

$$1 - 2i - (-i) = 1 - 2i + i = 1 - i$$

$$z_1 \cdot z_4 (z_3 - z_6) = (1+i) \cdot (3-2i) \cdot (1-i)$$

$$(1+i)(1-i) = 1^2 - i^2 = 1 - (-1) = 2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a=1, b=i$$

$$i(z_5 + 3z_6) \cdot (1-i)$$

$$= 2(3-2i)$$

$$(1-i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$$

$$-i(1-i)^2 = -i(-2i) = 2i^2 = -2$$

$$-i(1-i)(i^2) = -i(1-i)(-1) = i(1-i) = i - i^2 = i + 1 = 1+i$$

$$2(3-2i)$$

$$z_5 + 3z_6 = -2 + 5i + 3(-i) = -2 + 5i - 3i = -2 + 2i$$

$$\frac{-2 + 2i}{3 - 2i} = \frac{-2(3+2i)}{3^2 + 2^2} = \frac{-2(3+2i)}{9+4}$$

$$\frac{-2 + 2i}{3 - 2i} = \frac{-2(1-i)}{2(-1+i)}$$

$$= \frac{-6 - 4i}{13} = -\frac{6}{13} - \frac{4}{13}i$$

Division komplexer Zahlen

$$\begin{aligned}(1+i)(1-i) &= 1 \\ (a+b)(a-b) &= a^2 - b^2 \\ a &= 2, \quad b = i \\ i(25+32i) & \cdot (3+2i) \\ \hline 2(3-2i) & \\ \hline 25+32i &= -2+ \\ \hline &= \frac{-2(3+2i)}{3^2+2^2}\end{aligned}$$

Definition:

Unter dem Quotient zweier komplexer Zahlen

$$z_1 = x_1 + i y_1, \quad z_2 = x_2 + i y_2$$

wird die komplexe Zahl

$$\frac{z_1}{z_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i \frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

verstanden.

$$\frac{z_1}{z_2} = \frac{z_1 \cdot z_2^*}{z_2 \cdot z_2^*}$$

$$\sqrt{z_2 \cdot z_2^*} = |z_2| = \sqrt{x_2^2 + y_2^2}$$

Handwritten work on a chalkboard showing the derivation of the real denominator for the division of complex numbers. The work includes the following steps:

$$(1+i)(1-i) = 1 - i^2 = 1 - (-1) = 2$$
$$(a+b)(a-b) = a^2 - b^2$$
$$a=2, b=i$$
$$i(2+3i) \cdot (2-3i)$$
$$2(3-2i)$$
$$2+3i = -2+3i$$
$$\frac{-2(3+2i)}{3^2+2^2}$$

Wir machen den Nenner der folgenden Brüchen reell

$$a) \frac{1}{1-i}, \quad b) \frac{1}{2+i}, \quad c) \frac{1+2i}{2+3i}$$

$$a) \frac{1}{1-i} = \frac{1 \cdot (1-i)^*}{(1-i) \cdot (1-i)^*} = \frac{1 \cdot (1+i)}{(1-i) \cdot (1+i)} = \frac{1}{2} + \frac{i}{2}$$

$$b) \frac{1}{2+i} = \frac{1 \cdot (2+i)^*}{(2+i) \cdot (2+i)^*} = \frac{1 \cdot (2-i)}{(2+i) \cdot (2-i)} = \frac{2}{5} - \frac{i}{5}$$

$$c) \frac{1+2i}{2+3i} = \frac{(1+2i)(2-3i)}{(2+3i)(2-3i)} = \frac{8}{13} + \frac{i}{13}$$

Aufgabe 1:

