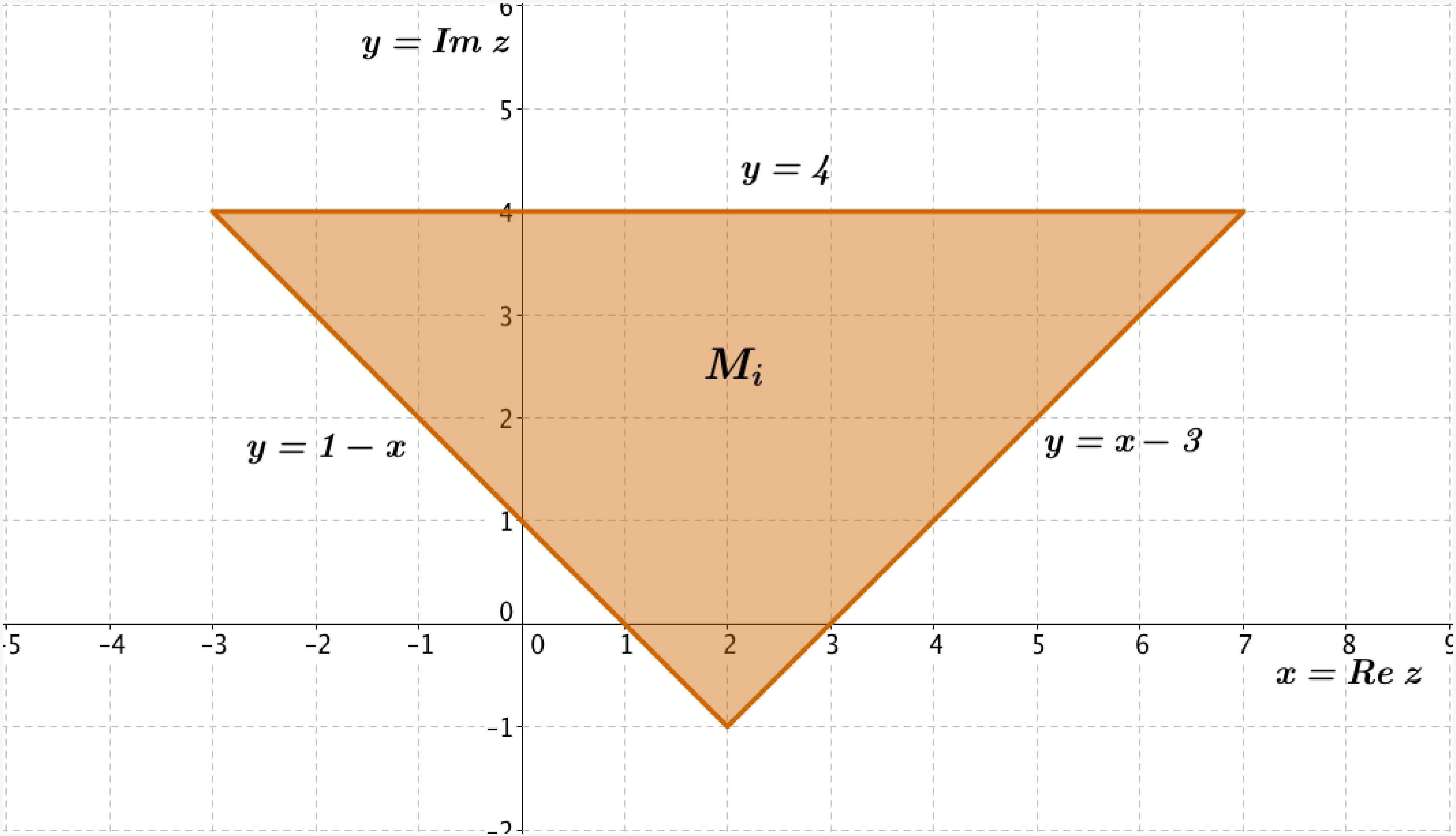


Abb. L-1i: Graphische Darstellung der Aufgabe

$$M_i = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) + \operatorname{Im}(z) \geqslant 1 \cap -\operatorname{Re}(z) + \operatorname{Im}(z) \geqslant -3 \cap \operatorname{Im}(z) \leqslant 4 \}$$

Stellen Sie die folgende Menge in der Gaußschen Zahlenebene dar:

