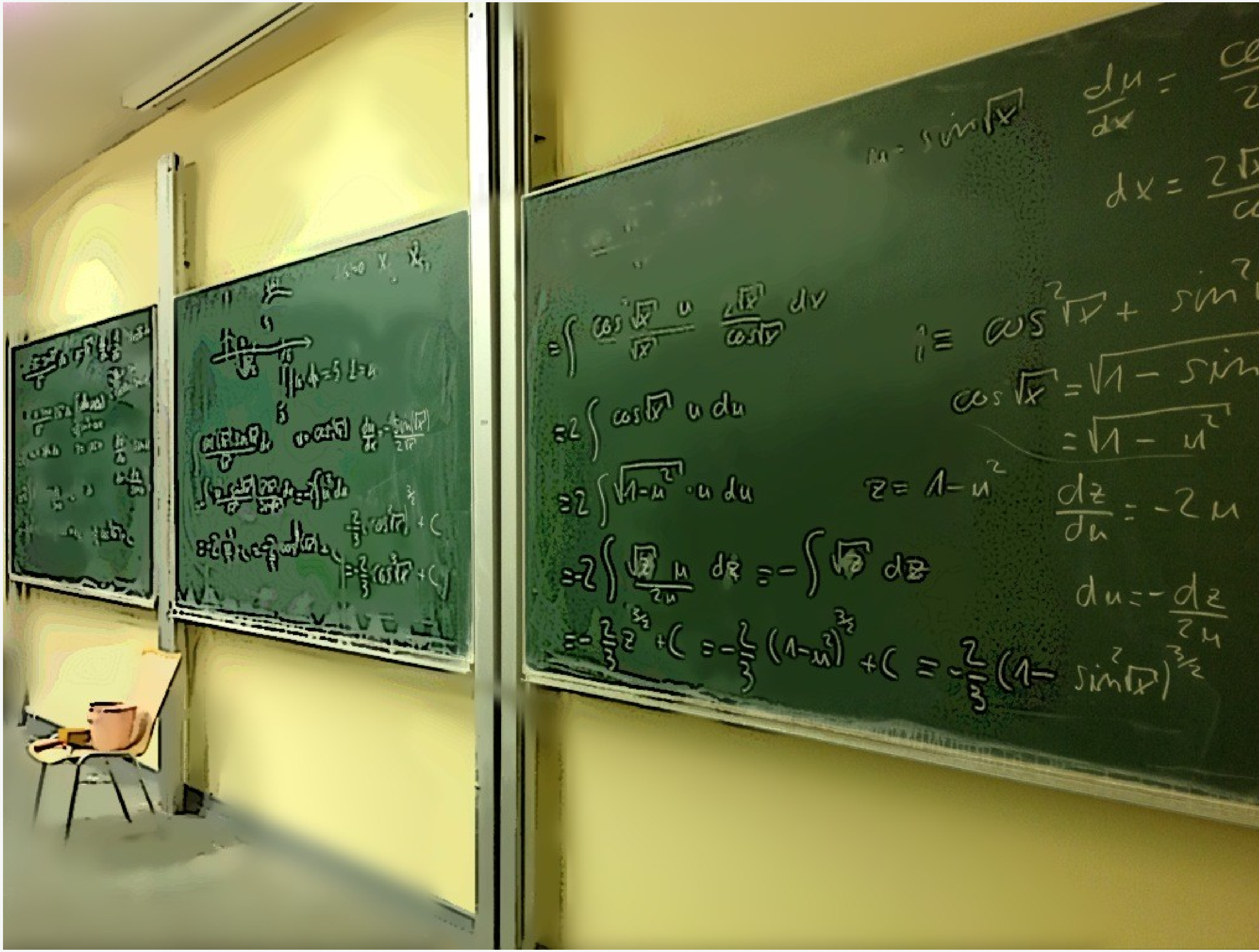


Die Integration durch Substitution

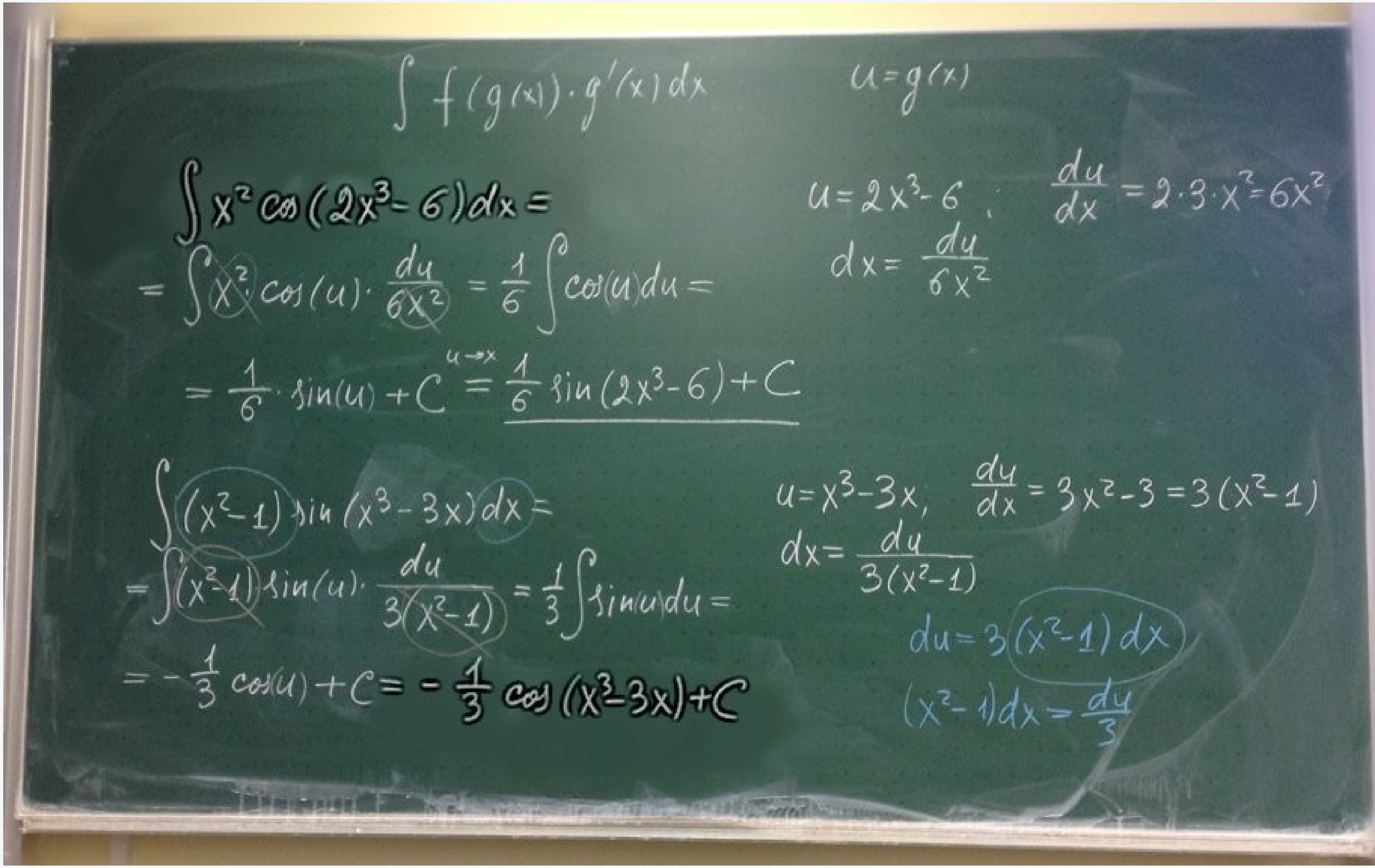
Berechnen Sie folgende Integrale:

$a)\ \int x^2 \cos(2x^3 - 6)\, dx, \qquad \int (x^2 - 1)\sin(x^3 - 3x)\, dx$

$b)\ \int \frac{1}{x^2} \sin\left(\frac{1}{x}\right), \qquad \int \frac{\cos(\sqrt{x})}{\sqrt{x}}\, dx$

