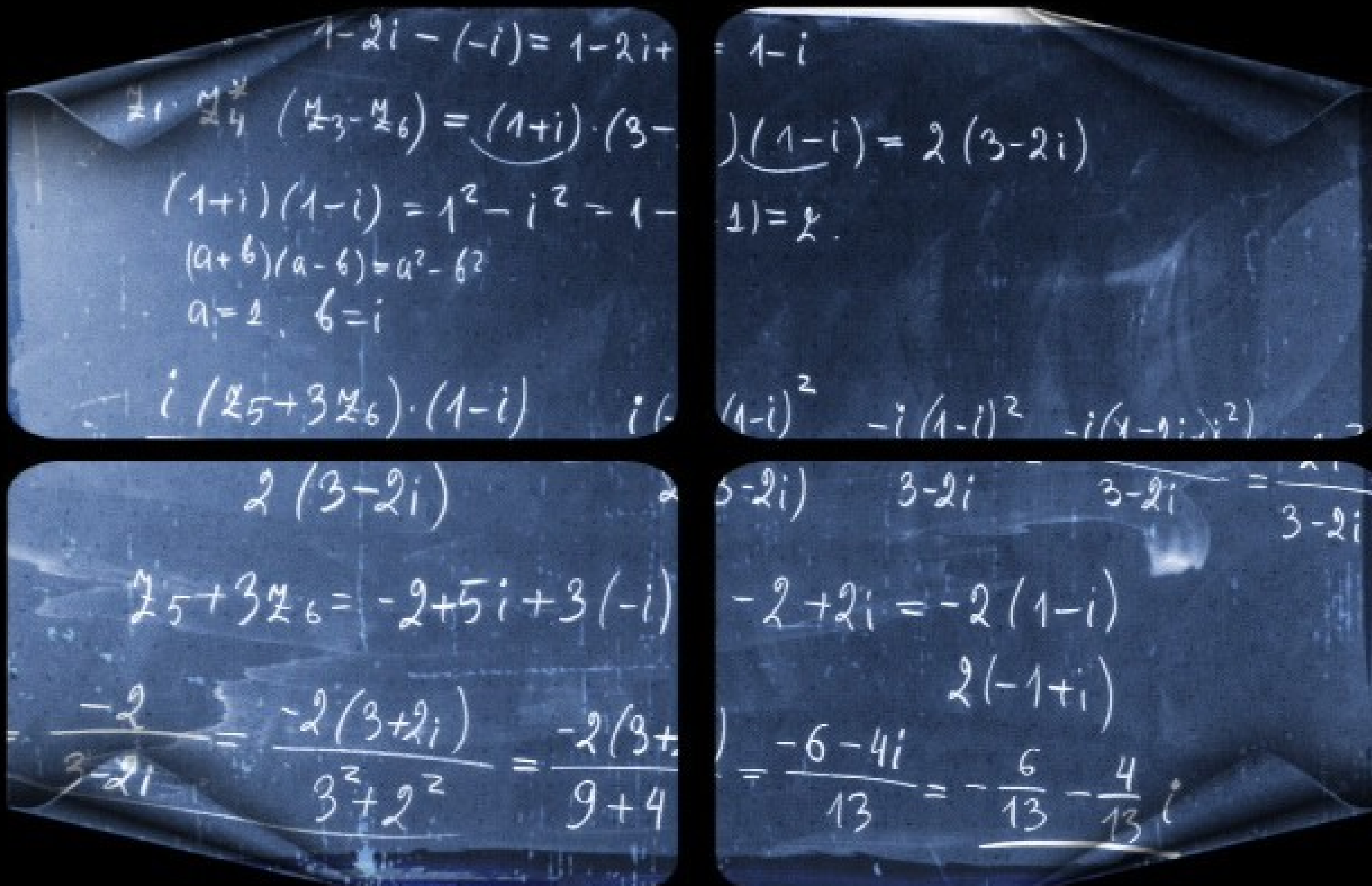
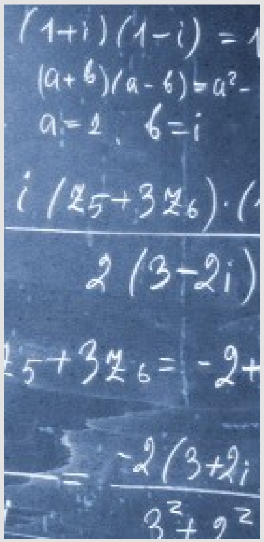


