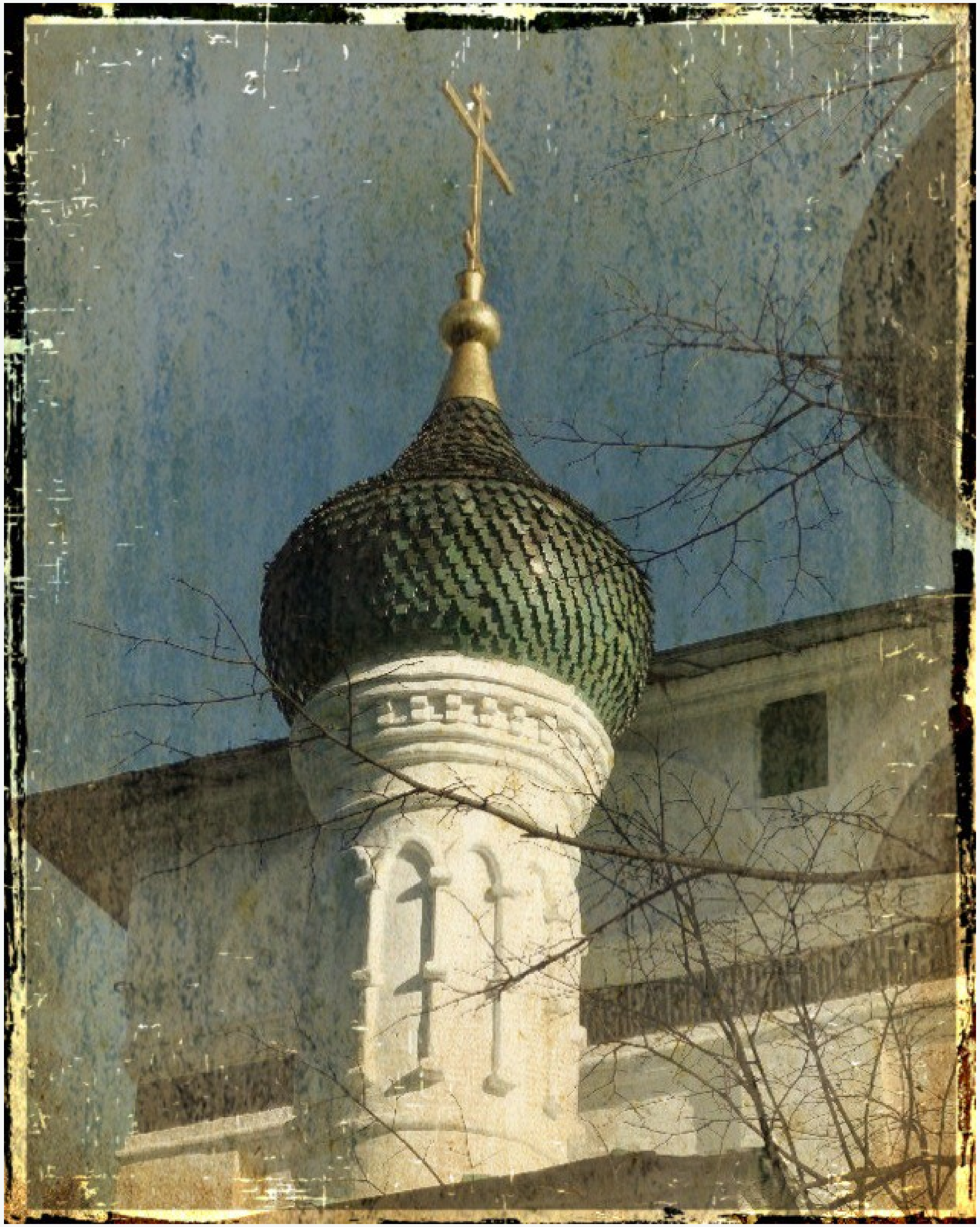

Schwerpunkt eines Rotationskörpers: Aufgaben

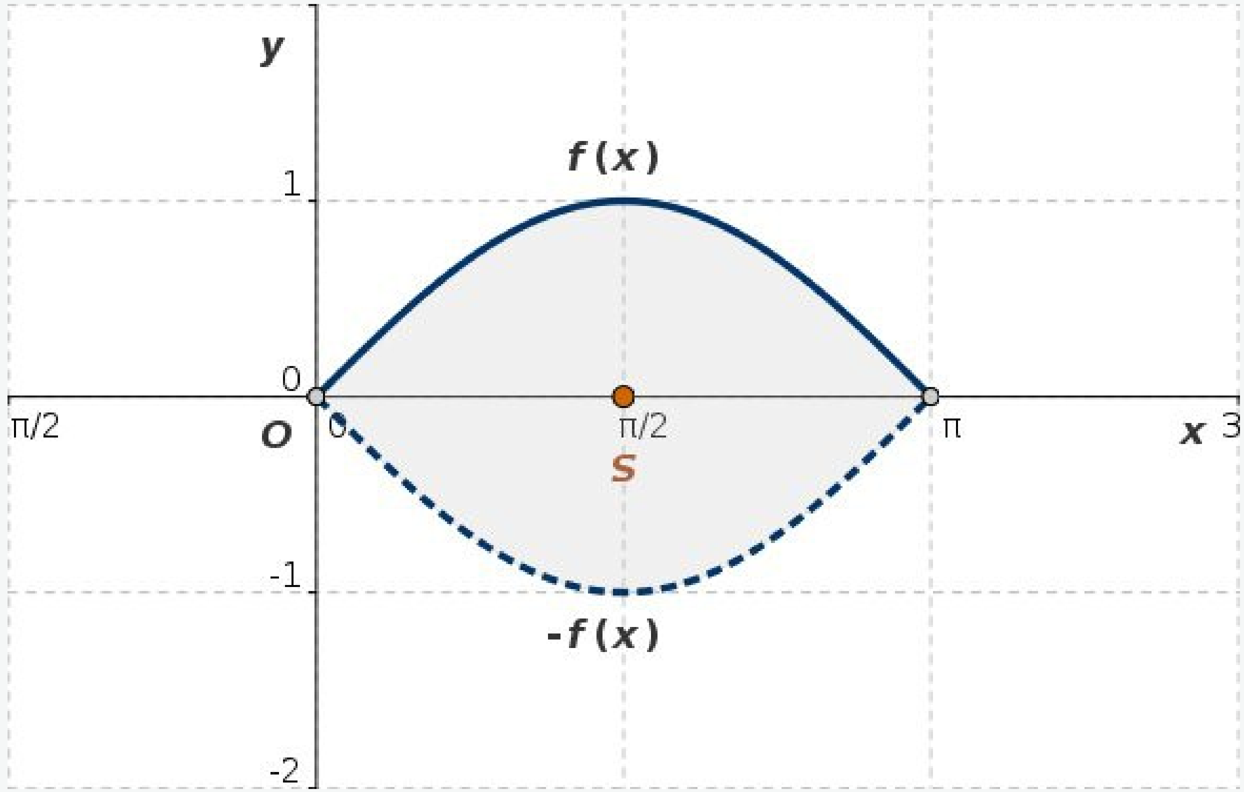

Abb. 13a-1: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$y = \sin x$$

$$V_x = \pi \cdot \int_0^\pi y^2 \, dx = \pi \cdot \int_0^\pi \sin^2 x \, dx = \frac{\pi}{2} \left[\int_0^\pi dx - \int_0^\pi \cos(2x) \, dx \right] =$$