$$M_a = \{ z \in \mathbb{C} \mid |z| = 2 \}$$

$$M_b = \{ z \in \mathbb{C} \mid |z| < 2 \}$$

$$M_c = \{ z \in \mathbb{C} \mid 1 \leqslant |z| \leqslant 2 \}$$

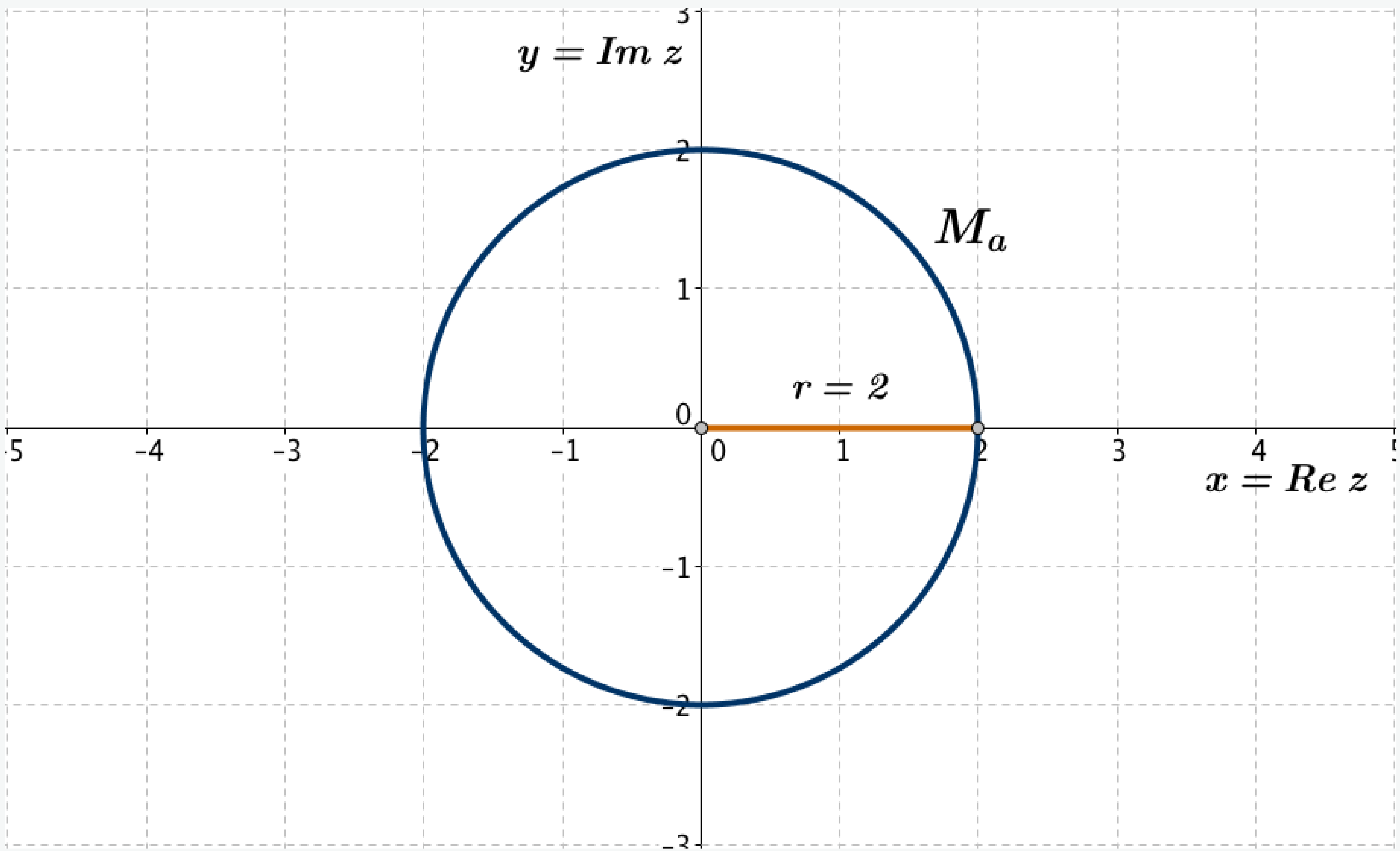


Abb. L-2a: Graphische Darstellung der Aufgabe

$$M_a = \{ z \in \mathbb{C} \mid |z| = 2 \}$$

$$|z| = \sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2} = r = 2$$

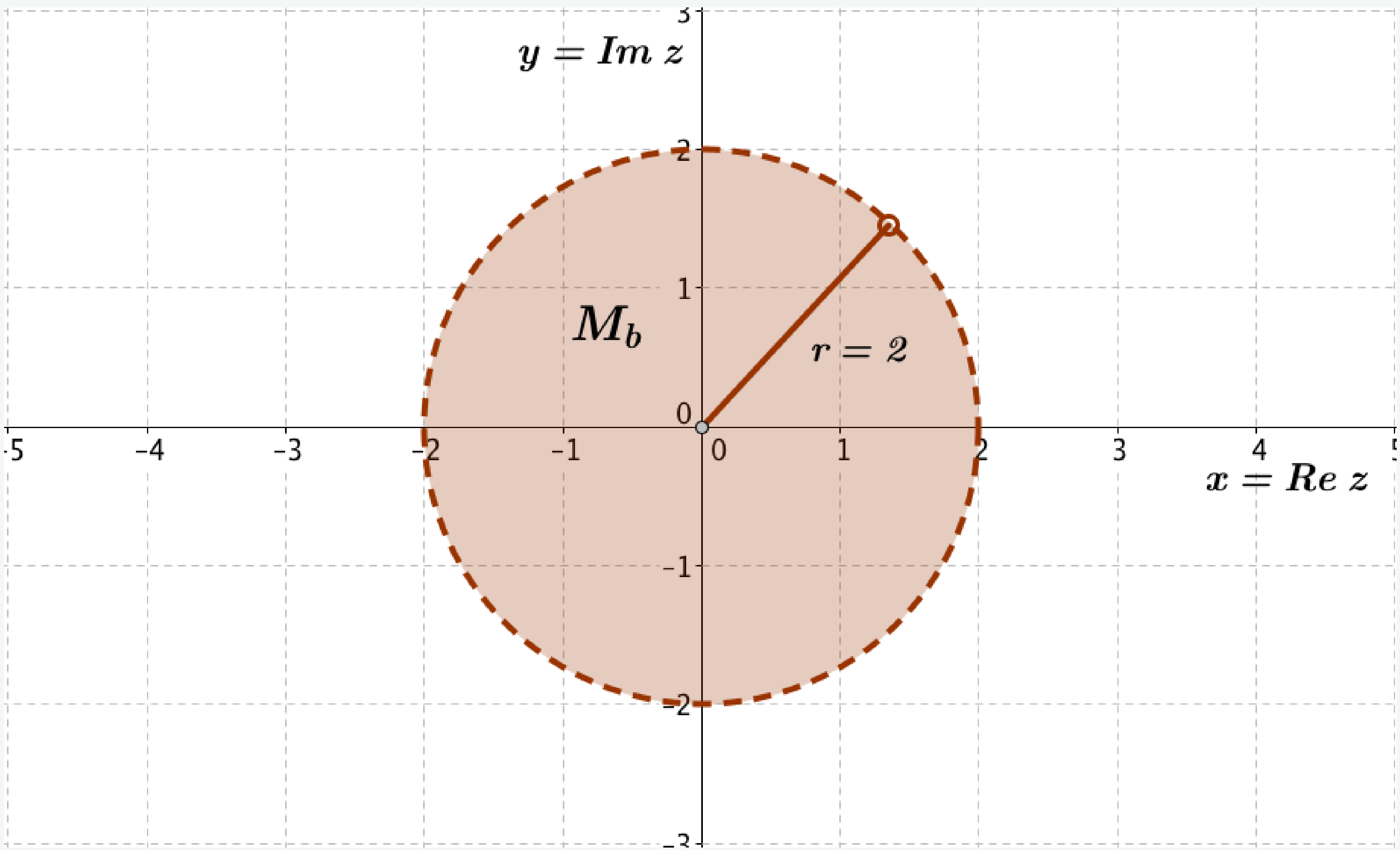


Abb. L-2i: Graphische Darstellung der Aufgabe

$$M_i = \{ z \in \mathbb{C} \mid |z| < 2 \}, \qquad |z| = \sqrt{x^2 + y^2} < 2$$

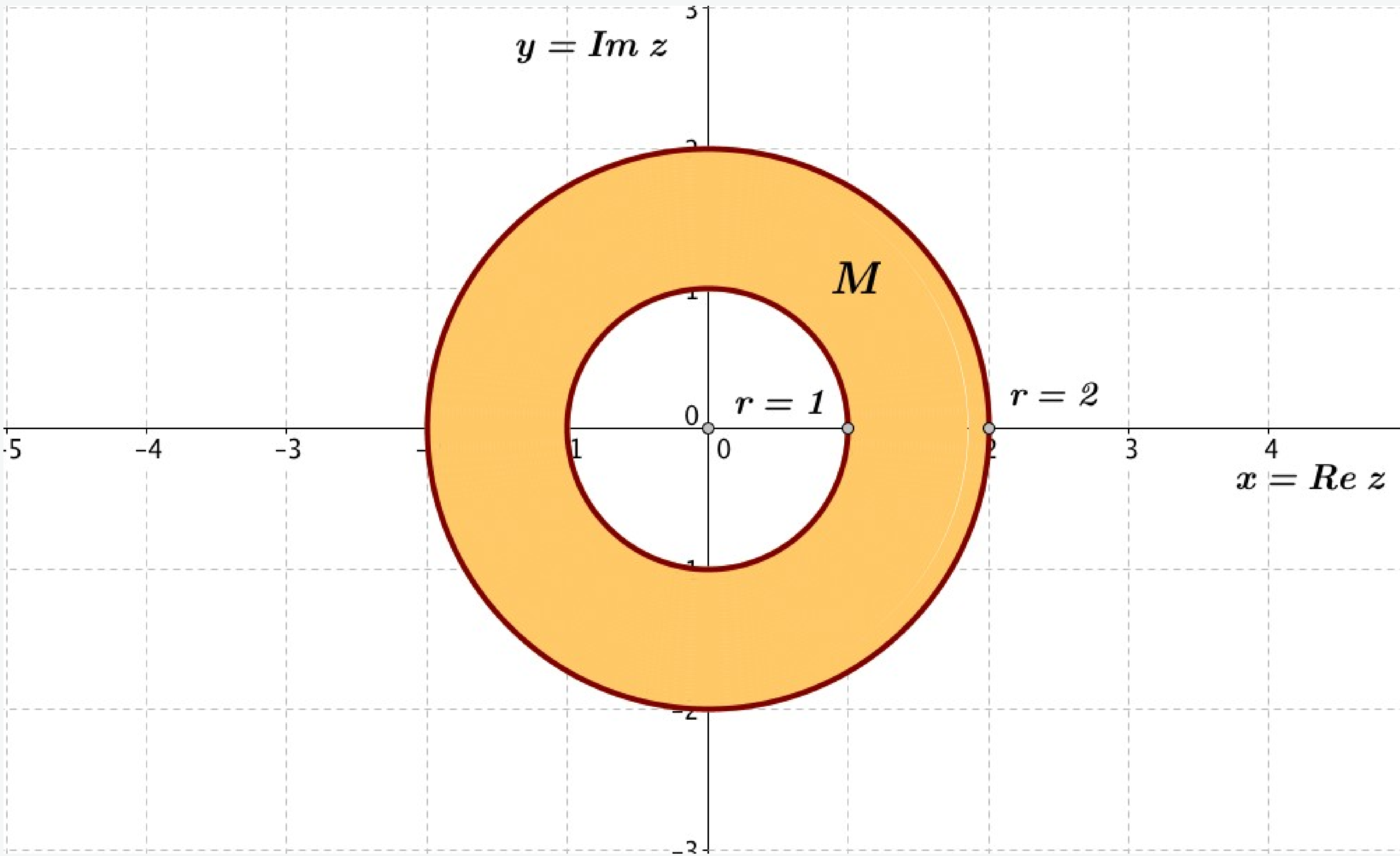
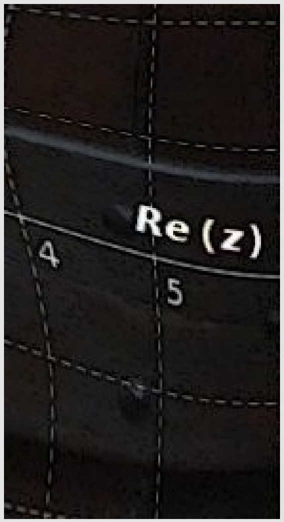


Abb. L-2c: Graphische Darstellung der Aufgabe

$$M_c = \{ z \in \mathbb{C} \mid 1 \leqslant |z| \leqslant 2 \}$$



Stellen Sie die folgende Menge in der Gaußschen Zahlenebene dar:

$$M_a = \{ z \in \mathbb{C} \mid |z| \leqslant 3, \quad \mathrm{Re}(z) \geqslant 0, \quad \mathrm{Im}(z) \geqslant 0 \}$$

$$M_b = \{ z \in \mathbb{C} \mid |z| \leqslant 2, \quad \mathrm{Re}(z) \leqslant 0, \quad \mathrm{Im}(z) \leqslant 0 \}$$

$$M_c = \{ z \in \mathbb{C} \mid |z| \leqslant 2, \quad \mathrm{Im}(z) \geqslant 0 \}$$

$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant |z + 2 + i| \leqslant 2 \}$$

