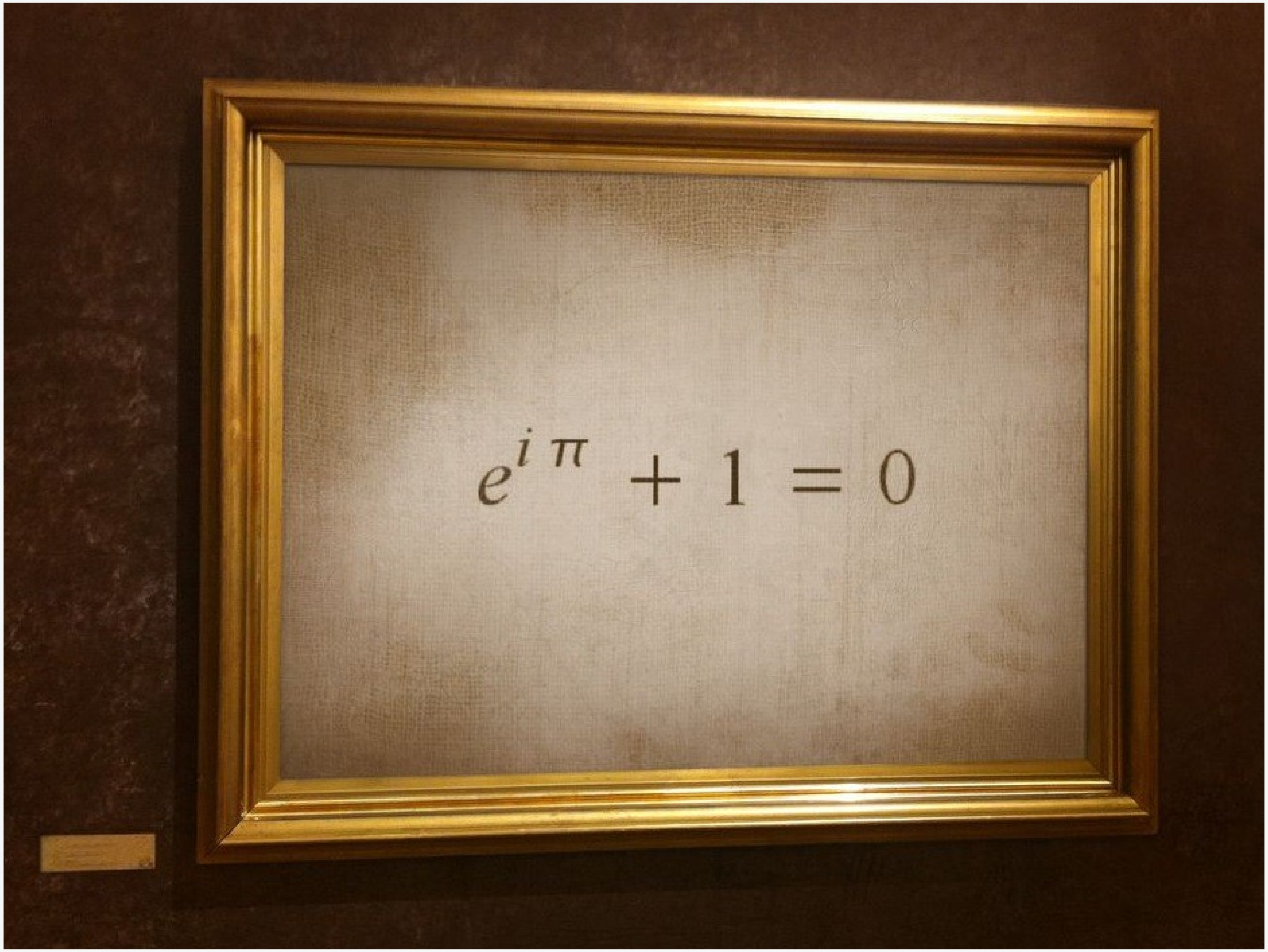


Exponentialform einer komplexen Zahl

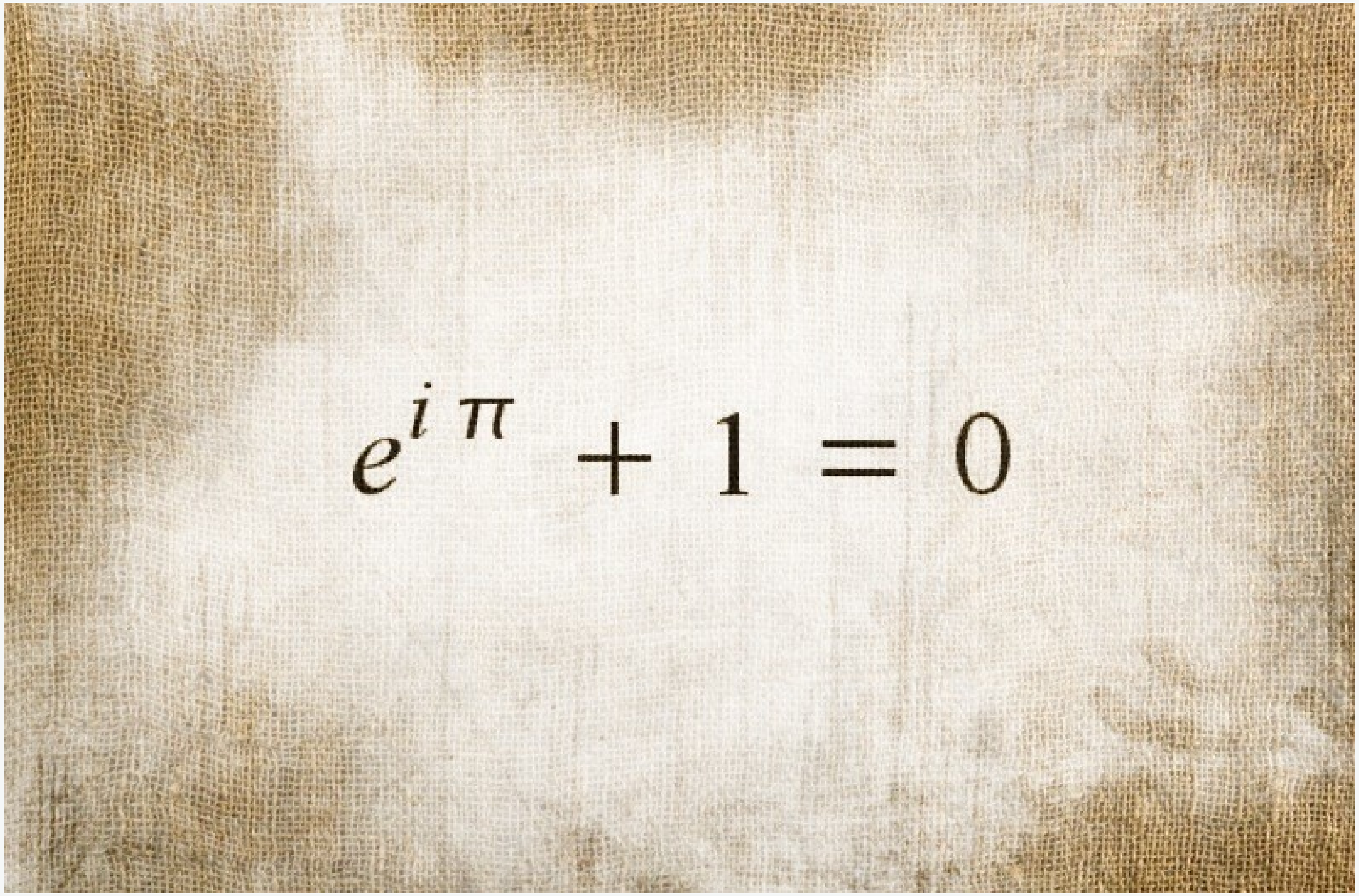
Die Eulersche Formel

$$e^{i\,\varphi} = \cos\varphi + i\,\sin\varphi$$

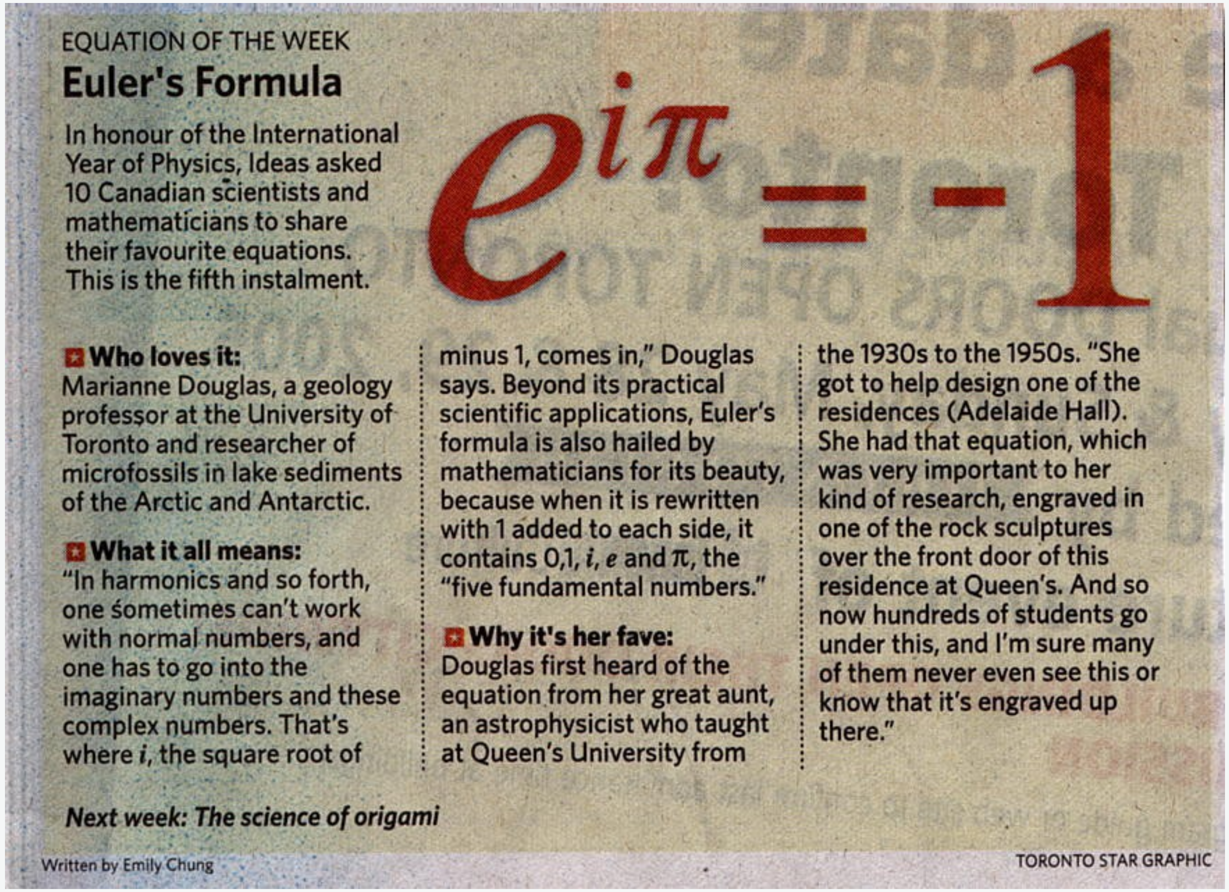
bildet das Bindeglied zwischen trigonometrischen Funktionen und den komplexen Zahlen.

Für den Winkel π ergibt sich

$$e^{i\,\pi} = -1 \quad \Leftrightarrow \quad e^{i\,\pi} + 1 = 0$$



Die Formel stellt einen verblüffend einfachen Zusammenhang zwischen fünf der bedeutendsten mathematischen Konstanten her: der Eulerschen Zahl ***e***, der imaginären Einheit ***i*** der komplexen Zahlen, der Kreiszahl ***π***, der Einheit **1** der reellen Zahlen und der Null **0**.