$$= \frac{\pi}{2} \left[x - \frac{1}{2} \sin(2x) \right]_0^\pi = \frac{\pi^2}{2} \; (\mathrm{VE})$$

$$x_S = \frac{\pi}{V_x} \int_0^{\pi} x\, y^2\, dx = \frac{2}{\pi} \int_0^{\pi} x \sin^2 x \; dx = \frac{1}{\pi} \int_0^{\pi} x\,(1 - \cos(2x)) \; dx =$$

$$= \frac{1}{\pi} \int_0^{\pi} x\, dx = \frac{\pi}{2} \; (\mathrm{LE})$$

$$\vec{r}_S = \left(\frac{\pi}{2}, \; 0, \; 0 \right)$$

$$\int_0^{\pi} x \; \cos(2x) \; dx = \frac{1}{4} \int_0^{2\pi} u \; \cos u \; du = \frac{1}{4} \left[\cos u + u \; \sin u \right]_0^{2\pi} = 0$$

$$0 \leqslant x \leqslant \pi \,, \qquad 0 \leqslant u = 2x \leqslant 2\pi$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

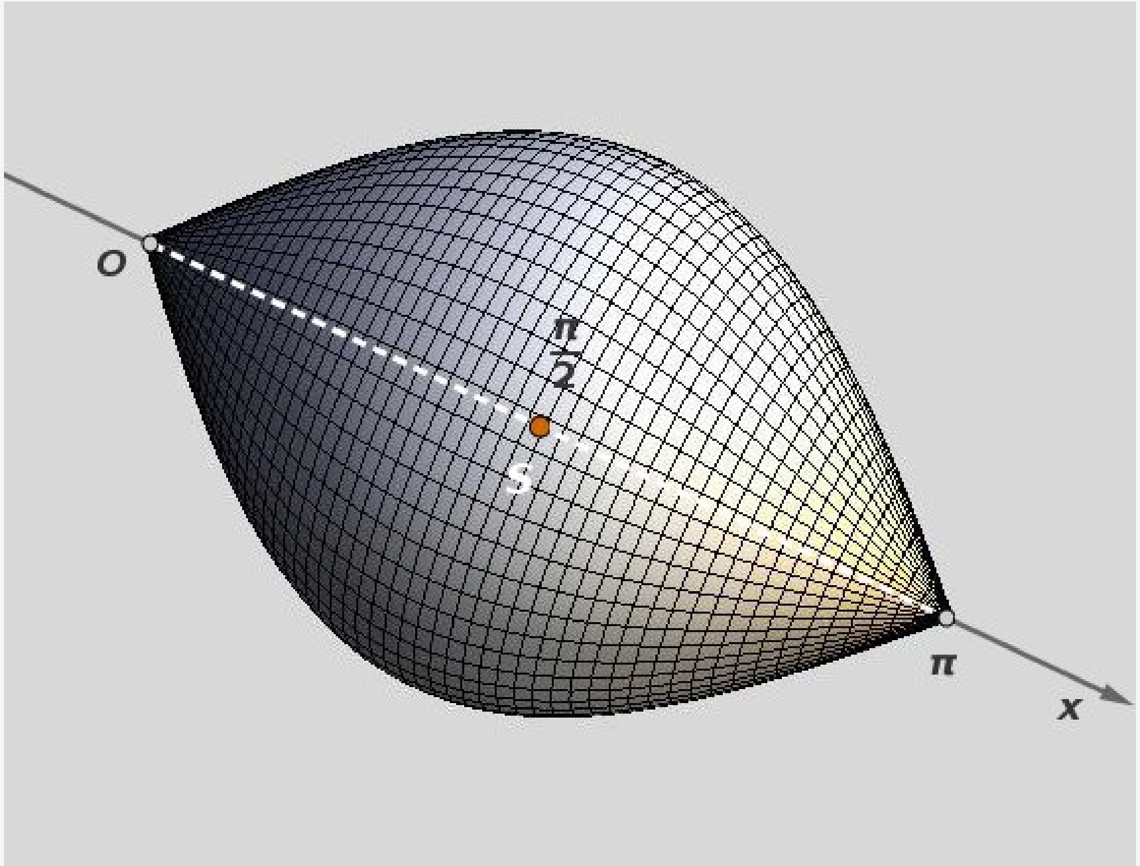

Abb. 13a-2: Durch Rotation des Kurvenstücks $y = f(x)$ ($\varphi = 2\pi$) um die x-Achse erzeugter Körper

