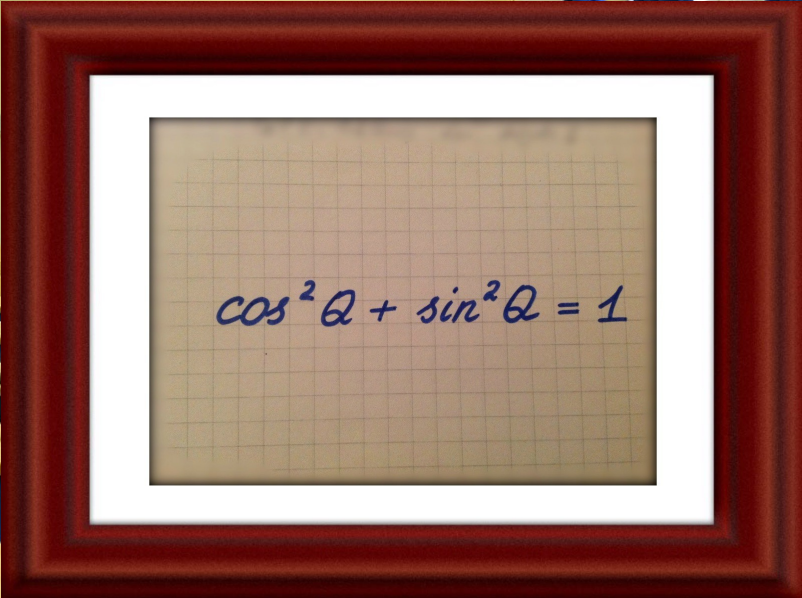


Test 2:

Matrizen, Determinanten und lineare Transformationen



$\cos^2 \alpha + \sin^2 \alpha = 1$

January

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

February

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29			

March

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

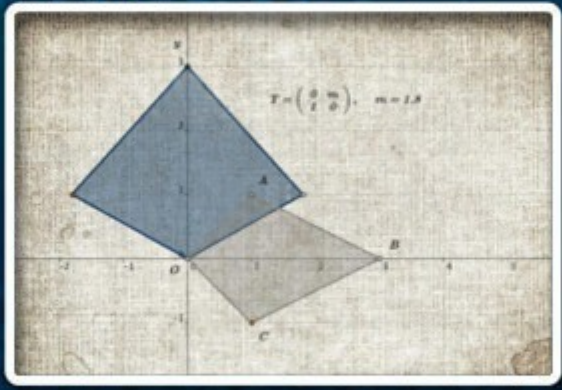
April

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

2012

May

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		



June

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

July

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

August

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

September

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

October

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

November

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

Dezember

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

$$= \begin{pmatrix} 1 & & & \\ & c & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \cdot \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\cos \varphi \sin \varphi & \sin^2 \varphi + \cos^2 \varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2 \varphi + \sin^2 \varphi & \cos \varphi \sin \varphi - \cos \varphi \sin \varphi \\ -\sin \varphi \cos \varphi + \cos \varphi \sin \varphi & \sin^2 \varphi + \cos^2 \varphi \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Aufgaben 1-3

Aufgabe 1:

Bestimmen Sie ob die Matrizen A und B vertauschbar sind:

$$A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad B = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

Aufgabe 2:

Bestimmen Sie die zweite Potenz der Matrix M , die Determinante, die Spur, Realteil und Imaginärteil von M^2

$$M = \begin{pmatrix} 1+i & 1-i \\ i & i^3 \end{pmatrix}$$

Aufgabe 3:

Bestimmen Sie die Determinante D

$$D = \begin{vmatrix} -2 & -1 & 0 & 1 \\ -2 & 0 & 5 & 0 \\ -3 & 1 & 1 & 1 \\ 0 & 4 & 0 & 0 \end{vmatrix}$$

Aufgaben 4, 5

Aufgabe 4:

Zeigen Sie, dass die Matrix B die inverse Matrix der Matrix A ist. Lösen Sie folgende Gleichung:

$$A = \begin{pmatrix} 1 & 0 & -3 \\ -2 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}, \quad B = \frac{1}{10} \begin{pmatrix} -2 & -6 & 0 \\ -2 & -1 & 5 \\ -4 & -2 & 0 \end{pmatrix}$$

$$A X = \begin{pmatrix} 1 & 0 & -3 \\ -2 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Aufgabe 5:

Eine Fläche sei durch die Eckpunkte Punkte O , A , B und C bestimmt. Wie ändert sich die Fläche durch die Transformation T ? Beschreiben Sie diese Transformation.