EQUATION OF THE WEEK

Euler's Formula

In honour of the International Year of Physics, Ideas asked 10 Canadian scientists and mathematicians to share their favourite equations. This is the fifth instalment.

$$e^{i\pi} = -1$$

Who loves it:
Marianne Douglas, a geology professor at the University of Toronto and researcher of microfossils in lake sediments of the Arctic and Antarctic.

What it all means:
"In harmonics and so forth, one sometimes can't work with normal numbers, and one has to go into the imaginary numbers and these complex numbers. That's where i, the square root of minus 1, comes in," Douglas says. Beyond its practical scientific applications, Euler's formula is also hailed by mathematicians for its beauty, because when it is rewritten with 1 added to each side, it contains 0,1, i, e and π, the "five fundamental numbers."

Why it's her fave:
Douglas first heard of the equation from her great aunt, an astrophysicist who taught at Queen's University from the 1930s to the 1950s. "She got to help design one of the residences (Adelaide Hall). She had that equation, which was very important to her kind of research, engraved in one of the rock sculptures over the front door of this residence at Queen's. And so now hundreds of students go under this, and I'm sure many of them never even see this or know that it's engraved up there."

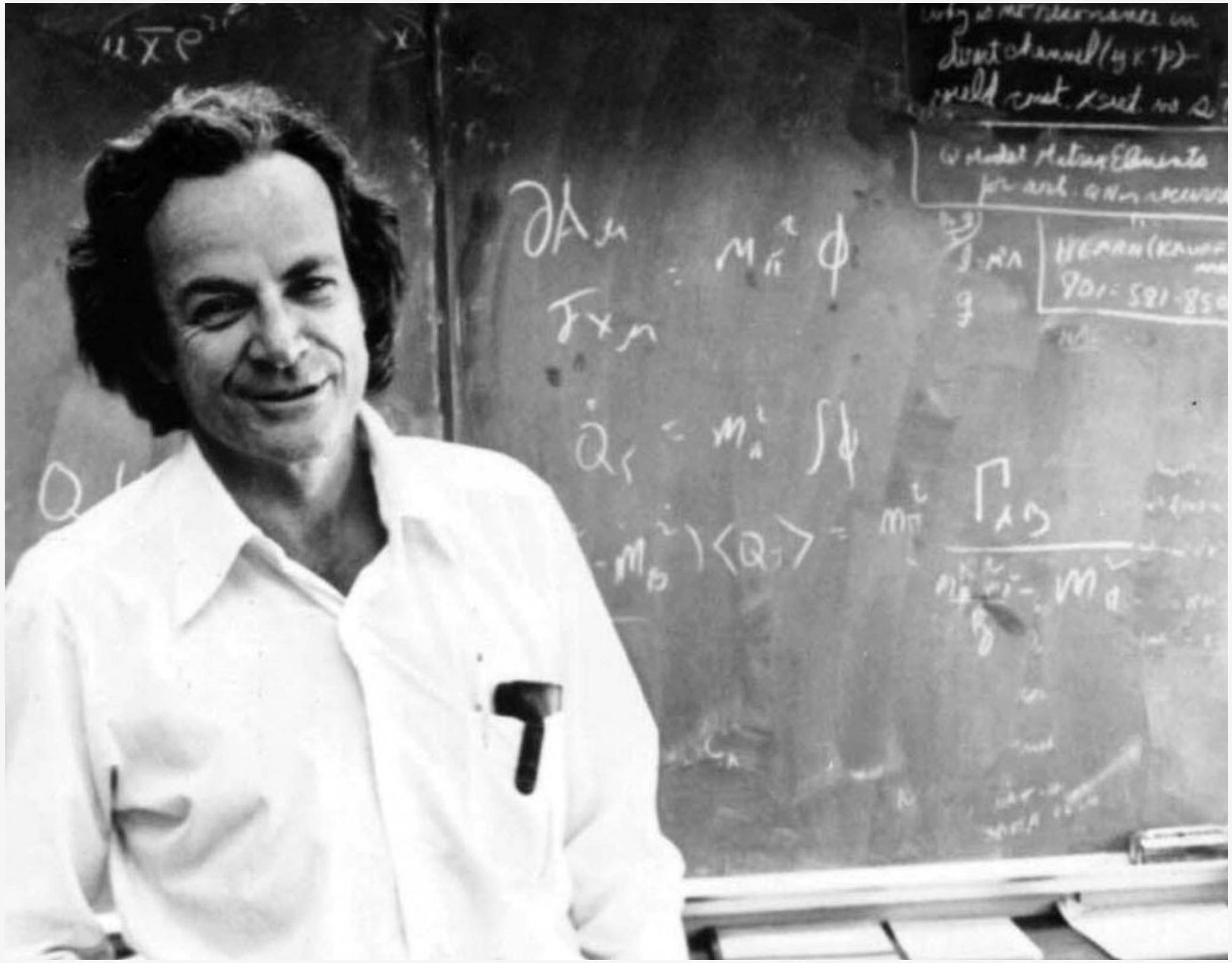
Next week: The science of origami

Written by Emily Chung