$$f(x) = \sin x\,, \qquad x \in [0,\ \pi]$$

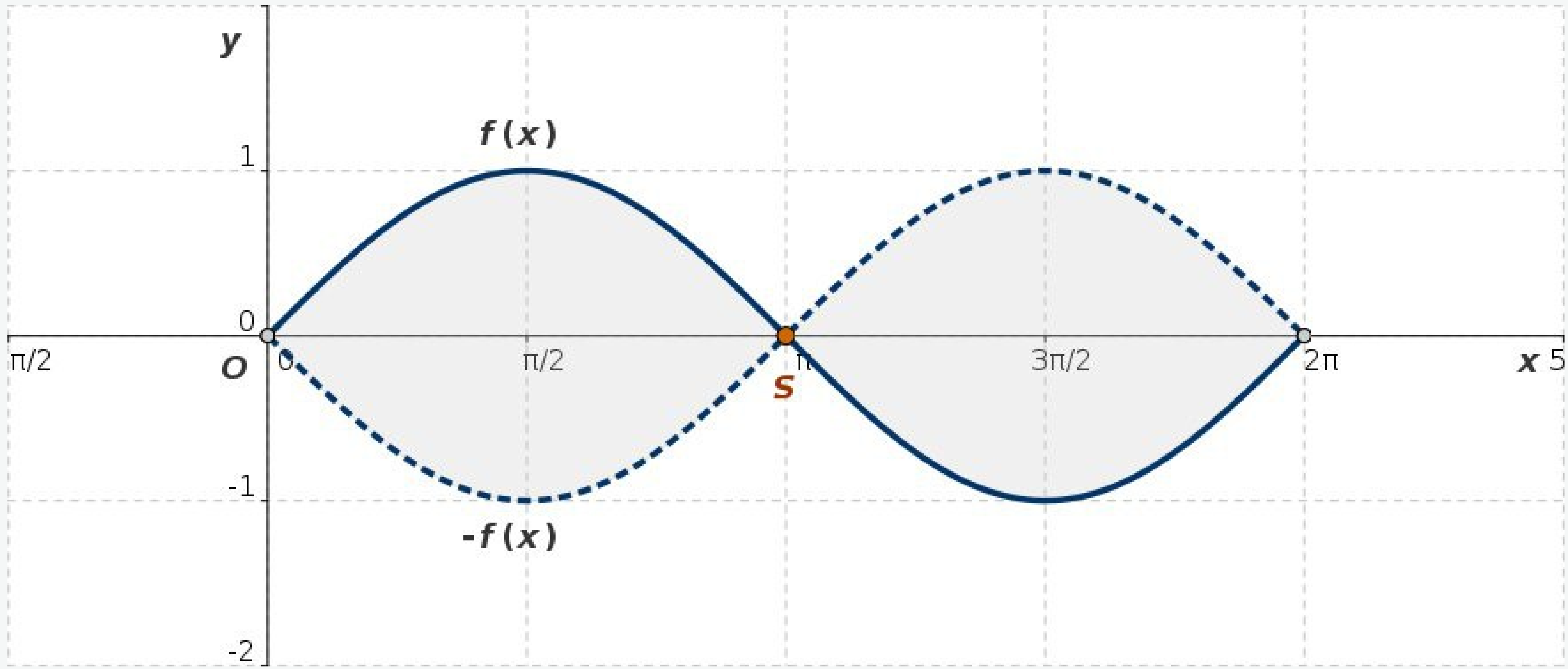

Abb. 13b-1: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$y = \sin x\,, \qquad V_x = \pi \int_0^{2\pi} y^2\, dx = \pi \int_0^{2\pi} \sin^2 x\, dx = \frac{\pi}{2}\left[\, x - \cos x \sin x\,\right]_0^{2\pi} = \pi^2 \ (\mathrm{VE})$$

$$x_S = \frac{\pi}{V_x} \int_0^{2\pi} x\, y^2\, dx = \frac{2}{\pi} \int_0^{2\pi} x \sin^2 x\, dx = -\frac{1}{2\pi}\left[2\, x \sin x \cos x - x^2 + \cos^2 x\right]_0^{2\pi} =$$

$$= \pi \ (\mathrm{LE})$$

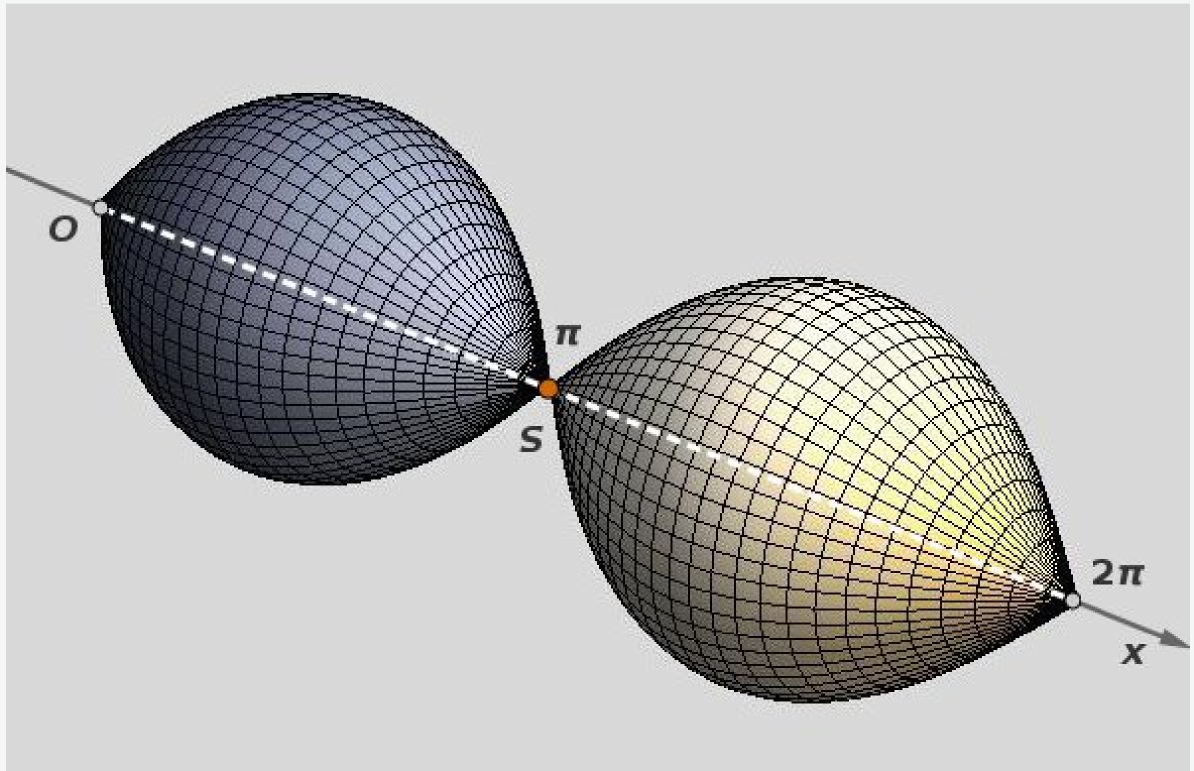

Abb. 13b-2: Durch Rotation des Kurvenstücks $y = f(x)$ $(\varphi = 2\pi)$ um die x-Achse erzeugter Körper

