

Berechnen Sie folgende Integrale:

$$a) \int x^2 \cos(2x^3 - 6) dx, \quad \int (x^2 - 1) \sin(x^3 - 3x) dx$$

$$b) \int \frac{1}{x^2} \sin\left(\frac{1}{x}\right), \quad \int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

$$c) \int \frac{\cos^2(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx, \quad \int \frac{\sin^2(\sqrt{x}) \cos(\sqrt{x})}{\sqrt{x}} dx$$

Die Integration durch Substitution: Lösung 5a

The chalkboard shows two integration problems solved using substitution. The general formula $\int f(g(x)) \cdot g'(x) dx$ is written at the top, with $u = g(x)$ to its right.

Problem 1:
$$\int x^2 \cos(2x^3 - 6) dx =$$

The solution shows the substitution $u = 2x^3 - 6$ and $\frac{du}{dx} = 2 \cdot 3 \cdot x^2 = 6x^2$, leading to $dx = \frac{du}{6x^2}$. The integral is then transformed to $\int \cancel{x^2} \cos(u) \cdot \frac{du}{\cancel{6x^2}} = \frac{1}{6} \int \cos(u) du = \frac{1}{6} \sin(u) + C = \frac{1}{6} \sin(2x^3 - 6) + C$.

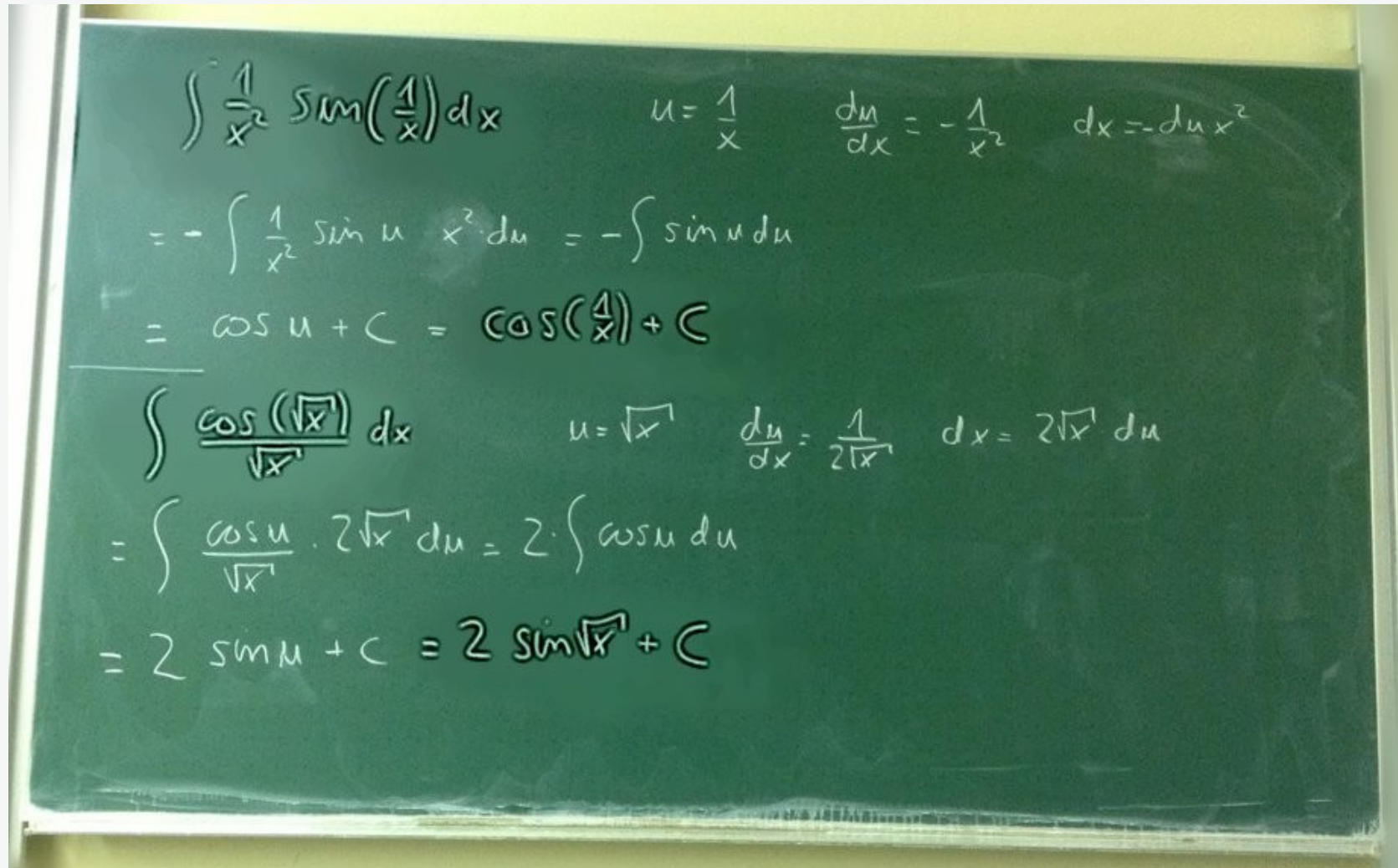
Problem 2:
$$\int (x^2 - 1) \sin(x^3 - 3x) dx =$$