TORONTO STAR GRAPHIC

http://www.cap.ca/wyp/mediaPhysics.asp

Bemerkenswerteste Formel der Welt



Richard Feynman (1918-1988)

Richard Feynman, amerikanischer Physiker und Nobelpreisträger des Jahres *1965* nannte diese Gleichung in seinem Notizbuch die “bemerkenswerteste Formel der Welt”.

Trigonometrische Form

$$z = r\,(\cos\varphi + i\,\sin\varphi)$$

↓

$$e^{i\varphi} = \cos\varphi + i\,\sin\varphi$$

↓

Exponentialform

$$z = r\,e^{i\varphi}$$

$$z = r\,(\cos\varphi + i\,\sin\varphi), \qquad z^* = r\,(\cos\varphi - i\,\sin\varphi)$$

$$z = r\,e^{i\varphi}, \qquad z^* = r\,e^{-i\varphi}$$

Normalform: $(x\,,\ y):\quad \mathrm{Re}\,(z) = x\,,\quad \mathrm{Im}\,(z) = y$

$$z = x + i\,y$$

Algebraische (kartesische) Form

Polarform: $(r\,,\ \varphi):\quad r$ – Betrag von z, φ – Argument von z

$$z = r\,(\cos\varphi + i\,\sin\varphi)$$

Trigonometrische Form

$$z = r\,e^{i\,\varphi}$$

Exponentialform

konjugiert komplex: $\mathrm{Im}\,(z^*) = -\,\mathrm{Im}\,(z)$

Stellen Sie folgende komplexe Zahlen in der kartesischen Form dar:

$$a)\quad z = 2\,e^{i\,\frac{\pi}{6}}$$

$$b)\quad z = 2\sqrt{3}\,e^{i\,\frac{\pi}{3}}$$

$$c)\quad z = 4\,e^{3\pi i}$$

$$d)\quad z = 4\,e^{i\,\frac{\pi}{2}}$$

$$e)\quad z = \sqrt{2}\;e^{i\,\frac{3\pi}{4}}$$

$$f)\quad z = 2\sqrt{3}\,e^{i\,\frac{2\pi}{3}}$$

$$g)\quad z = \sqrt{3}\,e^{i\,\frac{13\pi}{6}}$$

$$a)\quad z = 2\,e^{i\,\frac{\pi}{6}} = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = \sqrt{3} + i$$