$$f(x) = \sin x\,, \qquad x \in [0,\ 2\pi]$$

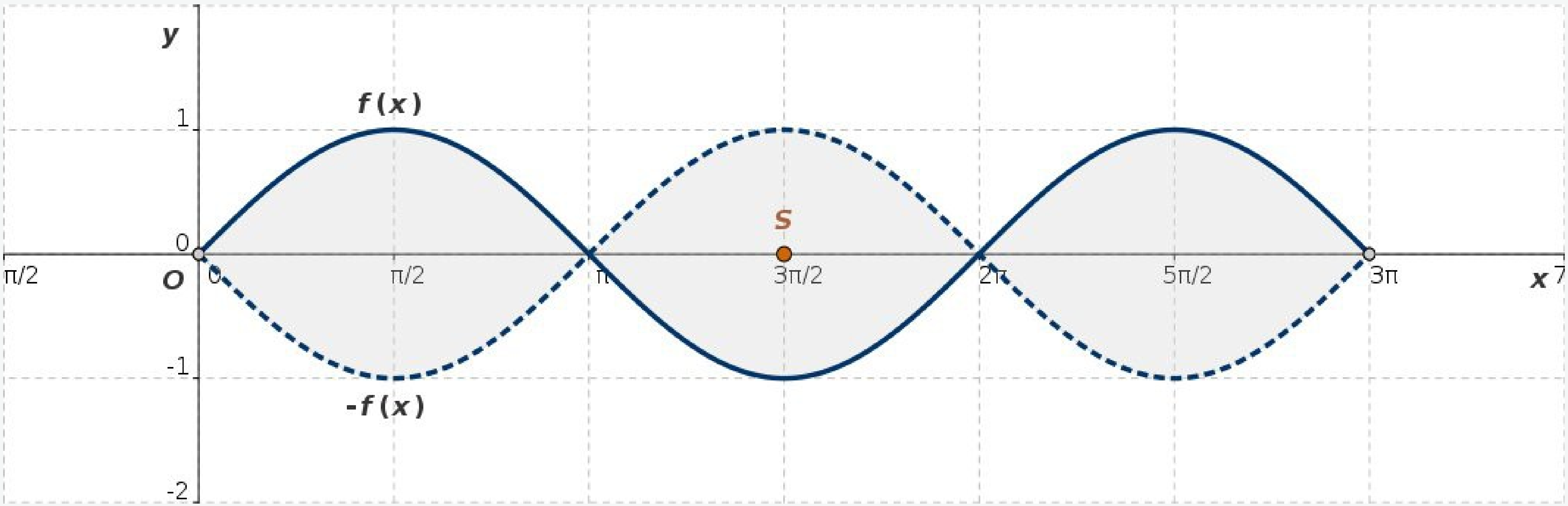

Abb. 13c-1: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$y = \sin x, \qquad V_x = \pi \int_0^{3\pi} y^2\, dx = \pi \int_0^{3\pi} \sin^2 x\, dx = \frac{3}{2}\pi^2 \text{ (VE)}$$

$$x_S = \frac{\pi}{V_x} \int_0^{3\pi} x\, y^2\, dx = \frac{2}{3\pi} \int_0^{3\pi} x \sin^2 x\, dx = \frac{3}{2}\pi \text{ (LE)}$$

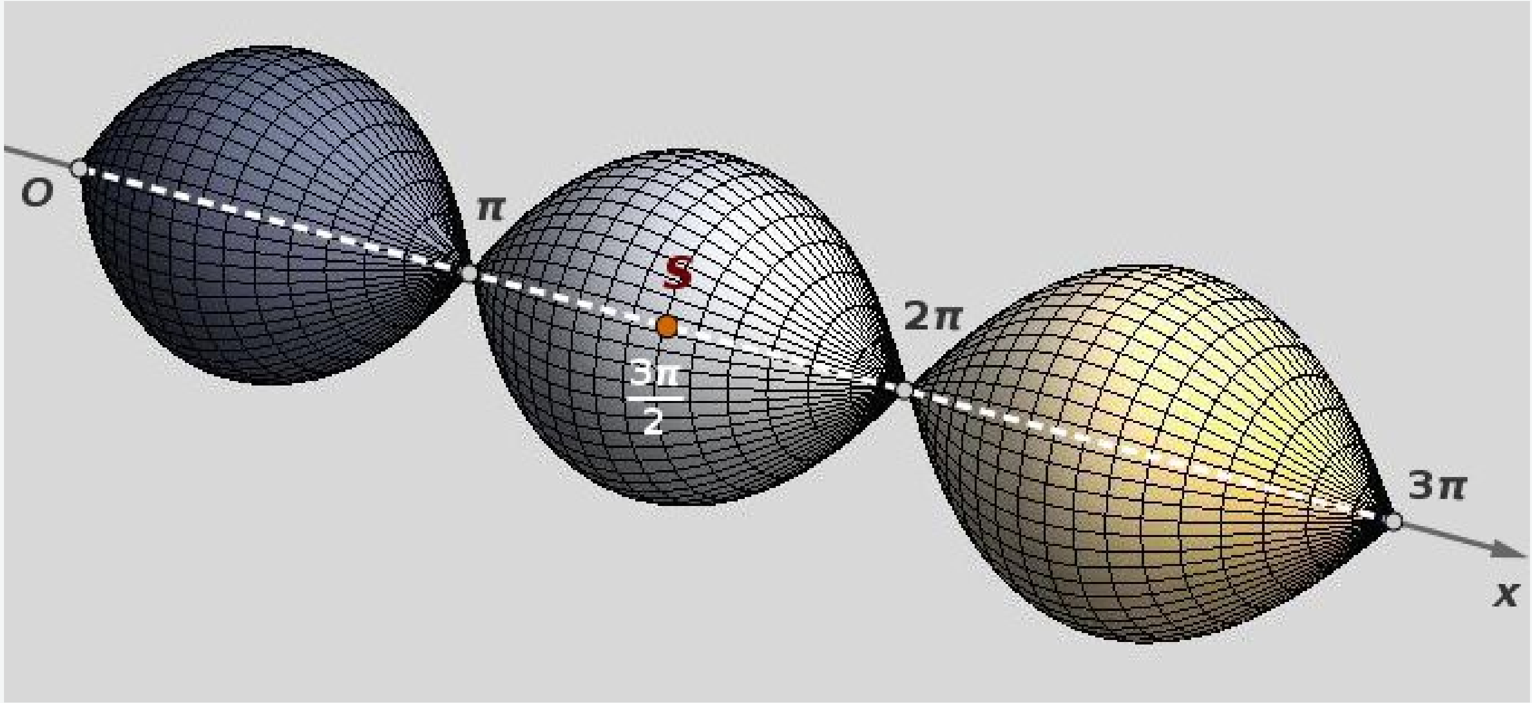

Abb. 13c-2: Durch Rotation des Kurvenstücks $y = f(x)$ $(\varphi = 2\pi)$ um die x-Achse erzeugter Körper