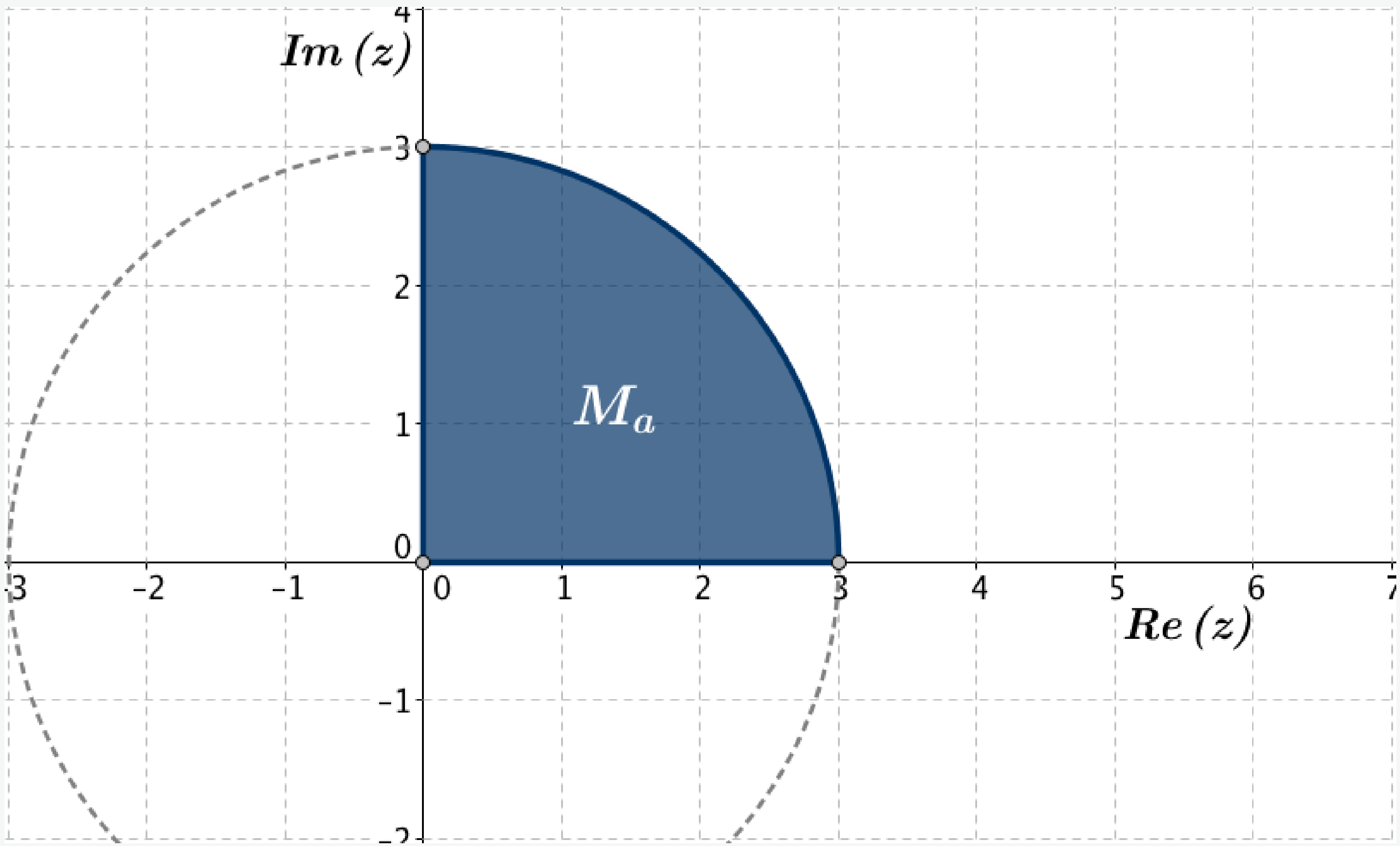


Abb. L-3a: Graphische Darstellung der Aufgabe

$$M_a = \{ z \in \mathbb{C} \ \mid \ |z| \leqslant 3, \quad \operatorname{Re}(z) \geqslant 0, \ \ \operatorname{Im}(z) \geqslant 0 \}$$

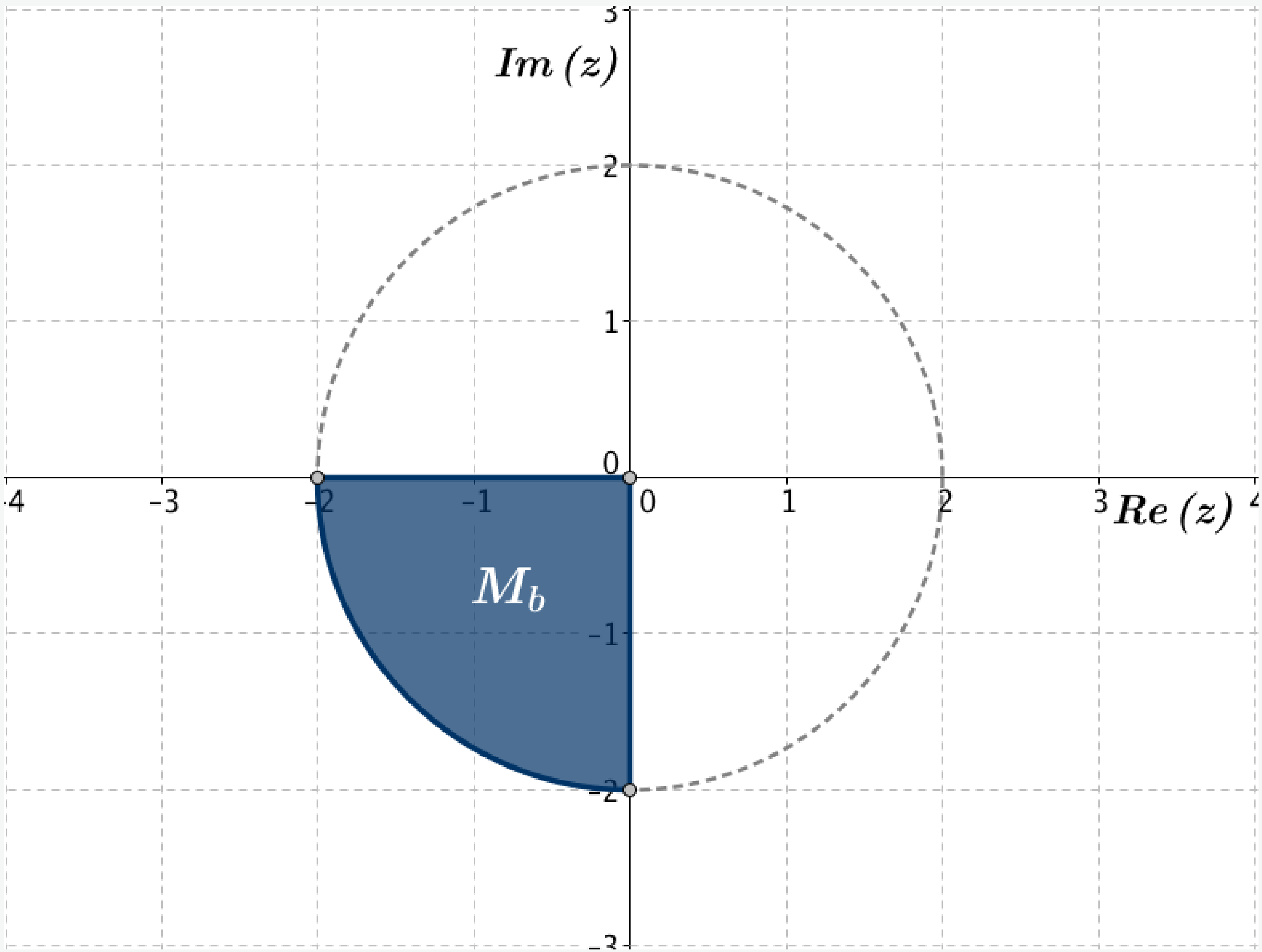


Abb. L-3b: Graphische Darstellung der Aufgabe

$$M_b = \{ z \in \mathbb{C} \ | \ |z| \leqslant 2, \quad \mathrm{Re}(z) \leqslant 0, \quad \mathrm{Im}(z) \leqslant 0 \}$$