$$b)\quad z = 2\sqrt{3}\,e^{i\,\frac{\pi}{3}} = \sqrt{3} + 3\,i$$

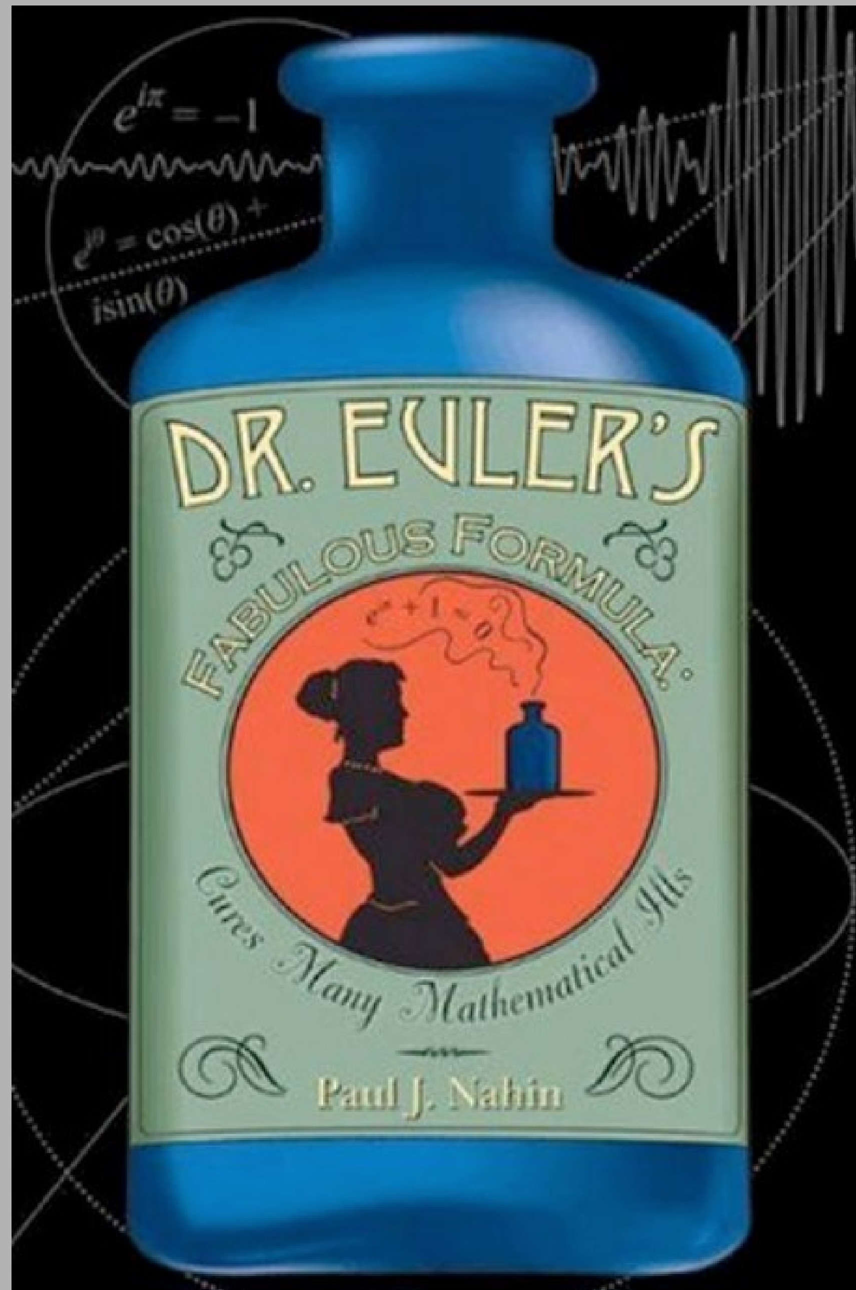
$$c)\quad z = 4\,e^{3\pi i} = -4$$

$$d)\quad z = 4\,e^{i\,\frac{\pi}{2}} = 4\,i$$

$$e)\quad z = \sqrt{2}\;e^{i\,\frac{3\pi}{4}} = -1 + i$$

$$f)\quad z = 2\sqrt{3}\,e^{i\,\frac{2\pi}{3}} = -\sqrt{3} + 3\,i$$

$$g)\quad z = \sqrt{3}\,e^{i\,\frac{13\pi}{6}} = \frac{3}{2} + \frac{\sqrt{3}}{2}\,i = \frac{1}{2}\left(3 + \sqrt{3}\,i\right)$$



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