$c)\ \int \frac{\cos^2(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\, dx, \qquad \int \frac{\sin^2(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}\, dx$

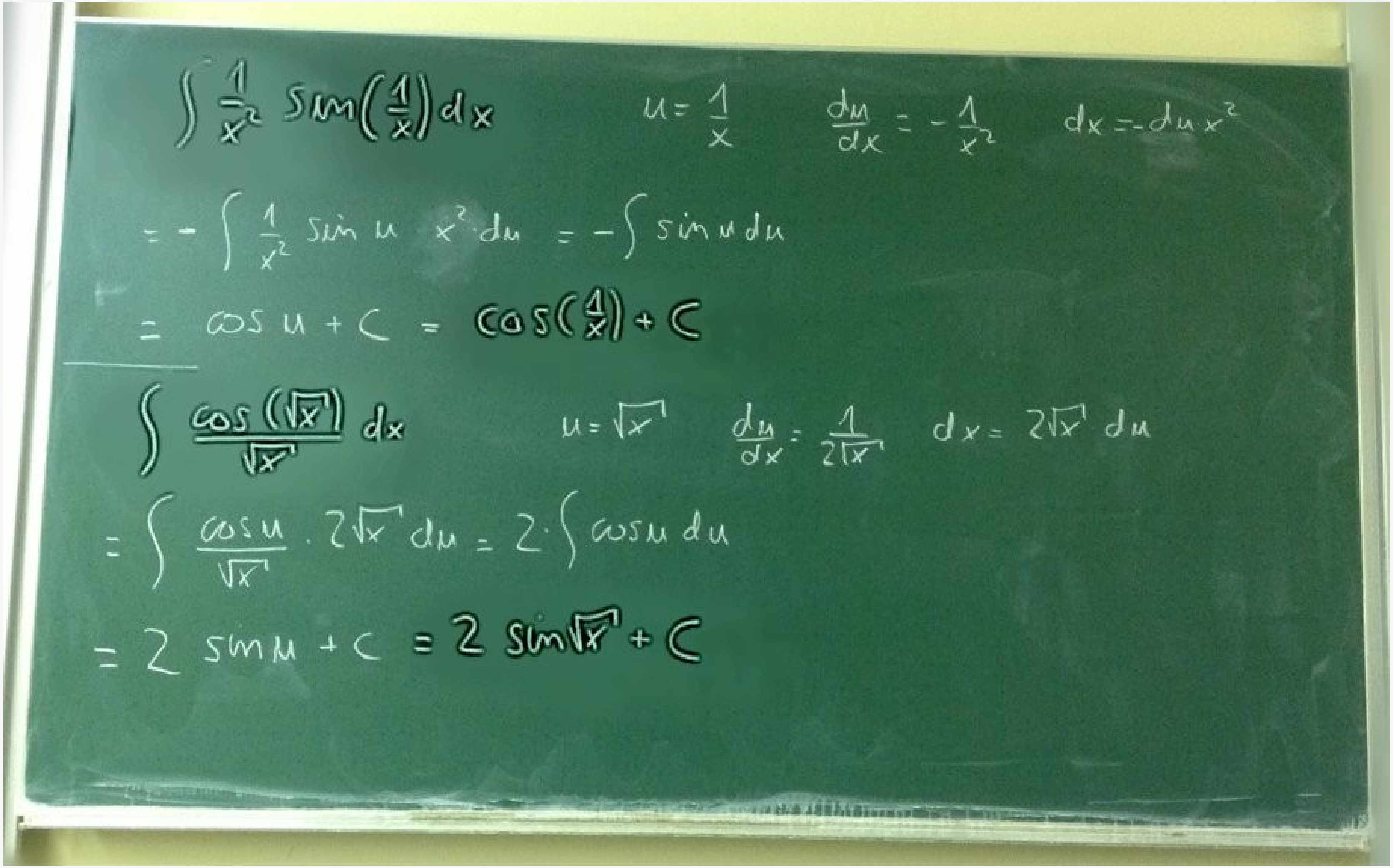
Die Integration durch Substitution: Lösung 5a



$$\int x^2 \cos(2x^3 - 6)\, dx = \frac{1}{6} \sin(2x^3 - 6) + C$$

$$\int (x^2 - 1) \sin(x^3 - 3x)\, dx = -\frac{1}{3} \cos(x^3 - 3x) + C$$

Die Integration durch Substitution: Lösung 5b



$$\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) = \cos\left(\frac{1}{x}\right) + C$$