$$f(x) = \sin x\,, \qquad x \in [0,\ 3\pi]$$

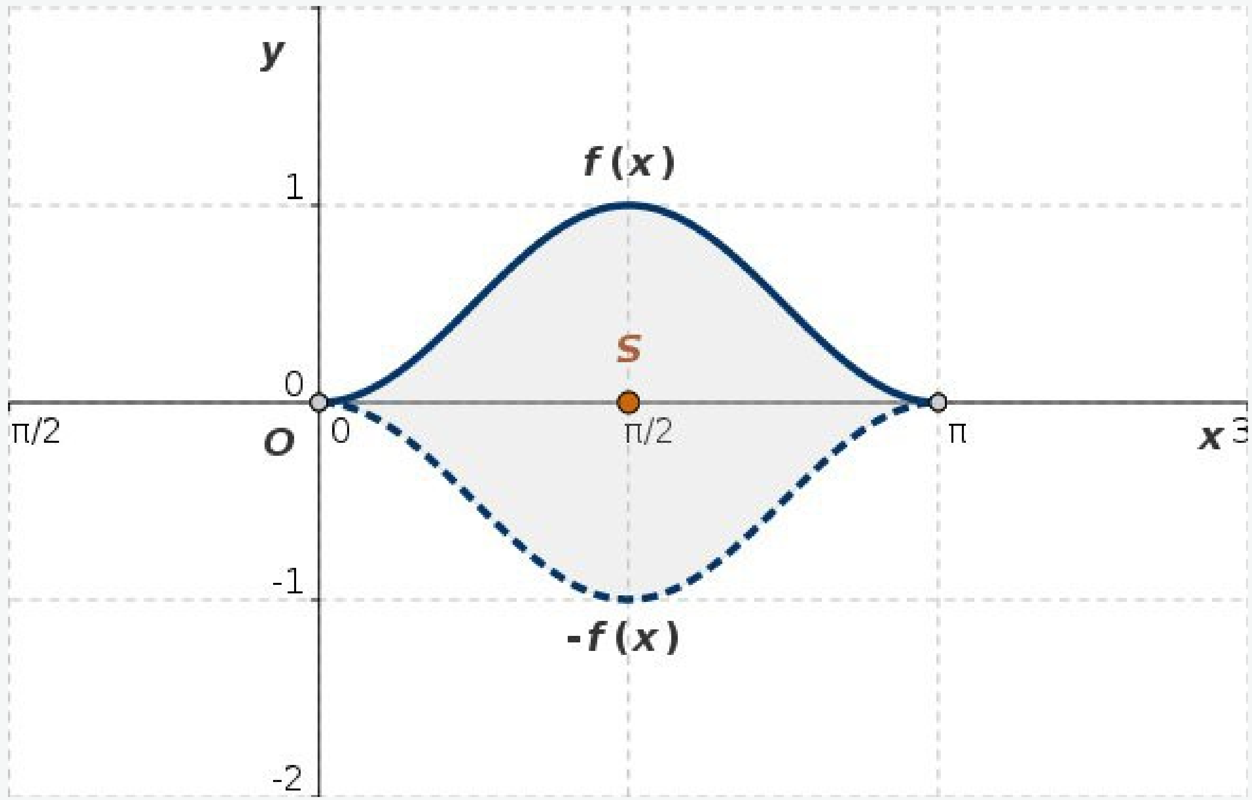

Abb. 14a-1: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$y = \sin^2 x\,, \qquad V_x = \pi \int_0^{\pi} y^2\, dx = -\frac{\pi}{8}\left[2\sin^3 x \cos x + 3\cos x \sin x - 3\,x\right]_0^{\pi} =$$

$$= \frac{3}{8}\pi^2 \ (\text{VE})$$

$$x_S = \frac{\pi}{V_x}\int_0^{\pi} x\, y^2\, dx = \frac{8}{3\,\pi}\int_0^{\pi} x\ \sin^4 x\ dx = \frac{\pi}{2}\ (\text{LE})$$

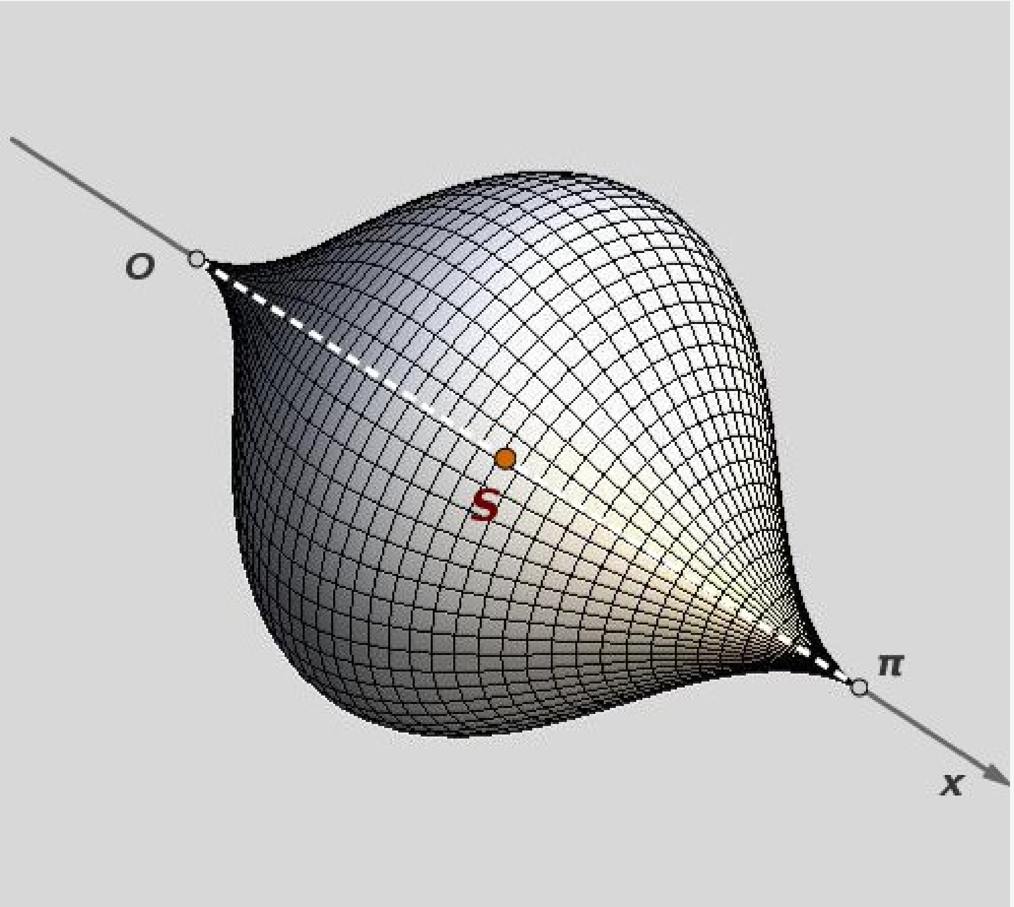

Abb. 14a-2: Durch Rotation des Kurvenstücks $y = f(x)$ $(\varphi = 2\pi)$ um die x-Achse erzeugter Körper