Division komplexer Zahlen



Definition:

Unter dem Quotient zweier komplexer Zahlen

$$z_1 = x_1 + i\, y_1 , \qquad z_2 = x_2 + i\, y_2$$

wird die komplexe Zahl

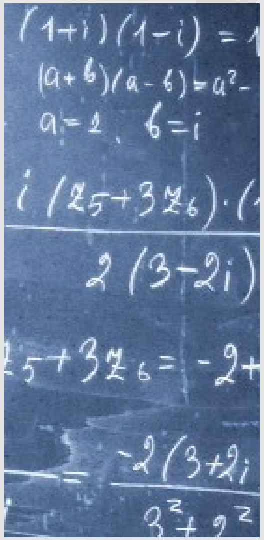
$$\frac{z_1}{z_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\, \frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

verstanden.

$$\frac{z_1}{z_2} = \frac{z_1 \cdot z_2^*}{z_2 \cdot z_2^*}$$

$$\sqrt{z_2 \cdot z_2^*} = | z_2 | = \sqrt{x_2^2 + y_2^2}$$

Division komplexer Zahlen: Beispiele



Wir machen den Nenner der folgenden Brüchen reell

$$a)\quad \frac{1}{1-i}, \qquad b)\quad \frac{1}{2+i}, \qquad c)\quad \frac{1+2i}{2+3i}$$

$$a)\quad \frac{1}{1-i} = \frac{1\cdot(1-i)^*}{(1-i)\cdot(1-i)^*} = \frac{1\cdot(1+i)}{(1-i)\cdot(1+i)} = \frac{1}{2} + \frac{i}{2}$$

$$b)\quad \frac{1}{2+i} = \frac{1\cdot(2+i)^*}{(2+i)\cdot(2+i)^*} = \frac{1\cdot(2-i)}{(2+i)\cdot(2-i)} = \frac{2}{5} - \frac{i}{5}$$

$$c)\quad \frac{1+2i}{2+3i} = \frac{(1+2i)(2-3i)}{(2+3i)(2-3i)} = \frac{8}{13} + \frac{i}{13}$$

Aufgabe 1:

Machen Sie den Nenner folgender Brüche reell

$$a)\quad \frac{1}{1-2i}, \quad \frac{1}{1+2i}, \qquad b)\quad \frac{1}{2-i}, \quad \frac{1}{2+i}$$

$$c)\quad \frac{2+i}{2-i}, \quad \frac{2-i}{2+i} \qquad d)\quad \frac{3+2i}{3-i}, \quad \frac{3-i}{3+2i}$$

$$e)\quad \frac{1}{(1+i)(2-i)}, \quad \frac{1}{(1-i)(2+i)}$$

Lösung 1:

$$a)\quad \frac{1}{1-2i} = \frac{1}{5} + \frac{2i}{5}, \qquad \frac{1}{1+2i} = \frac{1}{5} - \frac{2i}{5}$$

$$b)\quad \frac{1}{2-i} = \frac{2}{5} + \frac{i}{5}, \qquad \frac{1}{2+i} = \frac{2}{5} - \frac{i}{5}$$

$$c)\quad \frac{2+i}{2-i} = \frac{3}{5} + \frac{4i}{5}, \qquad \frac{2-i}{2+i} = \frac{3}{5} - \frac{4i}{5}$$

$$d)\quad \frac{3+2i}{3-i} = \frac{7}{10} + \frac{9i}{10}, \qquad \frac{3-i}{3+2i} = \frac{7}{13} - \frac{9i}{13}$$

$$e)\quad \frac{1}{(1+i)(2-i)} = \frac{3}{10} - \frac{i}{10}, \qquad \frac{1}{(1-i)(2+i)} = \frac{3}{10} + \frac{i}{10}$$

Berechnen Sie mit den komplexen Zahlen

$z_1 = 1 + i, \quad z_2 = 2 + i, \quad z_3 = 1 - 2i$

$z_4 = 3 + 2i, \quad z_5 = -2 + 5i, \quad z_6 = -i$

die folgenden Terme

$$a)\quad \frac{z_1 + z_2}{z_3}, \qquad \frac{z_2 + z_3}{z_4 + z_6}, \qquad \frac{z_1 + z_2 + z_3}{z_4 - z_2}$$