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\, dx = 2 \sin\left(\sqrt{x}\right) + C$$

1 Variante:

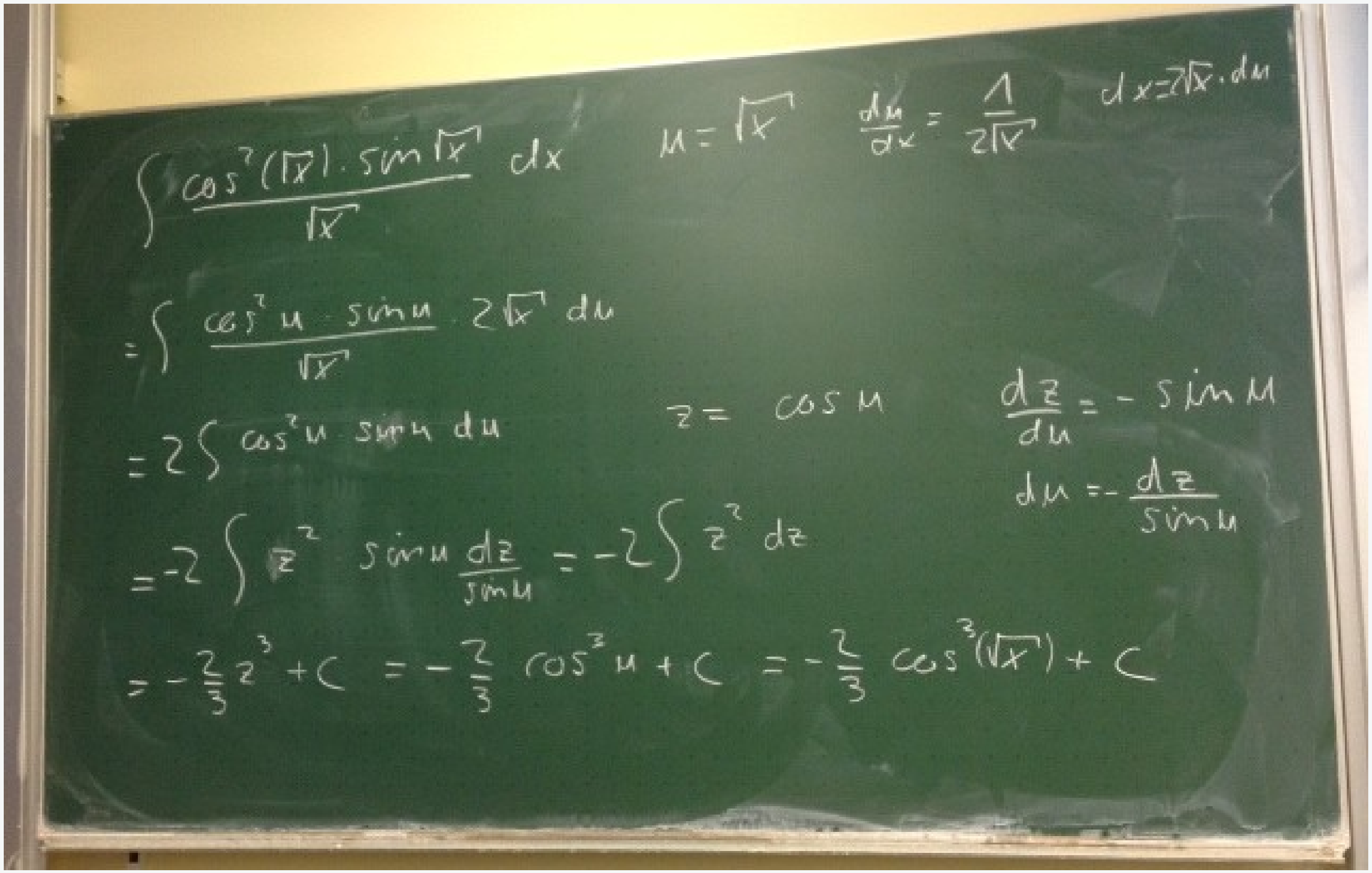
$$\int \frac{\cos^2(\sqrt{x}\,)\sin(\sqrt{x}\,)}{\sqrt{x}}\,dx\,, \qquad u = \cos(\sqrt{x}\,)$$