The solution shows the substitution $u = x^3 - 3x$ and $\frac{du}{dx} = 3x^2 - 3 = 3(x^2 - 1)$, leading to $dx = \frac{du}{3(x^2 - 1)}$. The integral is then transformed to $\int \cancel{(x^2 - 1)} \sin(u) \cdot \frac{du}{\cancel{3(x^2 - 1)}} = \frac{1}{3} \int \sin(u) du = -\frac{1}{3} \cos(u) + C = -\frac{1}{3} \cos(x^3 - 3x) + C$.

$$\int x^2 \cos(2x^3 - 6) dx = \frac{1}{6} \sin(2x^3 - 6) + C$$

$$\int (x^2 - 1) \sin(x^3 - 3x) dx = -\frac{1}{3} \cos(x^3 - 3x) + C$$

Die Integration durch Substitution: Lösung 5b



$$\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx = \cos\left(\frac{1}{x}\right) + C$$

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = 2 \sin(\sqrt{x}) + C$$

1 Variante:

$$\int \frac{\cos^2(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx, \quad u = \cos(\sqrt{x})$$

$$\frac{du}{dx} = -\frac{\sin(\sqrt{x})}{2\sqrt{x}}, \quad \frac{\sin(\sqrt{x})}{\sqrt{x}} dx = -2 du, \quad \cos^2(\sqrt{x}) = u^2$$

$$\int \frac{\cos^2(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx = -2 \int u^2 du = -2 \frac{u^3}{3} + C = -\frac{2}{3} \cos^3(\sqrt{x}) + C$$

Die Integration durch Substitution: Lösung 5c

2 Variante:

The chalkboard shows the following steps for the integration:

$$\int \frac{\cos^2(\sqrt{x}) \cdot \sin(\sqrt{x})}{\sqrt{x}} dx \quad u = \sqrt{x} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad dx = 2\sqrt{x} \cdot du$$
$$= \int \frac{\cos^2 u \cdot \sin u \cdot 2\sqrt{x}}{\sqrt{x}} du$$
$$= 2 \int \cos^2 u \cdot \sin u \, du \quad z = \cos u \quad \frac{dz}{du} = -\sin u$$
$$= -2 \int z^2 \cdot \sin u \frac{dz}{\sin u} = -2 \int z^2 \, dz \quad du = -\frac{dz}{\sin u}$$
$$= -\frac{2}{3} z^3 + C = -\frac{2}{3} \cos^3 u + C = -\frac{2}{3} \cos^3(\sqrt{x}) + C$$

$$\int \frac{\cos^2(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx = -\frac{2}{3} \cos^3(\sqrt{x}) + C, \quad u = \sqrt{x}$$

$$\int \frac{\sin^2(\sqrt{x}) \cos(\sqrt{x})}{\sqrt{x}} dx = \frac{2}{3} \sin^3(\sqrt{x}) + C$$

Die Integration durch Substitution: Aufgabe 6

$$a) \int \frac{\cos x}{1 + \sin^2 x} dx, \quad \int \frac{\sin x}{1 + \cos^2 x} dx, \quad \int \frac{\cos x}{9 + \sin^2 x} dx$$

$$b) \int \frac{\sin(2x - 3)}{\cos^2(2x - 3)} dx, \quad \int \frac{\cos(2x)}{4 + \sin^2(2x)} dx, \quad \int \frac{\sin(3x - 2)}{6 + 2 \cos^2(3x - 2)} dx$$

$$c) \int \frac{\sin x}{\sqrt{\cos x}} dx, \quad \int \frac{\sin x}{\sqrt{4 - \cos x}} dx, \quad \int \frac{\cos x}{\sqrt{9 - 3 \sin x}} dx$$