Machen Sie den Nenner folgender Brüche reell

$$a) \frac{1}{1-2i}, \quad \frac{1}{1+2i}, \quad b) \frac{1}{2-i}, \quad \frac{1}{2+i}$$

$$c) \frac{2+i}{2-i}, \quad \frac{2-i}{2+i}, \quad d) \frac{3+2i}{3-i}, \quad \frac{3-i}{3+2i}$$

$$e) \frac{1}{(1+i)(2-i)}, \quad \frac{1}{(1-i)(2+i)}$$

Lösung 1:

$$a) \frac{1}{1-2i} = \frac{1}{5} + \frac{2i}{5}, \quad \frac{1}{1+2i} = \frac{1}{5} - \frac{2i}{5}$$

$$b) \frac{1}{2-i} = \frac{2}{5} + \frac{i}{5}, \quad \frac{1}{2+i} = \frac{2}{5} - \frac{i}{5}$$

$$c) \frac{2+i}{2-i} = \frac{3}{5} + \frac{4i}{5}, \quad \frac{2-i}{2+i} = \frac{3}{5} - \frac{4i}{5}$$

$$d) \frac{3+2i}{3-i} = \frac{7}{10} + \frac{9i}{10}, \quad \frac{3-i}{3+2i} = \frac{7}{13} - \frac{9i}{13}$$

$$e) \frac{1}{(1+i)(2-i)} = \frac{3}{10} - \frac{i}{10}, \quad \frac{1}{(1-i)(2+i)} = \frac{3}{10} + \frac{i}{10}$$

Division komplexer Zahlen: Aufgabe 2

Berechnen Sie mit den komplexen Zahlen

$$z_1 = 1 + i, \quad z_2 = 2 + i, \quad z_3 = 1 - 2i$$

$$z_4 = 3 + 2i, \quad z_5 = -2 + 5i, \quad z_6 = -i$$

die folgenden Terme

$$a) \quad \frac{z_1 + z_2}{z_3}, \quad \frac{z_2 + z_3}{z_4 + z_6}, \quad \frac{z_1 + z_2 + z_3}{z_4 - z_2}$$

$$b) \quad \frac{(z_1 - 2z_2)z_6}{z_2}, \quad \frac{z_1 \cdot z_2^*}{z_3^*}, \quad \frac{z_1 \cdot (z_2^* + 3z_6^*)}{z_5 + 7z_6}$$

$$c) \quad \frac{1}{z_1} + \frac{1}{z_2}, \quad \frac{z_2^*}{z_1} + \frac{z_1^*}{z_2}, \quad \frac{1}{z_1} + \frac{z_2}{z_1 \cdot z_2^*}$$

$$d) \quad \left(\frac{1}{z_3} - \frac{z_2}{z_3 \cdot z_2^*} \right) z_6, \quad \frac{i(z_5 + 3z_6)z_1^*}{z_1 \cdot z_4^* (z_3 - z_6)}, \quad \frac{i}{z_1 \cdot z_2^* \cdot z_6}$$

Division komplexer Zahlen: Lösung 2a

$$a) \quad \frac{z_1 + z_2}{z_3} = -\frac{1}{5} + \frac{8i}{5}, \quad \frac{z_2 + z_3}{z_4 + z_6} = \frac{4}{5} - \frac{3i}{5}$$

$$\frac{z_1 + z_2 + z_3}{z_4 - z_2} = 2 - 2i$$

The image shows a handwritten solution on a chalkboard for the division of complex numbers. The steps are as follows:

$$\begin{aligned} a) \quad \frac{z_1 + z_2}{z_3} &= \frac{1+i+2+i}{1-2i} = \frac{3+2i}{1-2i} = \frac{(3+2i)(1+2i)}{(1-2i)(1+2i)} \Rightarrow \curvearrowright \\ & \quad (1-2i)^* = 1+2i \\ & \quad (1-2i)(1+2i) = 1^2 + (-2)^2 = 1+4=5 \\ \curvearrowleft \ominus & \quad \frac{1}{5} (3+2i)(1+2i) = \frac{1}{5} (3(1+2i) + 2i(1+2i)) = \\ & \quad = \frac{1}{5} (3+6i+2i+4i^2) = \\ & \quad = \frac{1}{5} (-1+8i) = \frac{-1+8i}{5} = -\frac{1}{5} + \frac{8}{5}i \end{aligned}$$