$$\frac{du}{dx} = -\frac{\sin(\sqrt{x}\,)}{2\sqrt{x}}\,, \qquad \frac{\sin(\sqrt{x}\,)}{\sqrt{x}}\,dx = -2\,du\,, \qquad \cos^2(\sqrt{x}\,) = u^2$$

$$\int \frac{\cos^2(\sqrt{x}\,)\sin(\sqrt{x}\,)}{\sqrt{x}}\,dx = -2\int u^2\,du = -2\,\frac{u^3}{3} + C = -\frac{2}{3}\cos^3(\sqrt{x}\,) + C$$

Die Integration durch Substitution: Lösung 5c

2 Variante:



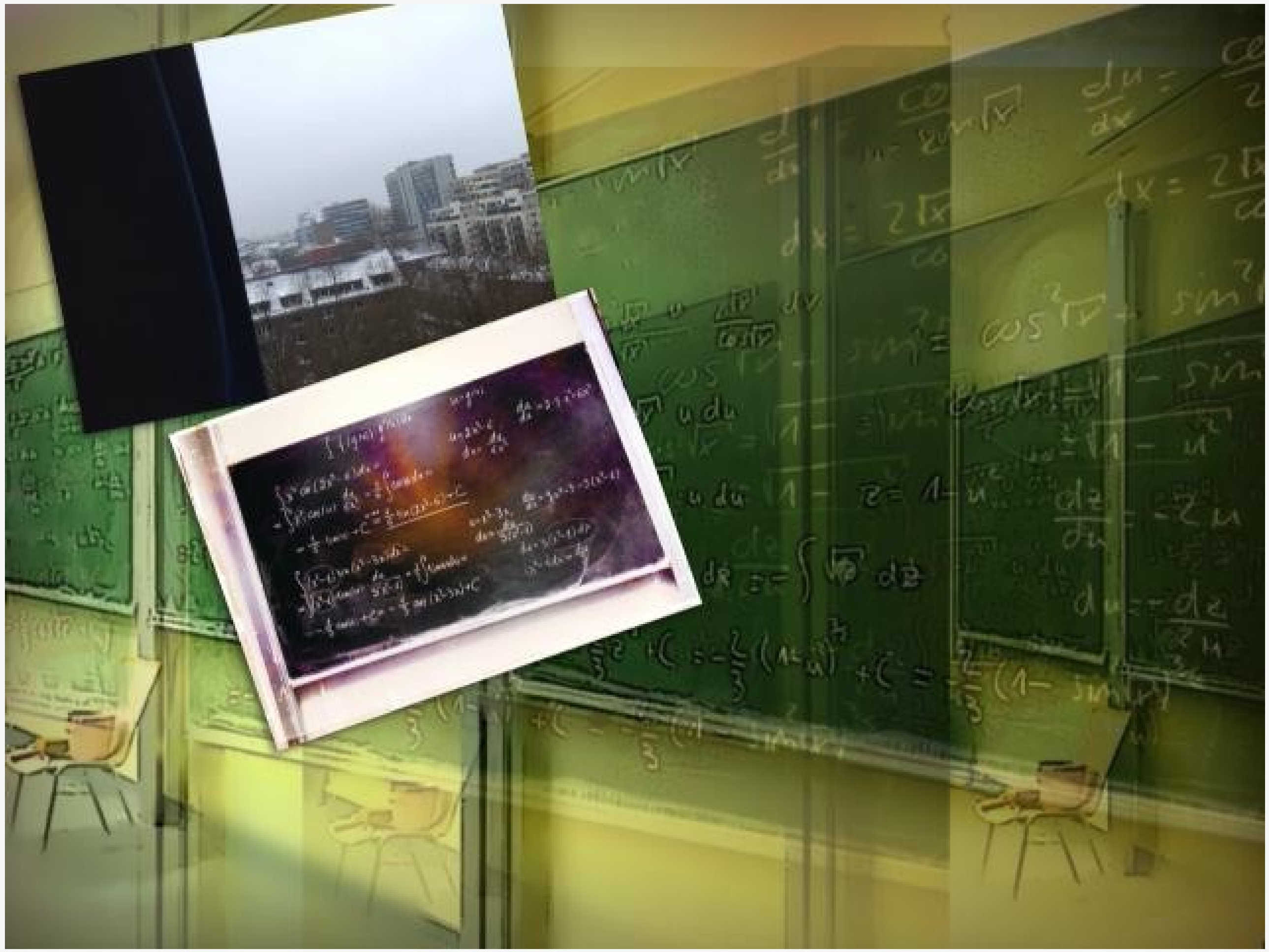
$$\int \frac{\cos^2(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\, dx = -\frac{2}{3}\cos^3(\sqrt{x}) + C\,, \qquad u = \sqrt{x}$$