$$O(0, 0), \quad A(1, 1), \quad B(3, 0), \quad C(1, -1)$$

$$T = \begin{pmatrix} 0 & m \\ 1 & 0 \end{pmatrix}, \quad a) \ m = [1, 3], \quad b) \ m = [-2, 2]$$

Lösung 1

$$AB \stackrel{?}{=} BA$$
$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \therefore AB = \underline{I}$$
$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \therefore BA = \underline{I}$$
$$\Rightarrow B = A^{-1}$$

Zur Abbildung: Man soll aufschreiben, dass $\cos^2 \theta + \sin^2 \theta = 1$

$$A \cdot B = B \cdot A = E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Zur Lösung 1

$$A \cdot B = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} =$$

$$= \begin{pmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \cos \theta \sin \theta \\ \cos \theta \sin \theta - \cos \theta \sin \theta & \sin^2 \theta + \cos^2 \theta \end{pmatrix} =$$

$$= \begin{pmatrix} \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} =$$

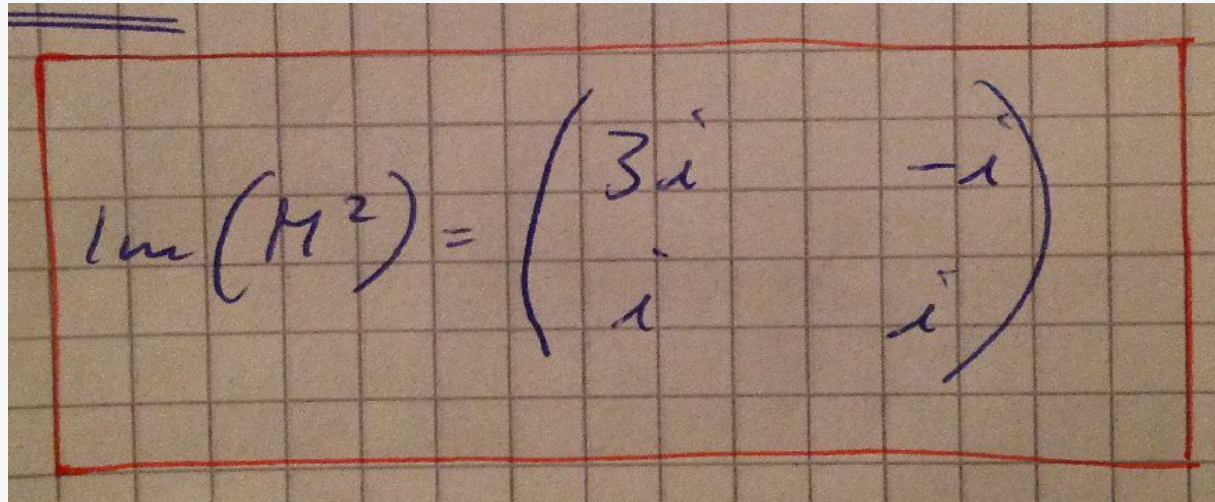
$$= \begin{pmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \cos \theta \sin \theta \\ \cos \theta \sin \theta - \cos \theta \sin \theta & \cos^2 \theta + \sin^2 \theta \end{pmatrix} =$$

$\cos^2 \theta + \sin^2 \theta = 1$ fehlt

Lösung 2

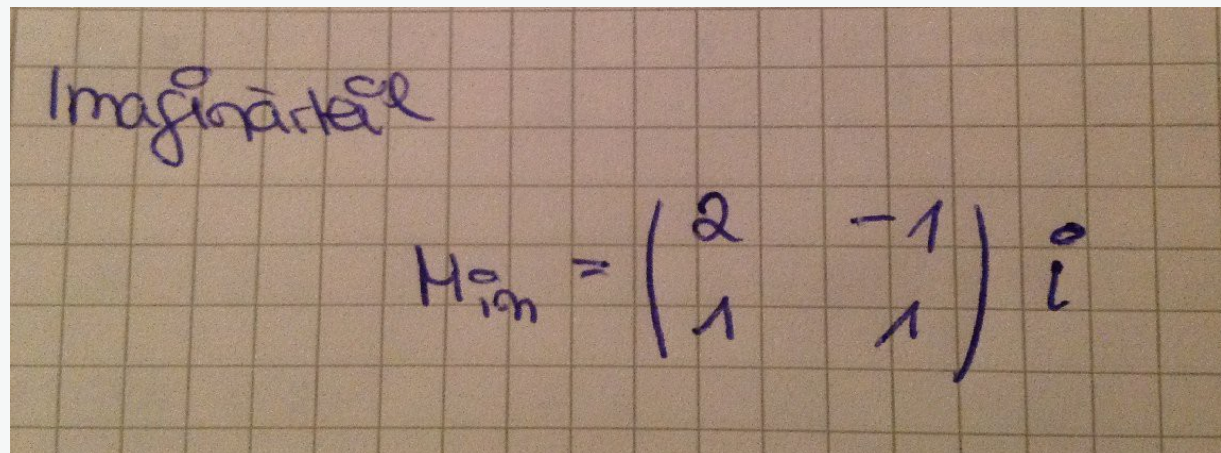
$$M = \begin{pmatrix} 1+i & 1-i \\ i & i^3 \end{pmatrix}, \quad M^2 = \begin{pmatrix} 1+3i & 1-i \\ i & i \end{pmatrix}, \quad \det M^2 = -4$$