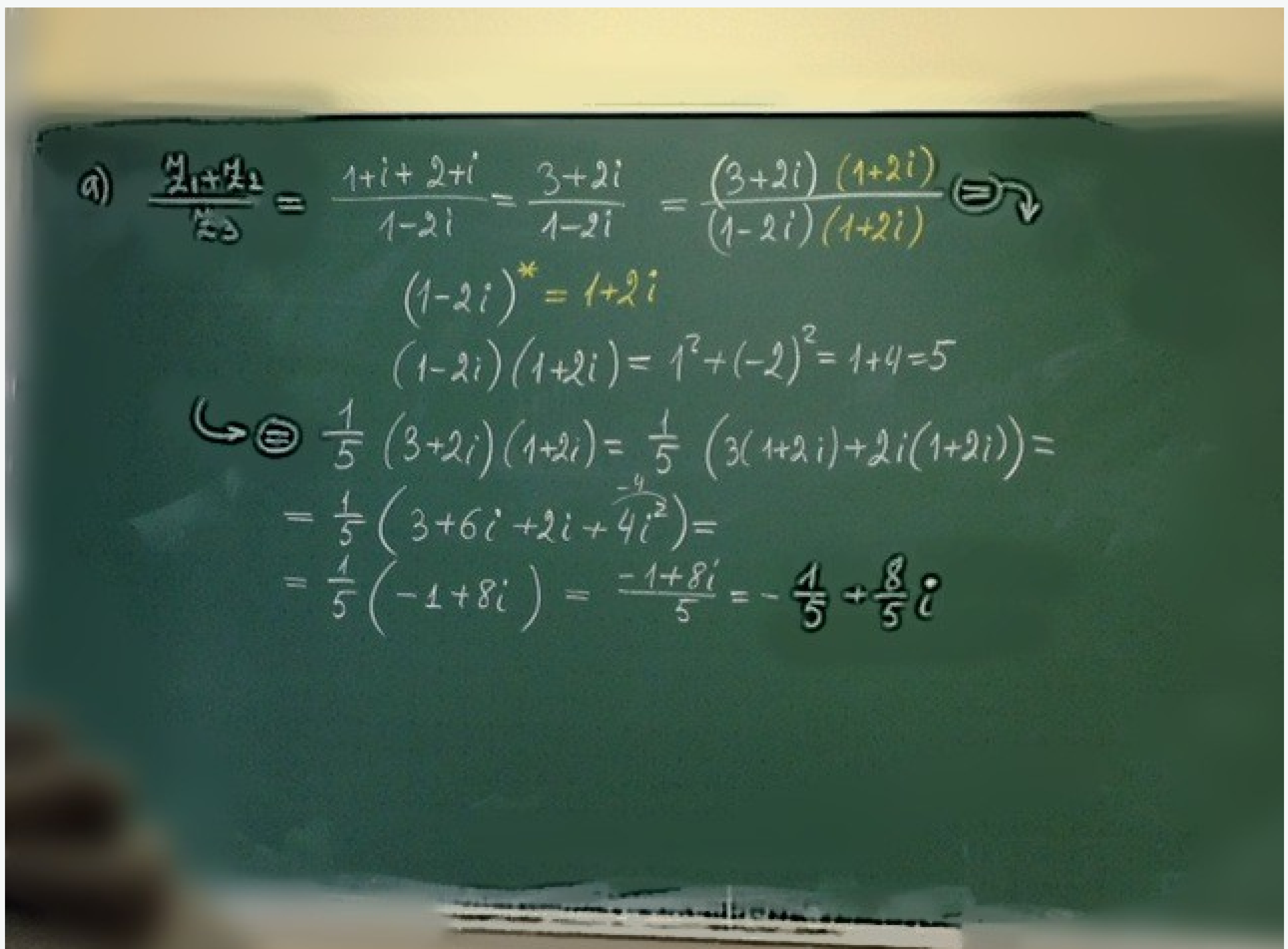
$$b)\quad \frac{(z_1 - 2z_2)\, z_6}{z_2}, \qquad \frac{z_1 \cdot z_2^*}{z_3^*}, \qquad \frac{z_1 \cdot (z_2^* + 3z_6^*)}{z_5 + 7z_6}$$