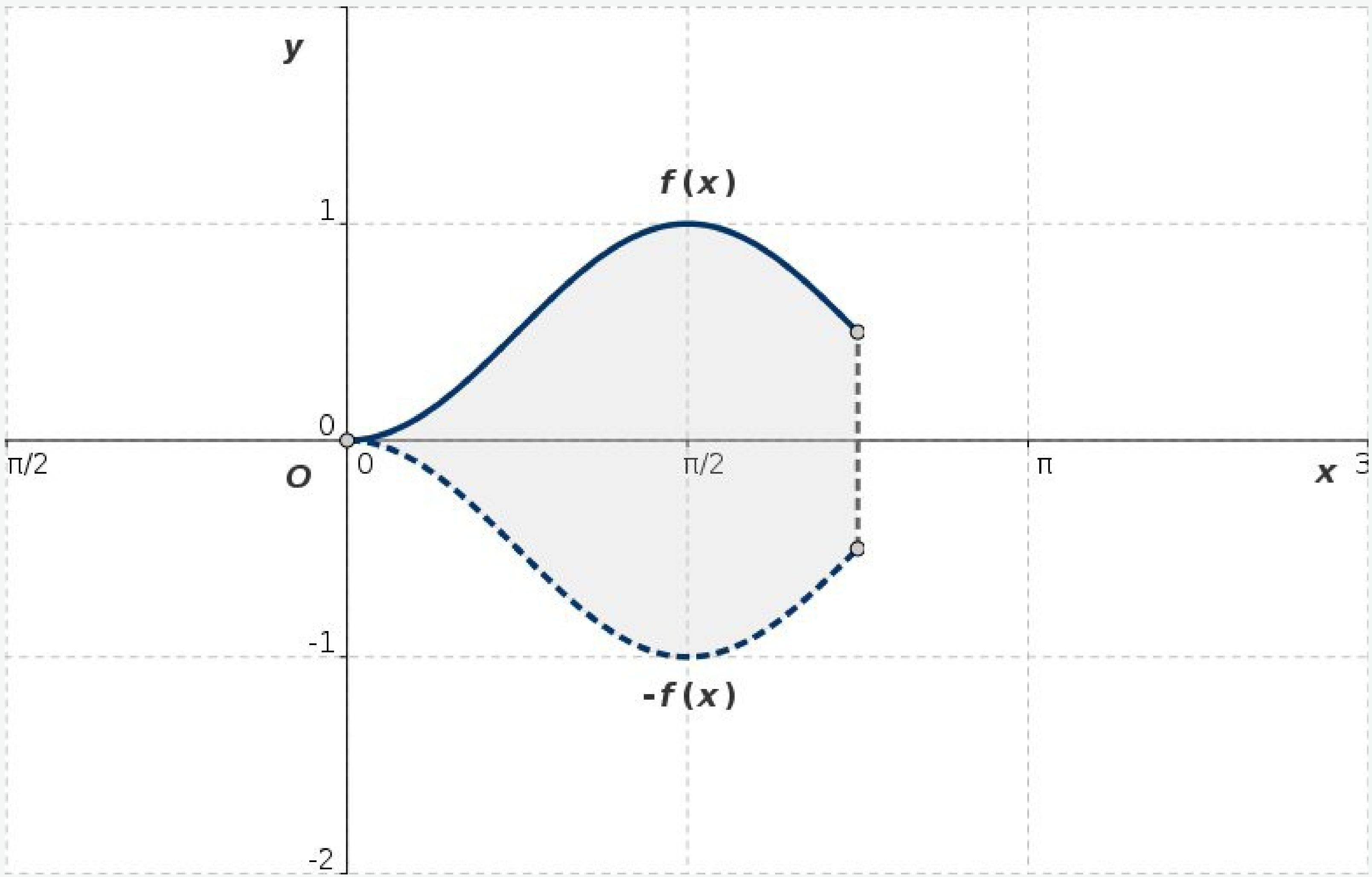

Abb. 14b-1: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$y = \sin^2 x\,, \qquad V_x = \pi \int_0^{3\pi/4} y^2\, dx = -\frac{\pi}{8}\left[2\sin^3 x \cos x + 3\cos x \sin x - 3x\right]_0^{3\pi/4} =$$