$$\int \frac{\sin^2(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}\, dx = \frac{2}{3}\sin^3(\sqrt{x}) + C$$

$$a)\ \int \frac{\cos x}{1+\sin^2 x}\,dx, \qquad \int \frac{\sin x}{1+\cos^2 x}\,dx, \qquad \int \frac{\cos x}{9+\sin^2 x}\,dx$$

$$b)\ \int \frac{\sin(2x-3)}{\cos^2(2x-3)}\,dx, \qquad \int \frac{\cos(2x)}{4+\sin^2(2x)}\,dx, \qquad \int \frac{\sin(3x-2)}{6+2\cos^2(3x-2)}\,dx$$

$$c)\ \int \frac{\sin x}{\sqrt{\cos x}}\,dx, \qquad \int \frac{\sin x}{\sqrt{4-\cos x}}\,dx, \qquad \frac{\int \cos x}{\sqrt{9-3\sin x}}\,dx$$



$\int f(g(x))\cdot g'(x)\,dx$ $u=g(x)$
$\int x^2\cos(2x^3-6)\,dx =$ $u=2x^3-6$
$= \int x^2\cos(u)\cdot\frac{du}{6x^2} = \frac{1}{6}\int\cos(u)\,du =$ $dx=\frac{du}{6x^2}$
$= \frac{1}{6}\sin(u)+C = \frac{1}{6}\sin(2x^3-6)+C$
$\int (x^2-1)\sin(x^3-3x)\,dx =$ $u=x^3-3x,$
$= \int (x^2-1)\sin(u)\cdot\frac{du}{3(x^2-1)} = \frac{1}{3}\int\sin(u)\,du =$ $dx=\frac{du}{3(x^2-1)}$
$= -\frac{1}{3}\cos(u)+C = -\frac{1}{3}\cos(x^3-3x)+C$
$\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx$ $u=\frac{1}{x}$ $\frac{du}{dx}=-\frac{1}{x^2}$ $dx=-du\,x^2$
$= -\int \frac{1}{x^2}\sin u\; x^2\,du = -\int \sin u\,du$
$= \cos u + C = \cos\left(\frac{1}{x}\right)+C$
$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$ $u=\sqrt{x}$ $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ $dx=2\sqrt{x}\,du$
$= \int \frac{\cos u}{\sqrt{x}}\cdot 2\sqrt{x}\,du = 2\cdot\int\cos u\,du$
$= 2\sin u + C = 2\sin\sqrt{x}+C$