$$c)\quad \frac{1}{z_1} + \frac{1}{z_2}, \qquad \frac{z_2^*}{z_1} + \frac{z_1^*}{z_2}, \qquad \frac{1}{z_1} + \frac{z_2}{z_1 \cdot z_2^*}$$

$$d)\quad \left(\frac{1}{z_3} - \frac{z_2}{z_3 \cdot z_2^*}\right) z_6, \qquad \frac{i\,(z_5 + 3z_6)\, z_1^*}{z_1 \cdot z_4^*\,(z_3 - z_6)}, \qquad \frac{i}{z_1 \cdot z_2^* \cdot z_6}$$

a) $\dfrac{z_1 + z_2}{z_3} = -\dfrac{1}{5} + \dfrac{8\,i}{5}\,,\qquad \dfrac{z_2 + z_3}{z_4 + z_6} = \dfrac{4}{5} - \dfrac{3\,i}{5}$

$$\frac{z_1 + z_2 + z_3}{z_4 - z_2} = 2 - 2\,i$$



b) $$\frac{(z_1 - 2\, z_2)\; z_6}{z_2} = \frac{1}{5} + \frac{7\, i}{5}\,, \qquad \frac{z_1 \cdot z_2^*}{z_3^*} = 1 - i$$

$$\frac{z_1 \cdot (z_2^* + 3\, z_6^*)}{z_5 + 7\, z_6} = -1 - i$$

c) $$\frac{1}{z_1} + \frac{1}{z_2} = \frac{9}{10} - \frac{7\, i}{10}\,, \qquad \frac{z_2^*}{z_1} + \frac{z_1^*}{z_2} = \frac{7}{10} - \frac{21\, i}{10}$$

$$\frac{1}{z_1} + \frac{z_2}{z_1 \cdot z_2^*} = \frac{6}{5} - \frac{2\, i}{5}$$

d) $$\left(\frac{1}{z_3} - \frac{z_2}{z_3 \cdot z_2^*}\right) z_6 = \frac{2}{5}\, z_6 = -\frac{2\, i}{5}\,,$$

$$\frac{i\,(z_5 + 3\, z_6)\; z_1^*}{z_1 \cdot z_4^*\,(z_3 - z_6)} = -\frac{6}{13} - \frac{4\, i}{13}\,, \qquad \frac{i}{z_1 \cdot z_2^* \cdot z_6} = -\frac{3}{10} + \frac{i}{10}\,.$$

$$\frac{i\,(z_5 + 3\,z_6)\,z_1^*}{z_1 \cdot z_4^*\,(z_3 - z_6)}$$

$$z_1 = 1 + i, \qquad z_2 = 2 + i, \qquad z_3 = 1 - 2i$$

$$z_4 = 3 + 2i, \qquad z_5 = -2 + 5i, \qquad z_6 = -i$$

1) $z_5 + 3\,z_6 = -2 + 5i - 3i = -2 + 2i = 2\,(-1 + i)$

2) $i\,z_1^* = i\,(1 - i) = i + 1$

3) $i\,(z_5 + 3\,z_6)\,z_1^* = 2\,(1 + i)(-1 + i) = 2\,(i + 1)(i - 1) = 2\,(i^2 - 1) =$

$$= 2\,(-1 - 1) = -4$$

4) $z_3 - z_6 = 1 - 2i - (-i) = 1 - i$

5) $z_1\,(z_3 - z_6) = (1 + i)\,(1 - i) = 1 - i^2 = 1 + 1 = 2$

$$\frac{i\,(z_5 + 3\,z_6)\,z_1^*}{z_1 \cdot z_4^*\,(z_3 - z_6)} = \frac{-4}{2 \cdot z_4^*} = -\frac{2}{z_4^*} = -\frac{2 \cdot z_4}{z_4^* \cdot z_4} = -\frac{2 \cdot (3 + 2i)}{3^2 + 2^2} = -\frac{6 + 4i}{13} =$$

$$= -\frac{6}{13} - \frac{4i}{13}$$