$$\operatorname{Re} M^2 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad \operatorname{Im} M^2 = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}, \quad \operatorname{Sp} M^2 = 1 + 4i$$



Handwritten calculation of the imaginary part of M^2 on grid paper. The matrix is enclosed in a red hand-drawn box.

$$\operatorname{Im}(M^2) = \begin{pmatrix} 3i & -1 \\ i & i \end{pmatrix}$$



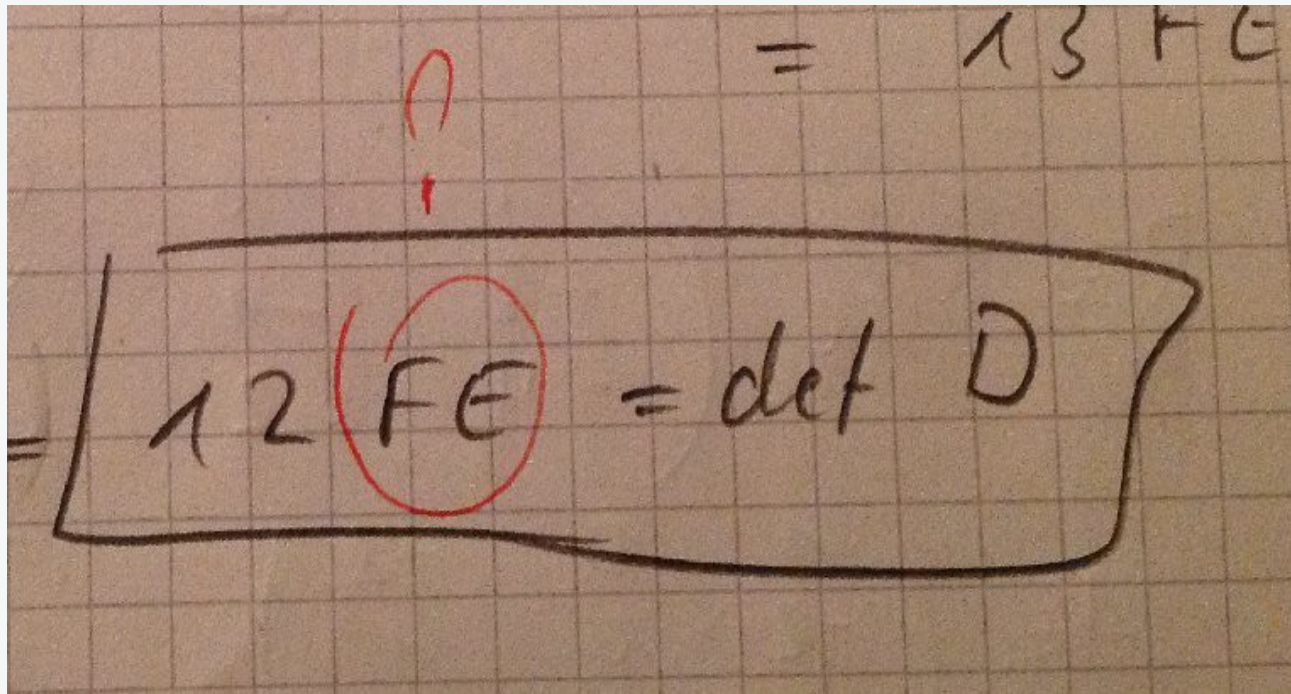
Handwritten calculation of the imaginary part of M^2 on grid paper. The text "Imaginärteil" is written above the matrix.