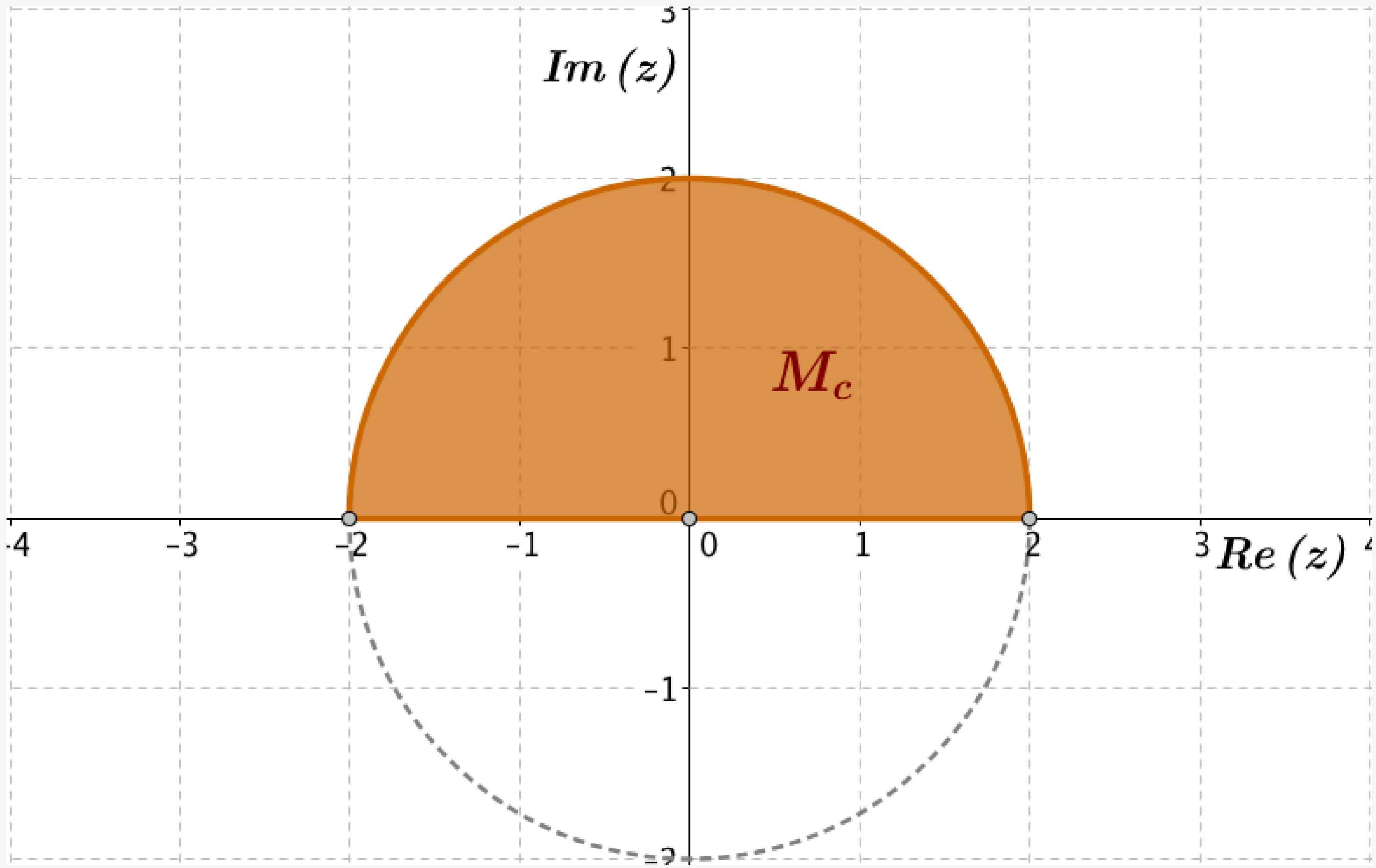


Abb. L-3c: Graphische Darstellung der Aufgabe

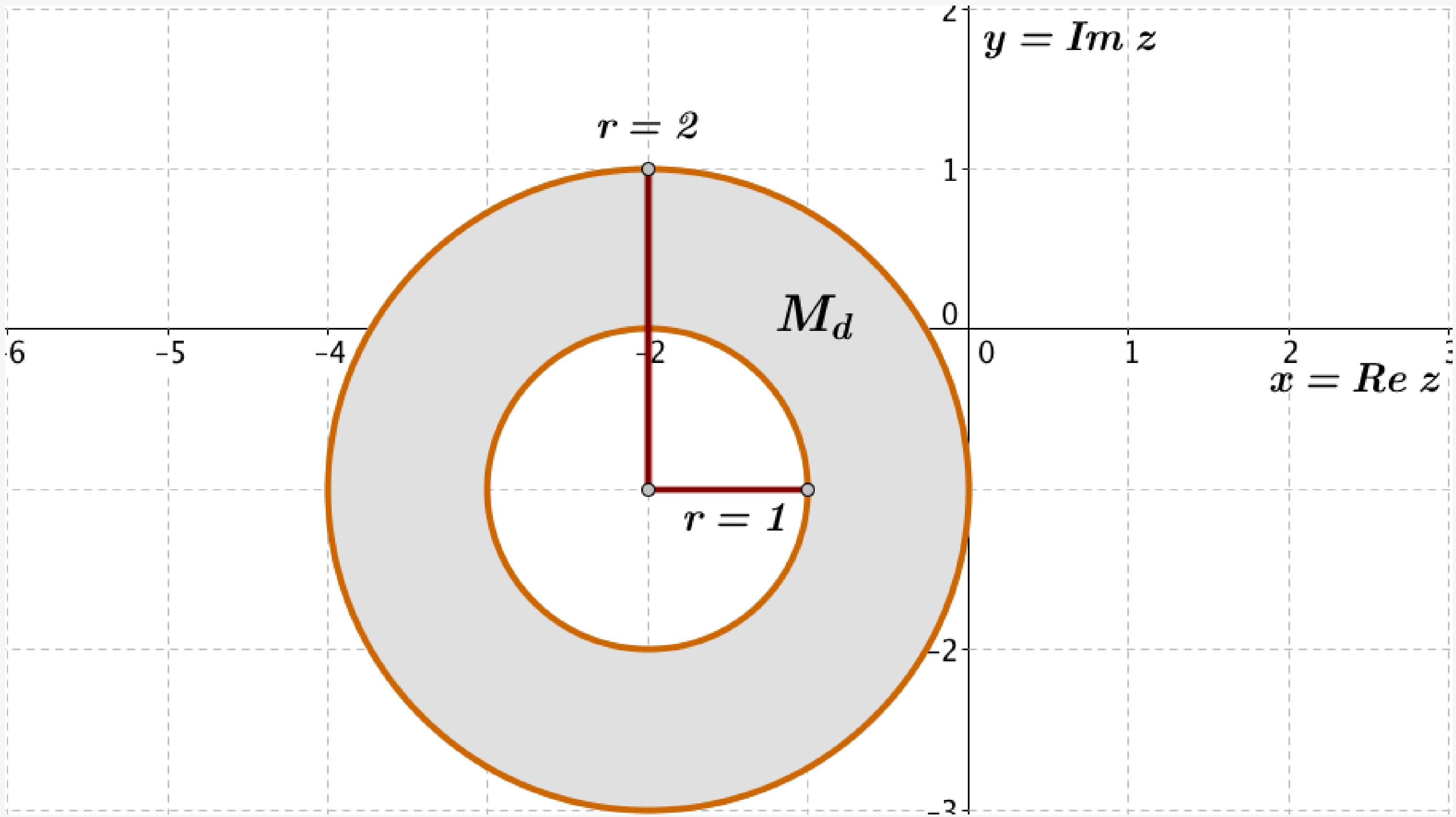


Abb. L-3d: Graphische Darstellung der Aufgabe

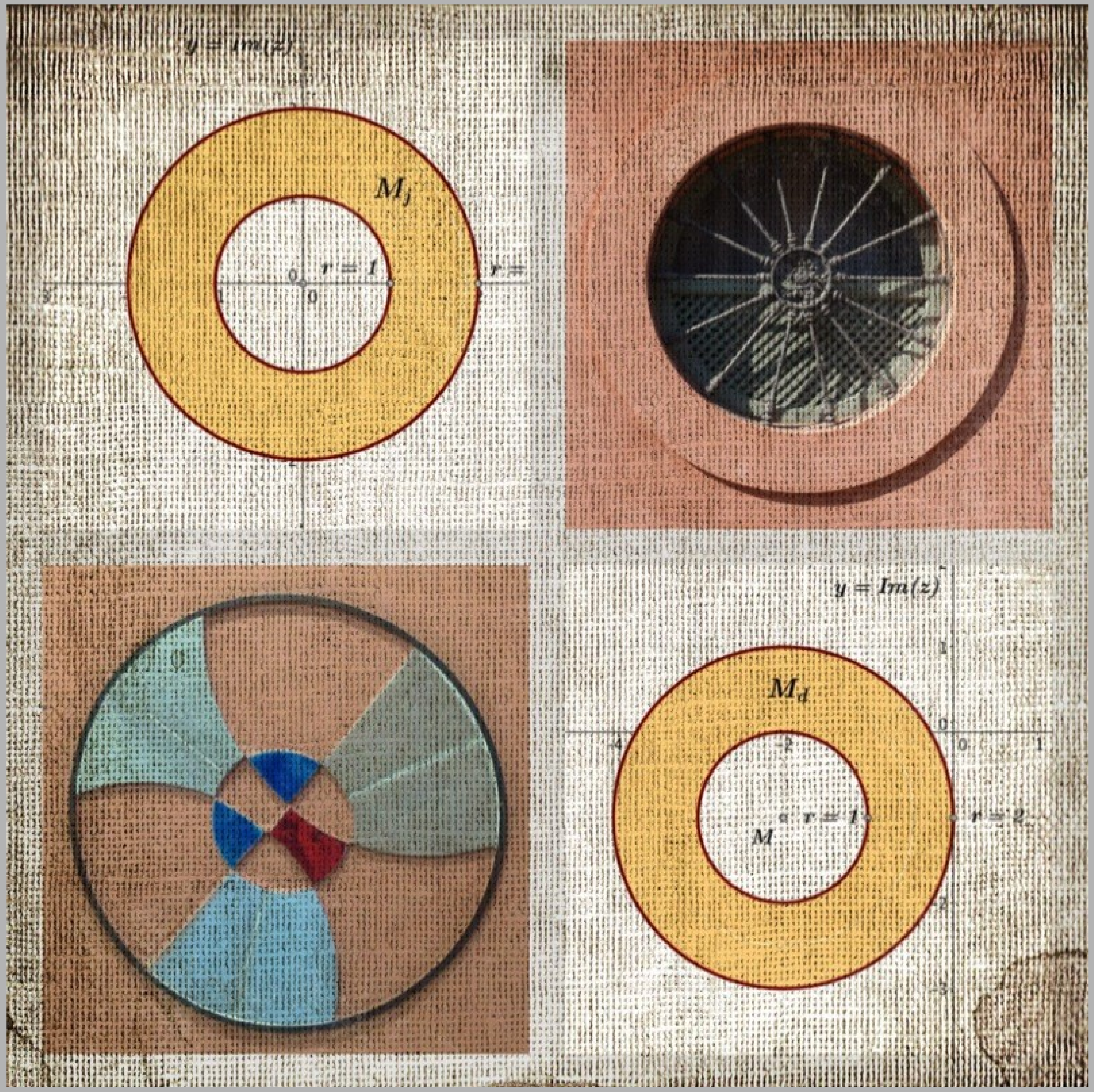
$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant |z + 2 + i| \leqslant 2 \}$$

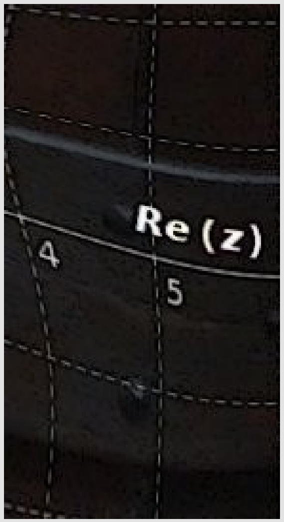
$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant |z + 2 + i| \leqslant 2 \}$$

$$z + 2 + i = (x + 2) + i\,(y + 1), \qquad |z + 2 + i| = \sqrt{(x + 2)^2 + (y + 1)^2}$$

$(x+2)^2 + (y+1)^2 = 1$ – der Kreis mit dem Mittelpunkt $P = (-2, -1)$ und $R = 1.$