$$= \frac{\pi}{32}\,(8 + 9\pi)\ (\text{VE})$$

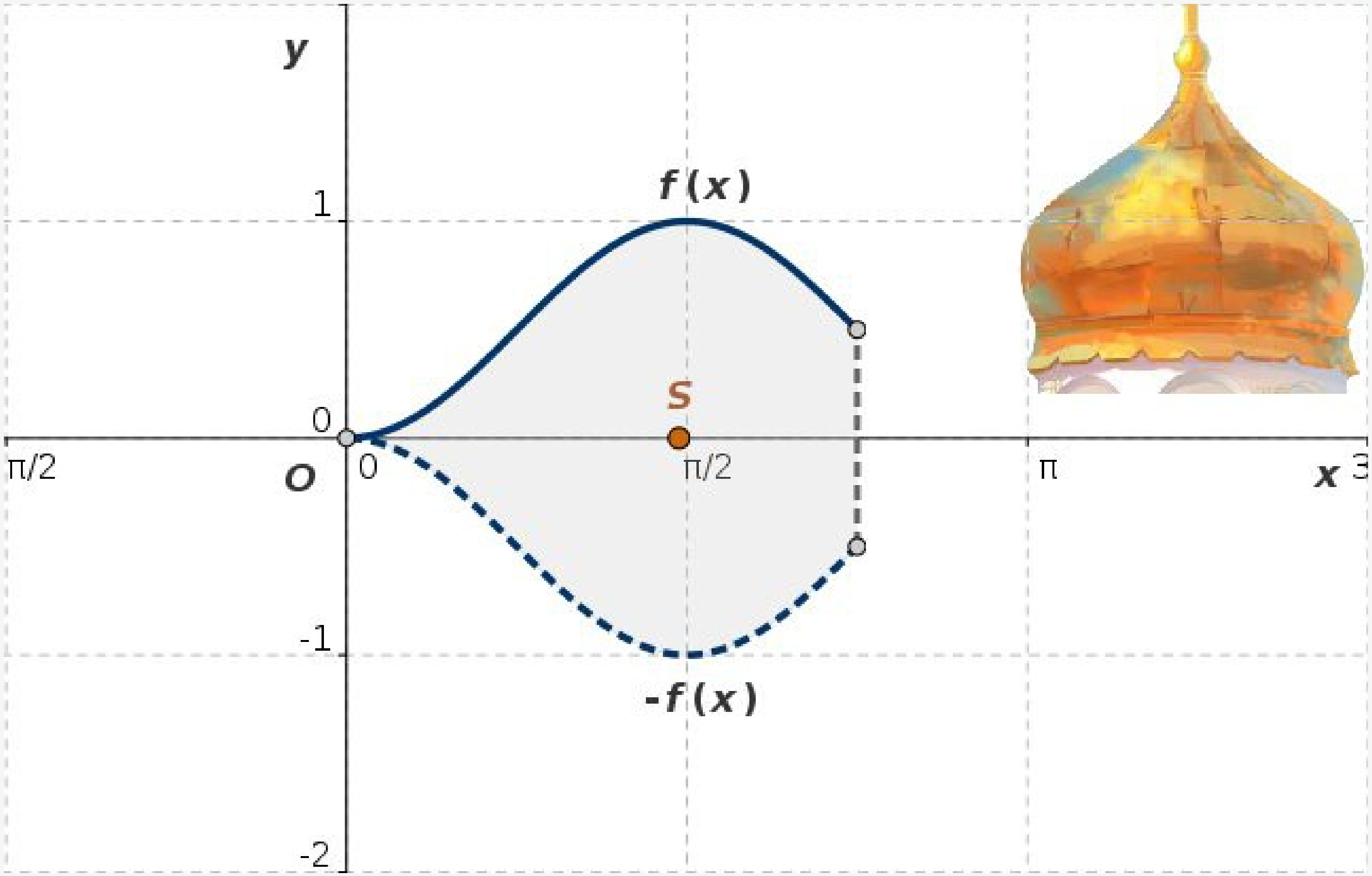

Abb. 14b-2: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$x_S = \frac{\pi}{V_x} \int_0^{3\pi/4} x\, y^2\, dx = \frac{32}{8 + 9\pi} \int_0^{3\pi/4} x \sin^4 x\, dx =$$

$$= \frac{1}{8} \, \frac{28 + 48\pi + 27\pi^2}{8 + 9\pi} \simeq 1.53 \ (\text{LE})$$

Abb. 14b-3: Durch Rotation des Kurvenstücks $y = f(x)$ $(\varphi = 2\pi)$ um die x-Achse erzeugter Körper

Abb. 14b-4: Durch Rotation des Kurvenstücks $y = f(x)$ ($\varphi = 2\pi$) um die x-Achse erzeugter Körper

$$f(x) = \sin^2 x, \qquad x \in \left[0, \ \frac{2\pi}{3}\right]$$

http://www.ellada-russia.org/images/Russian_Orthodox_Church_by_Steve_Jump2004.jpg

Abb. 14b-5: Durch Rotation des Kurvenstücks $y = f(x)$ $(\varphi = 2\pi)$ um die x-Achse erzeugter Körper

$$f(x) = \sin^2 x, \qquad x \in \left[0, \ \frac{5\pi}{8}\right]$$

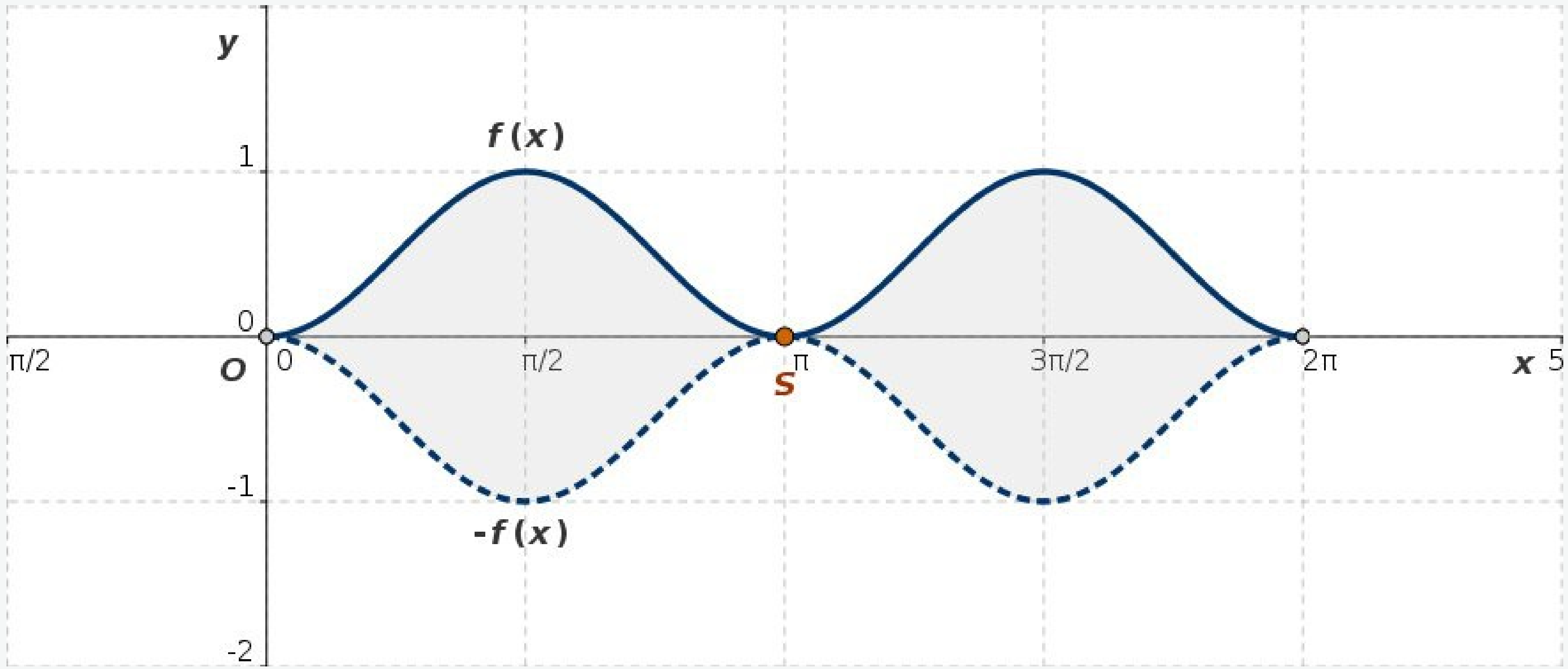

Abb. 14c-1: Die Schnittfläche des Rotationskörpers mit der x,y-Ebene

$$y = \sin^2 x$$

$$V_x = \pi \int_0^{2\pi} y^2 \, dx = -\frac{\pi}{8} \left[2 \sin^3 x \cos x + 3 \cos x \sin x - 3x \right]_0^{2\pi} =$$