Imaginärteil

$$M_{\operatorname{Im}} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} i$$

Lösung 3

$$D = \begin{vmatrix} -2 & -1 & 0 & 1 \\ -2 & 0 & 5 & 0 \\ -3 & 1 & 1 & 1 \\ 0 & 4 & 0 & 0 \end{vmatrix}, \quad D = 12$$



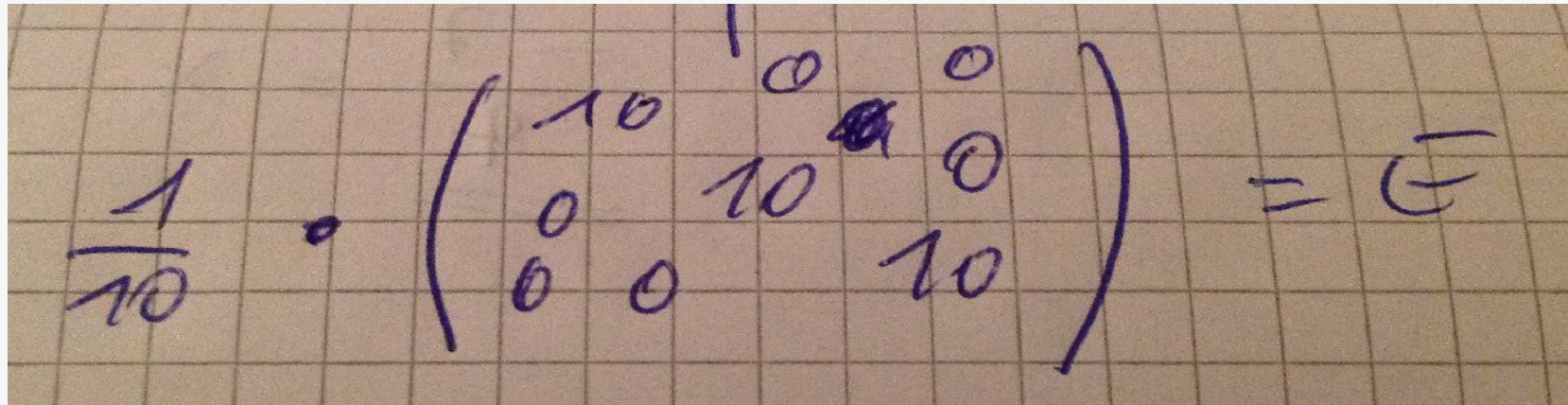
Welcher Flächeneinheit ?

Lösung 4

$$A X = \begin{pmatrix} 1 & 0 & -3 \\ -2 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = B \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} -2 & -6 & 0 \\ -2 & -1 & 5 \\ -4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

Lösung 4



A photograph of a handwritten equation on grid paper. The equation is $\frac{1}{10} \cdot \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix} = E$. The matrix is written with blue ink, and the scalar $\frac{1}{10}$ is written to the left of the matrix. The result E is written to the right of the matrix. There is a small scribble in the top right corner of the matrix.

besser in dieser Form: $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Die Transformation T ändert die Position der Punkte. Dabei ändern sich die Position des Punktes O nicht.

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & m \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} m y \\ x \end{pmatrix}$$

$$B: \quad T \begin{pmatrix} x_B \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & m \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_B \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ x_B \end{pmatrix}$$

$$A: \quad T \begin{pmatrix} x_A \\ y_A \end{pmatrix} = \begin{pmatrix} 0 & m \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_A \\ y_A \end{pmatrix} = \begin{pmatrix} m y_A \\ x_A \end{pmatrix}$$