$(x+2)^2 + (y+1)^2 = 4$ – der Kreis mit dem Mittelpunkt $P = (-2, -1)$ und $R = 2.$


$y = Im(z)$
M_j
$r = 1$
$y = Im(z)$
M_d
$r = 1$
$r = 2$
M



Aufgabe 4:

Bestimmen Sie die geometrische Bedeutung der folgenden Gleichung:

$$|z| = \mathrm{Re}\,(z) + 1$$

Aufgabe 5:

Bestimmen Sie die geometrische Bedeutung der folgenden Ungleichung:

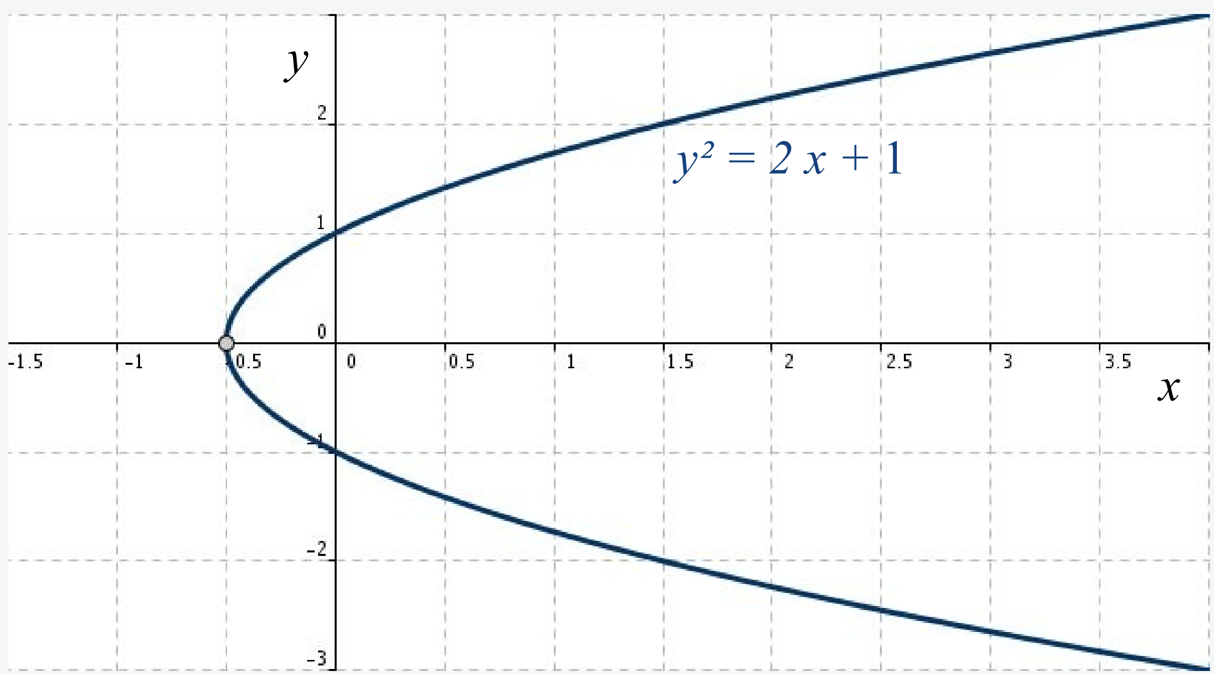
$$|z - 1| \geqslant 2\,|z - i|$$

$$|z| = \text{Re}\,(z) + 1$$

$$z = x + i\,y, \qquad |z| = \sqrt{x^2 + y^2}, \qquad \text{Re}(z) + 1 = x + 1$$

$$\Rightarrow \quad \sqrt{x^2 + y^2} = x + 1, \quad x^2 + y^2 = (x+1)^2 = x^2 + 2x + 1 \quad \Rightarrow$$