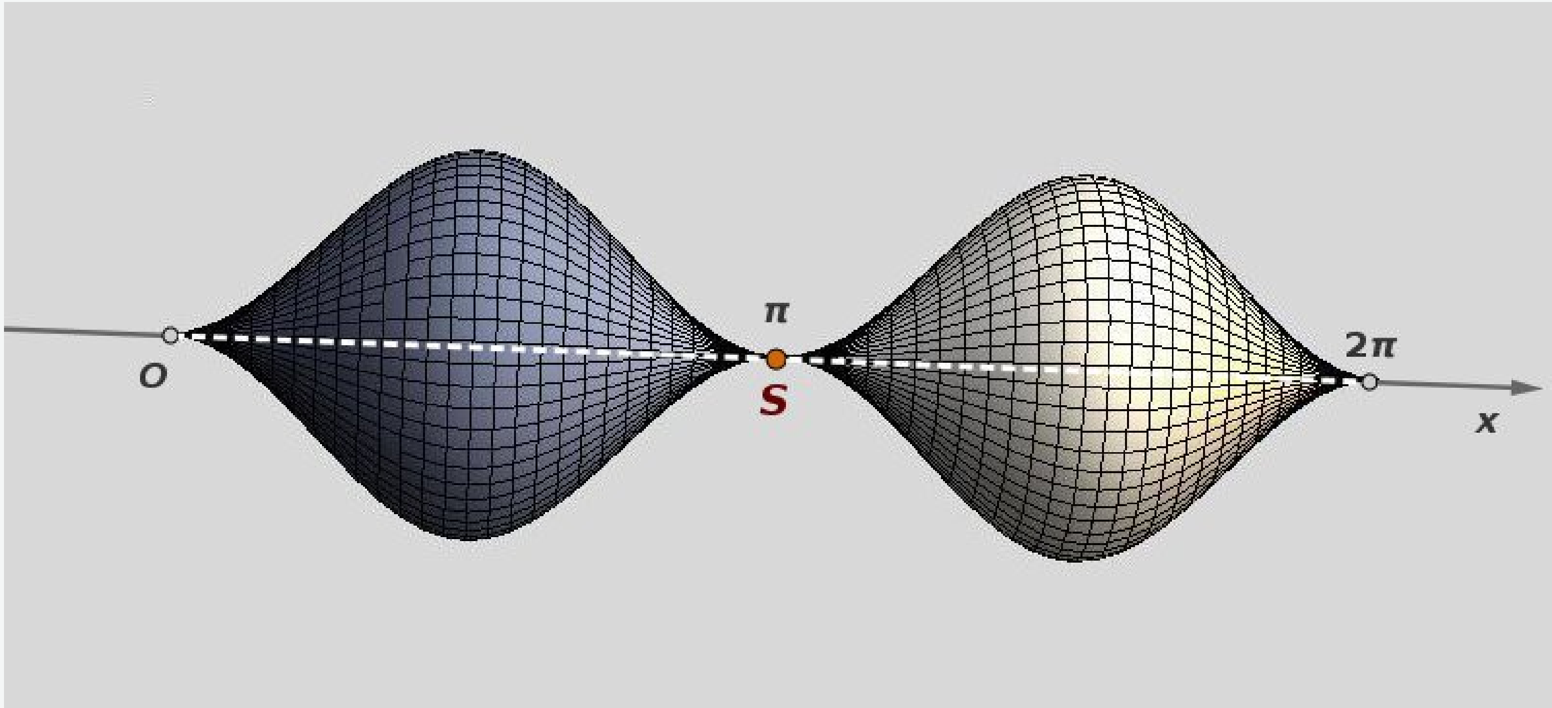

Abb. 14c-2: Durch Rotation des Kurvenstücks $y = f(x)$ ($\varphi = 2\pi$) um die x-Achse erzeugter Körper

$$x_S = \frac{\pi}{V_x} \int_0^{2\pi} x\, y^2\, dx = \frac{8}{3\pi} \int_0^{2\pi} x \sin^4 x \, dx = \pi \ (\text{LE})$$

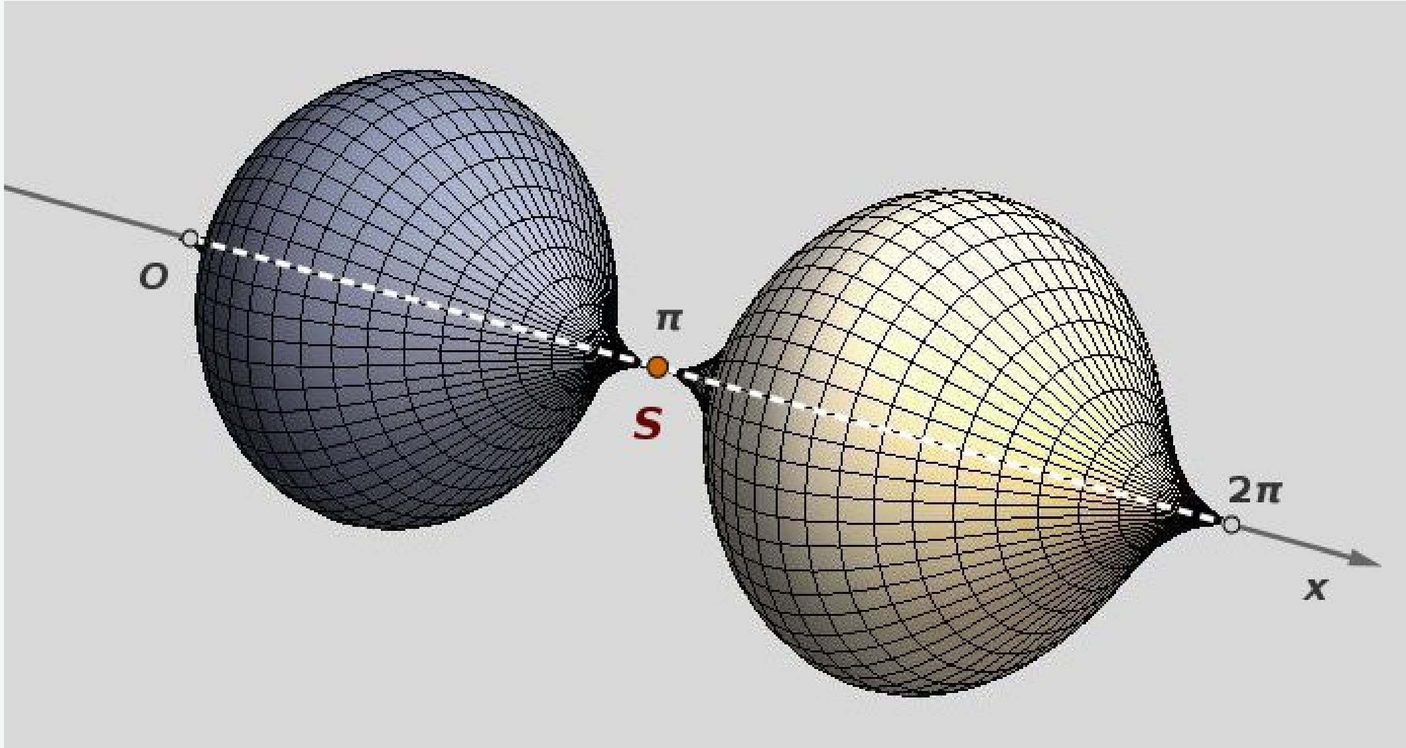

Abb. 14c-3: Durch Rotation des Kurvenstücks $y = f(x)$ ($\varphi = 2\pi$) um die x-Achse erzeugter Körper