Lösung 5a

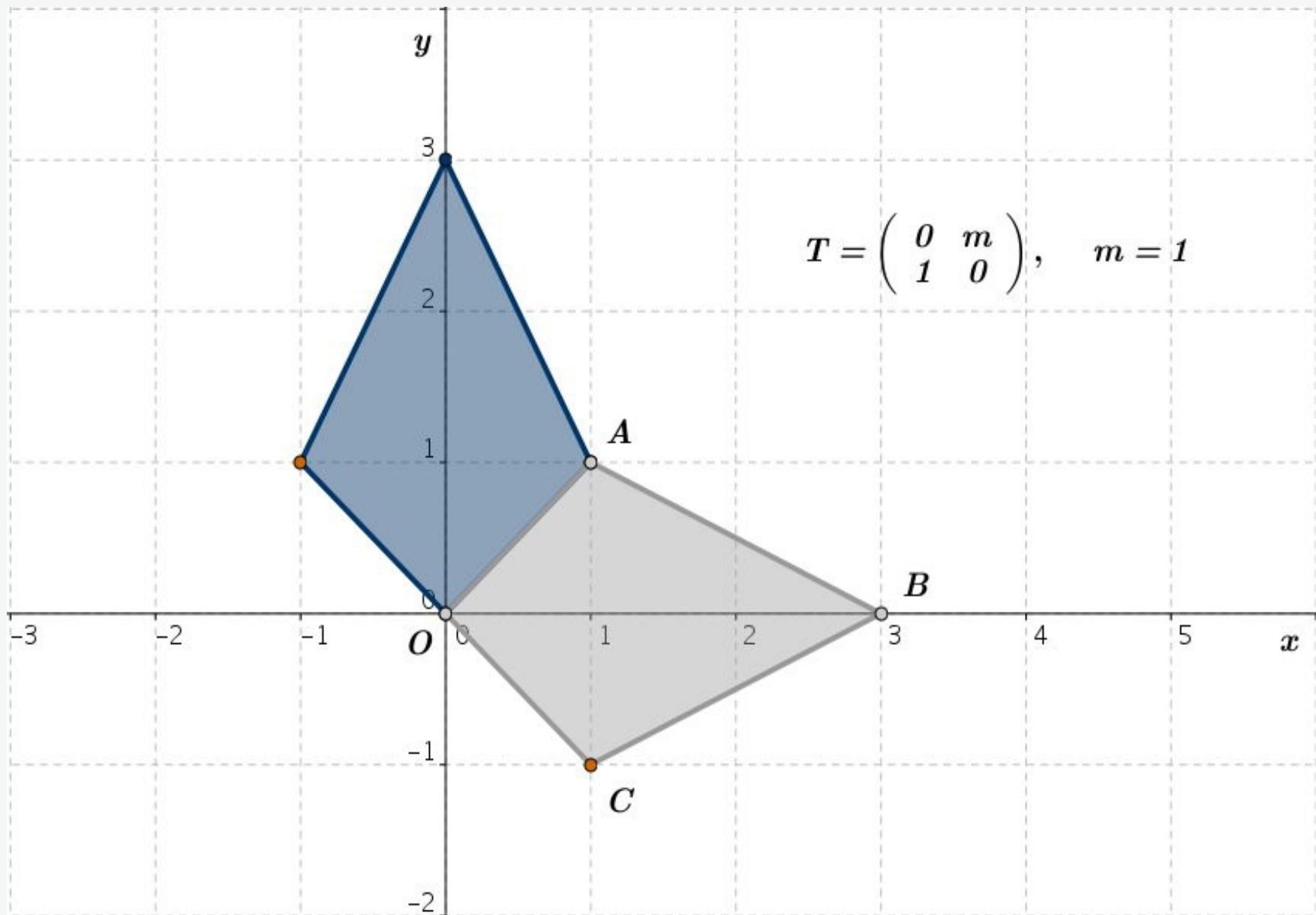


Abb. L5-1: Das Viereck $OABC$ (grau) und das transformierte Viereck (blau), $m = 1$