$$y^2 = 2x + 1 \quad - \quad \text{Gleichung einer Parabel}$$



$$| z - 1 | \geqslant 2 \, | z - i |$$

$$| z - 1 | \geqslant 2 \, | z - i | \quad \Leftrightarrow \quad | x - 1 + i \, y | \geqslant 2 \, | x + i \, (y - 1) | \quad \Rightarrow$$

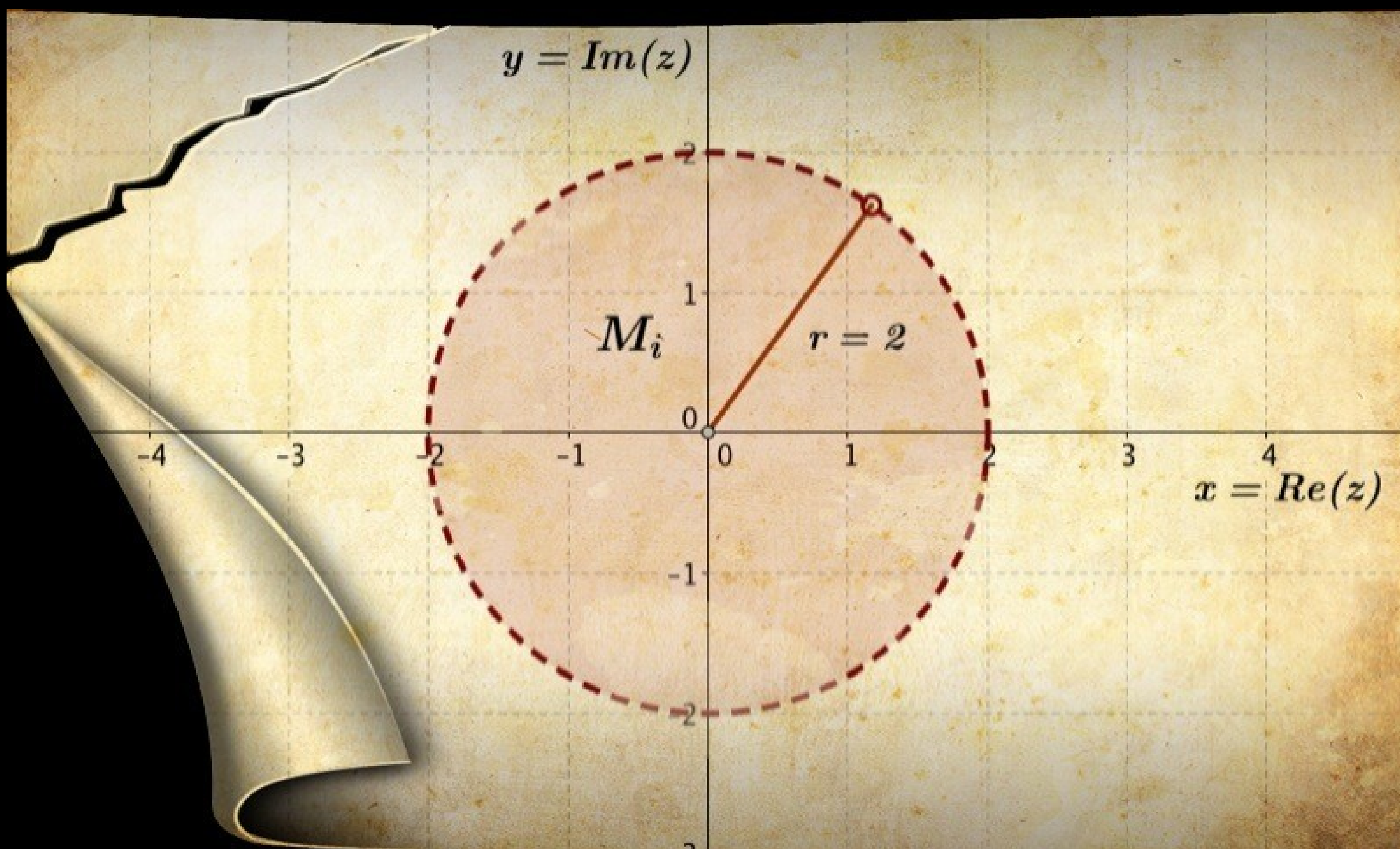
$$\sqrt{(x - 1)^2 + y^2} \geqslant 2 \sqrt{x^2 + (y - 1)^2}$$