$$b) \frac{(z_1 - 2z_2)z_6}{z_2} = \frac{1}{5} + \frac{7i}{5}, \quad \frac{z_1 \cdot z_2^*}{z_3^*} = 1 - i$$

$$\frac{z_1 \cdot (z_2^* + 3z_6^*)}{z_5 + 7z_6} = -1 - i$$

$$c) \frac{1}{z_1} + \frac{1}{z_2} = \frac{9}{10} - \frac{7i}{10}, \quad \frac{z_2^*}{z_1} + \frac{z_1^*}{z_2} = \frac{7}{10} - \frac{21i}{10}$$

$$\frac{1}{z_1} + \frac{z_2}{z_1 \cdot z_2^*} = \frac{6}{5} - \frac{2i}{5}$$

$$d) \left(\frac{1}{z_3} - \frac{z_2}{z_3 \cdot z_2^*} \right) z_6 = \frac{2}{5} z_6 = -\frac{2i}{5},$$

$$\frac{i(z_5 + 3z_6)z_1^*}{z_1 \cdot z_4^* (z_3 - z_6)} = -\frac{6}{13} - \frac{4i}{13}, \quad \frac{i}{z_1 \cdot z_2^* \cdot z_6} = -\frac{3}{10} + \frac{i}{10}.$$

$$\frac{i(z_5 + 3z_6) z_1^*}{z_1 \cdot z_4^* (z_3 - z_6)}$$

$$z_1 = 1 + i, \quad z_2 = 2 + i, \quad z_3 = 1 - 2i$$
$$z_4 = 3 + 2i, \quad z_5 = -2 + 5i, \quad z_6 = -i$$

$$1) \quad z_5 + 3z_6 = -2 + 5i - 3i = -2 + 2i = 2(-1 + i)$$

$$2) \quad i z_1^* = i(1 - i) = i + 1$$

$$3) \quad i(z_5 + 3z_6) z_1^* = 2(1 + i)(-1 + i) = 2(i + 1)(i - 1) = 2(i^2 - 1) =$$
$$= 2(-1 - 1) = -4$$

$$4) \quad z_3 - z_6 = 1 - 2i - (-i) = 1 - i$$

$$5) \quad z_1 (z_3 - z_6) = (1 + i)(1 - i) = 1 - i^2 = 1 + 1 = 2$$

$$\frac{i(z_5 + 3z_6) z_1^*}{z_1 \cdot z_4^* (z_3 - z_6)} = \frac{-4}{2 \cdot z_4^*} = -\frac{2}{z_4^*} = -\frac{2 \cdot z_4}{z_4^* \cdot z_4} = -\frac{2 \cdot (3 + 2i)}{3^2 + 2^2} = -\frac{6 + 4i}{13} =$$
$$= -\frac{6}{13} - \frac{4i}{13}$$

$$z_1 \cdot z_4^* (z_3 - z_6) = (1+i) \cdot (3-2i) (1-i) = 2(3-2i)$$

$$(1+i)(1-i) = 1^2 - i^2 = 1 - (-1) = 2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a=1, \quad b=i$$

$$\frac{i(z_5 + 3z_6) \cdot (1-i)}{2(3-2i)} = \frac{i(-2)(1-i)}{2(3-2i)} = \frac{(1-i)^2}{3-2i} = \frac{-i(-2i+i^2)}{3-2i} = \frac{2i^2}{3-2i}$$

$$z_5 + 3z_6 = -2 + 5i + 3(-i) = -2 + 2i = -2(1-i)$$

$$\frac{-2}{3-2i} = \frac{-2(3+2i)}{3^2 + 2^2} = \frac{-2(3+2i)}{9+4} = \frac{-6-4i}{13} = -\frac{6}{13} - \frac{4}{13}i$$