Lösung 5a

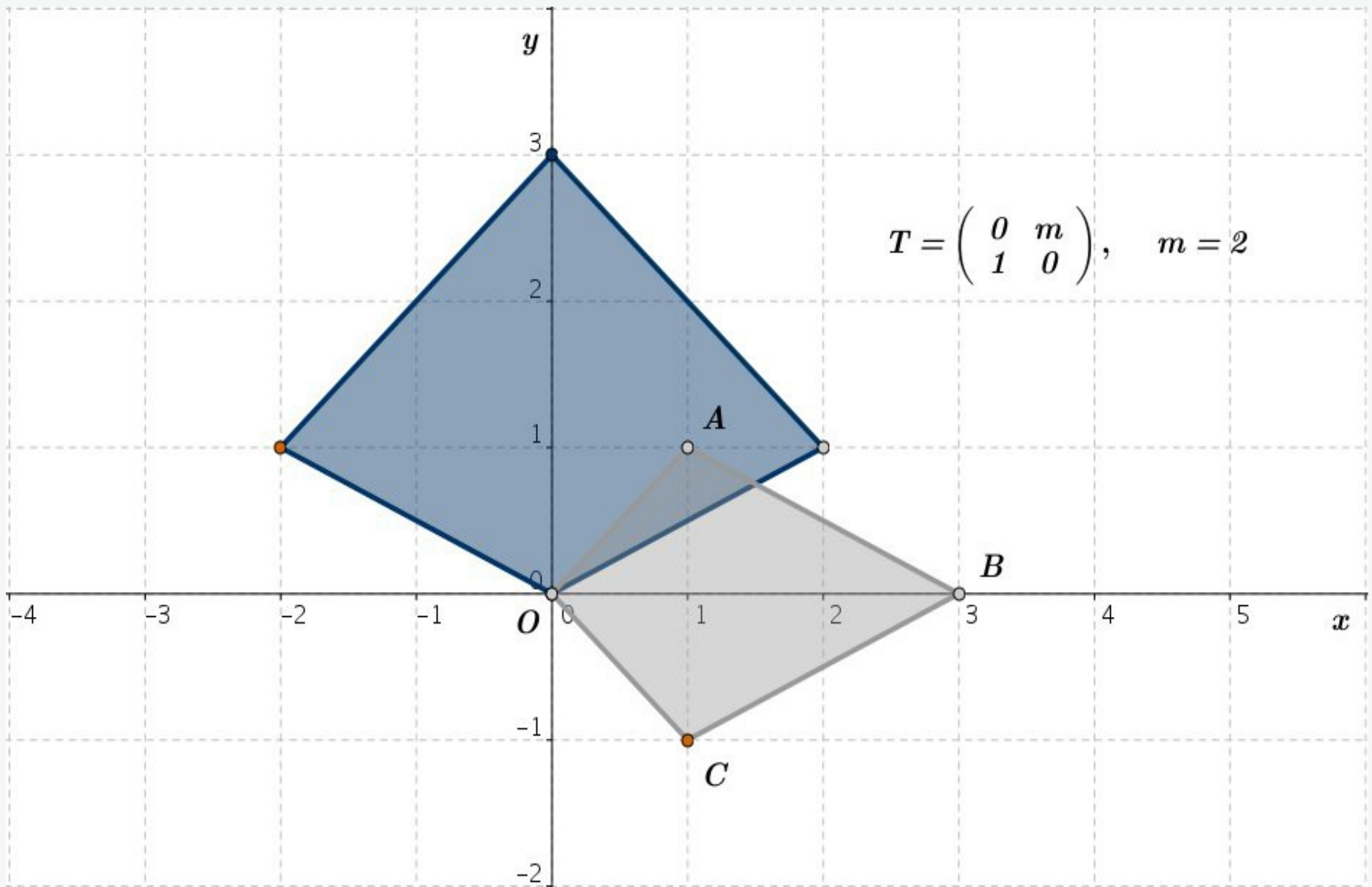


Abb. L5-2: Das Viereck $OABC$ (grau) und das transformierte Viereck (blau), $m=2$