$$(x - 1)^2 + y^2 \geqslant 4 \left(x^2 + (y - 1)^2 \right)$$

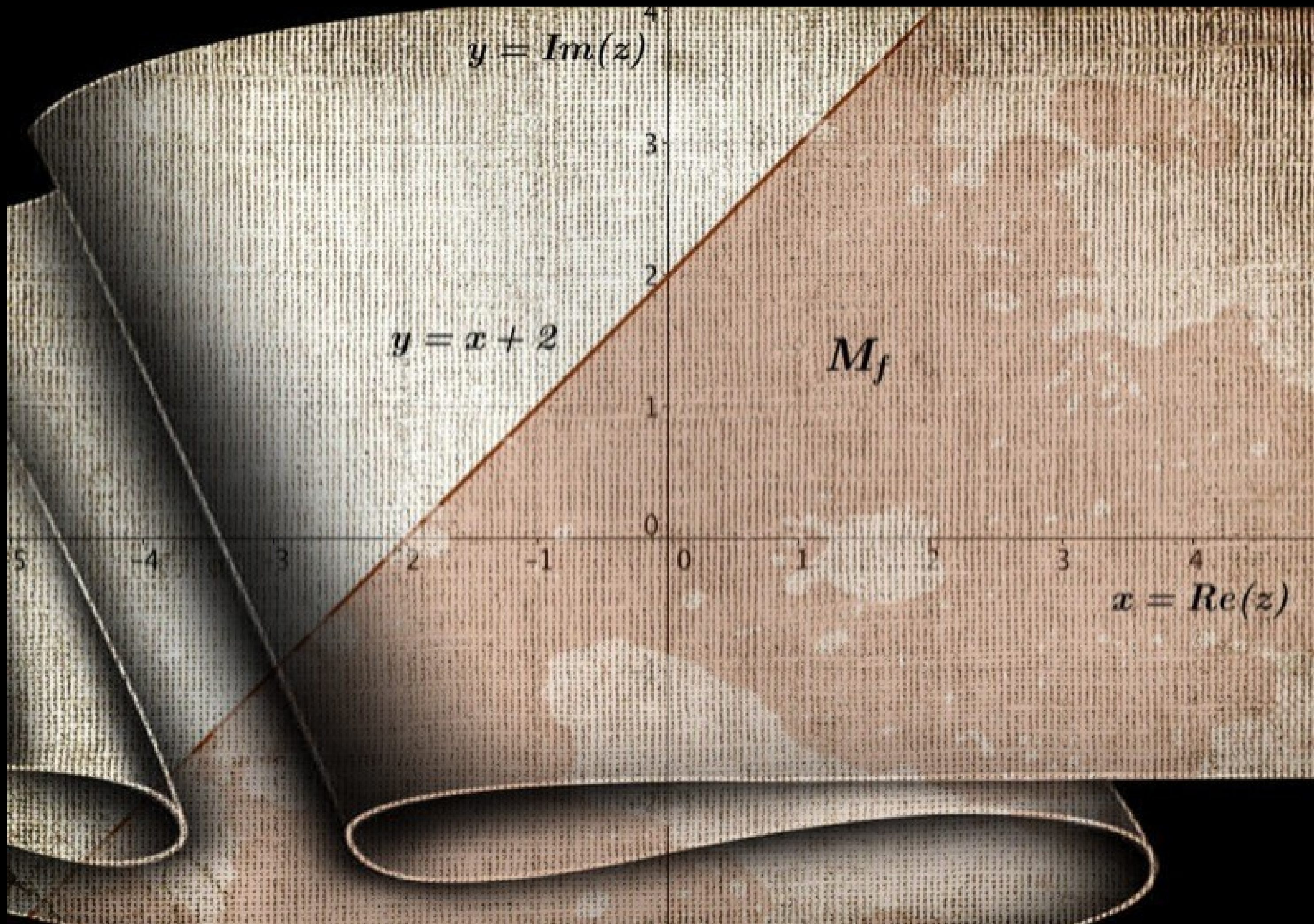
$$\left(x + \frac{1}{3} \right)^2 + \left(y - \frac{4}{3} \right)^2 \leqslant \frac{8}{9}$$

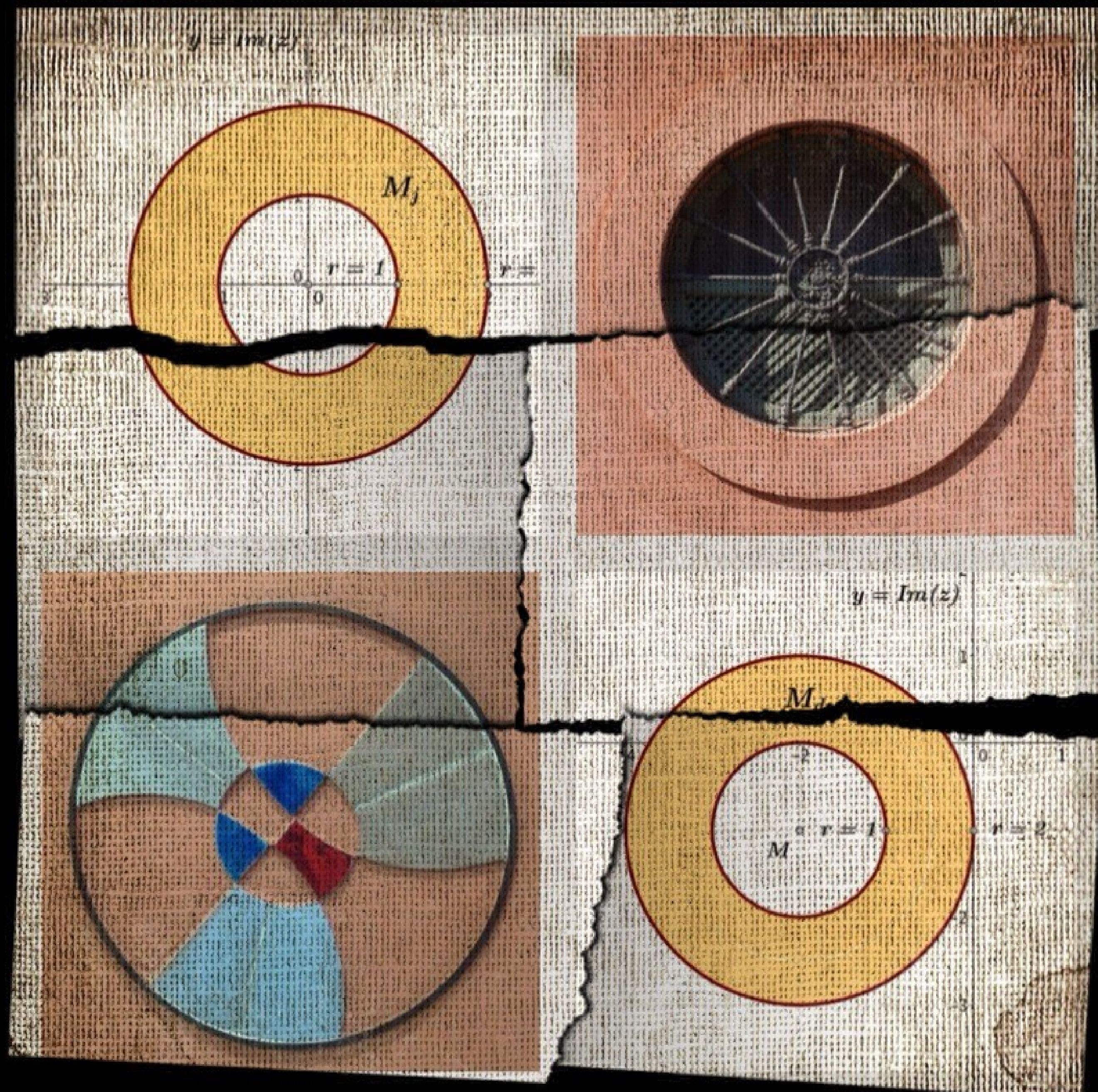
$$R = \sqrt{\frac{8}{9}} = \frac{2 \sqrt{2}}{3} , \qquad M = \left(- \frac{1}{3} , \frac{4}{3} \right)$$

Die Menge aller Punkte, die die Ungleichung erfüllen, befinden sich in einem Kreis mit dem Radius R und dem Mittelpunkt M.



$y = Im(z)$
M_i
$r = 2$
$x = Re(z)$
2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4


$y = Im(z)$
$y = x + 2$
M_f
$x = Re(z)$
-5
-4
-3
-2
-1
0
1
2
3
4
5
0
1
2
3
4


y = Im(z)
M_j
r = 1
y = Im(z)
r = 1
r = 2
M