Lösung 5a

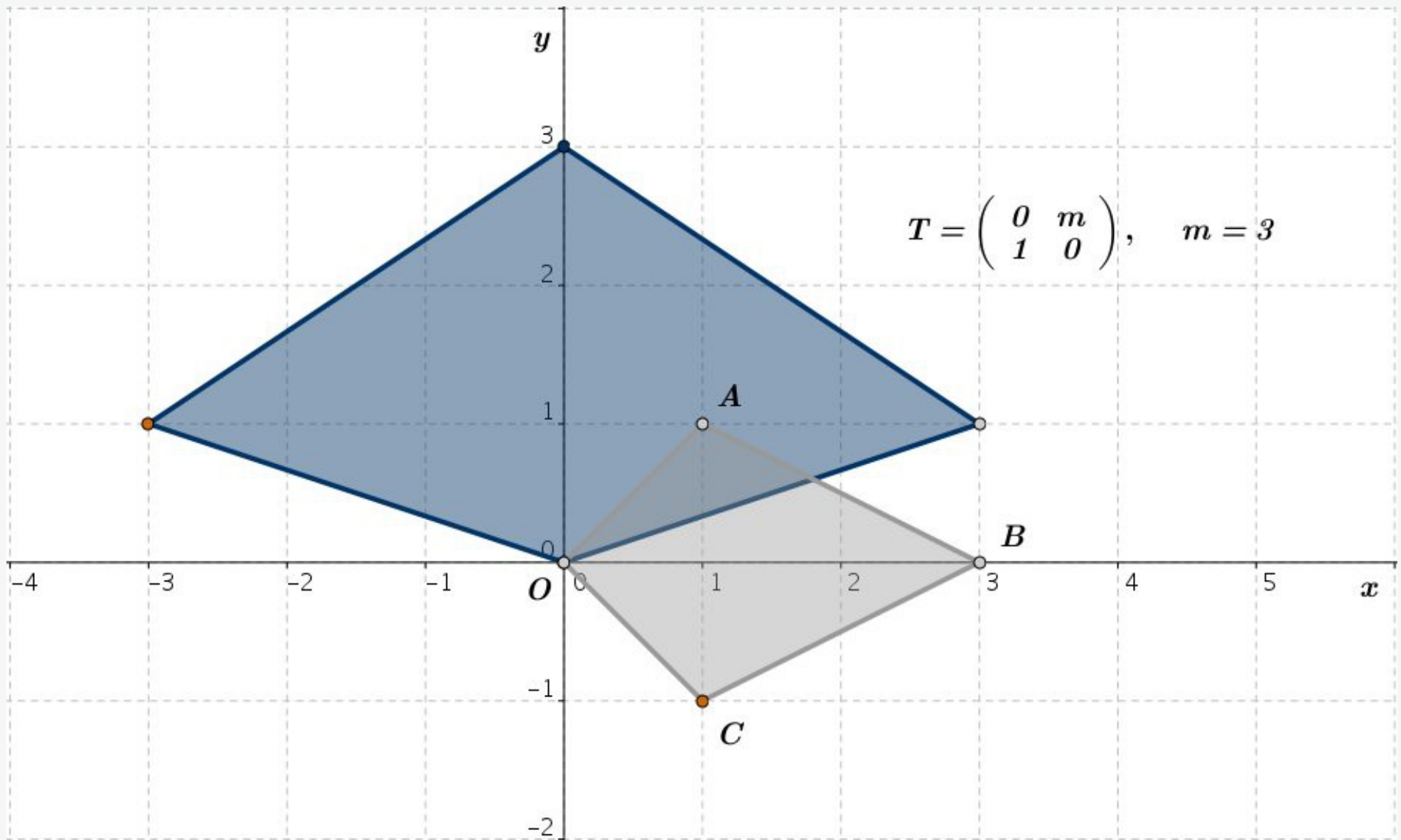


Abb. L5-3: Das Viereck $OABC$ (grau) und das transformierte Viereck (blau), $m=3$

Lösung 5b

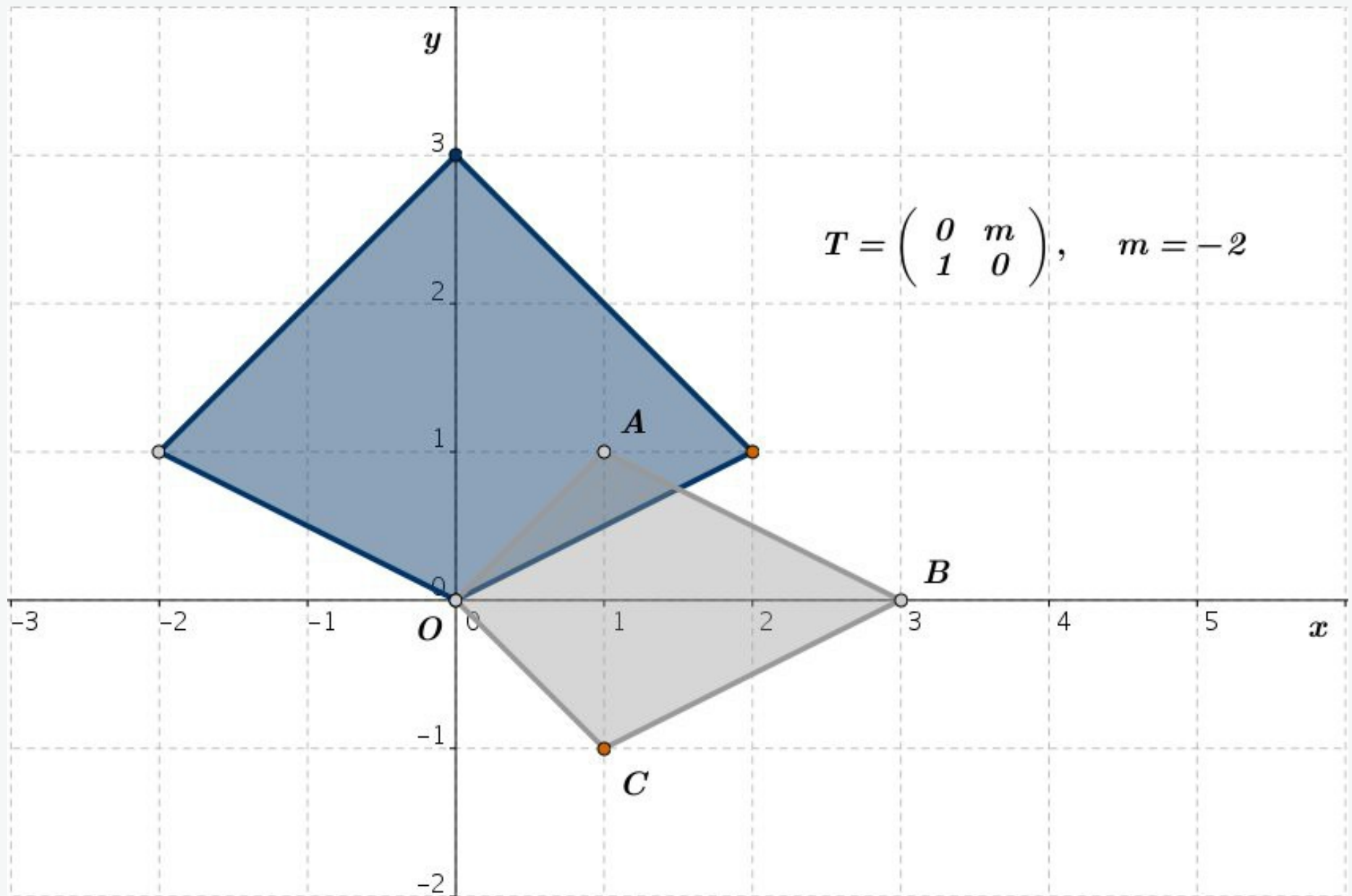
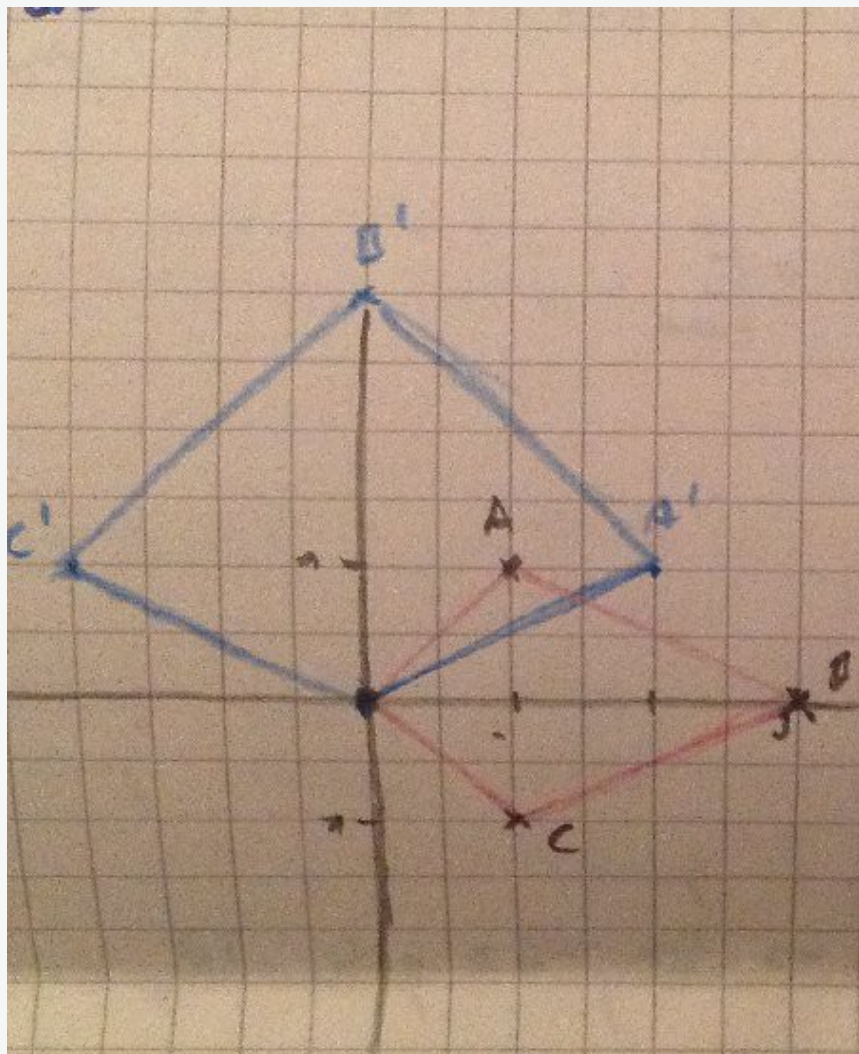


Abb. L5-4: Das Viereck $OABC$ (grau) und das transformierte Viereck (blau), $m = -2$



$\det T =$ Änderung der
Fläche

$$\det T = -2$$

$$F' = 2F$$

$$O' = O \cdot T \Rightarrow \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A' = T \cdot A = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$B' = T \cdot B = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$C' = T \cdot C = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\vec{O\bar{C}} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \vec{O\bar{A}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$2F_{OAC} = \det(\vec{O\bar{C}}, \vec{O\bar{A}}) = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$F_{OAC} = 1$$

$$\vec{B\bar{C}} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad \vec{B\bar{A}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$2F_{BAC} = \det(\vec{B\bar{A}}, \vec{B\bar{C}}) = \begin{vmatrix} -2 & 1 \\ -2 & -1 \end{vmatrix} = 2 - (-2) = 4$$