

$$3) f(x, y, z) = \ln(\sqrt{x} \cdot y^3 \cdot z^2) = \frac{1}{2} \ln x + 3$$
$$\underline{\underline{f_x = \frac{1}{2x}}} \quad \underline{\underline{f_y = \frac{3}{y}}} \quad \underline{\underline{f_z = \frac{2}{z}}}$$
$$4) f(x, y) = e^{\cos x} + y^2$$
$$\underline{\underline{f_x = -\sin x e^{\cos x}}} \quad \underline{\underline{f_y = 2y}}$$

Partielle Ableitungen: Schriftliche Arbeit 1

Bestimmen Sie die partiellen Ableitungen 1. Ordnung der Funktion f

$$1) f(x, y) = (2x + y) e^x$$

$$2) f(x, y, z) = \ln(xy) + e^{z^2}$$

$$3) f(x, y, z) = \ln(\sqrt{x} y^3 z^2)$$

$$4) f(x, y) = e^{\cos x} + y^2$$

$$5) f(x, y) = \cos(x^2 + \sqrt{y})$$

$$1) f(x, y) = (2x + y) e^x$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= e^x \frac{\partial}{\partial x} (2x + y) + (2x + y) \frac{\partial}{\partial x} e^x = 2e^x + (2x + y) e^x = \\ &= (2 + 2x + y) e^x \end{aligned}$$

$$\frac{\partial f}{\partial y} = e^x \frac{\partial}{\partial y} (2x + y) = e^x$$

$$2) f(x, y, z) = \ln(xy) + e^{z^2} = \ln x + \ln y + e^{z^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial f}{\partial y} = \frac{1}{y}, \quad \frac{\partial f}{\partial z} = \frac{\partial}{\partial z} e^{z^2} = e^{z^2} \frac{\partial}{\partial z} (z^2) = 2z e^{z^2}$$

Handwritten mathematical derivation of partial derivatives for the function $f(x, y, z) = \ln(\sqrt{x} \cdot y^3 \cdot z^2)$. The work is shown on a grid background.

$$\textcircled{c} f(x, y, z) = \ln(\sqrt{x} \cdot y^3 \cdot z^2)$$
$$f_x = \frac{1}{\sqrt{x} \cdot y^3 \cdot z^2} \cdot \left(y^3 \cdot z^2 \cdot \frac{1}{2\sqrt{x}} \right) = \frac{y^3 z^2}{2x \cdot y^3 z^2} = \underline{\underline{\frac{y z}{2x}}}$$
$$f_y = \frac{1}{\sqrt{x} \cdot y^3 \cdot z^2} \cdot (\sqrt{x} \cdot 3y^2 \cdot z^2) = \frac{\sqrt{x} \cdot 3y^2 \cdot z^2}{\sqrt{x} \cdot y^3 \cdot z^2} = \underline{\underline{\frac{3}{y}}}$$
$$f_z = \frac{1}{\sqrt{x} \cdot y^3 \cdot z^2} \cdot (2z \cdot \sqrt{x} \cdot y^3) = \frac{2z \cdot \sqrt{x} \cdot y^3}{\sqrt{x} \cdot y^3 \cdot z^2} = \underline{\underline{\frac{2}{z}}}$$

$$3) f(x, y, z) = \ln(\sqrt{x} y^3 z^2) = \frac{1}{2} \ln x + 3 \ln y + 2 \ln z$$

$$\frac{\partial f}{\partial x} = \frac{1}{2x}, \quad \frac{\partial f}{\partial y} = \frac{3}{y}, \quad \frac{\partial f}{\partial z} = \frac{2}{z}$$

$$4) f(x, y) = e^{\cos x} + y^2$$

$$\frac{\partial f}{\partial x} = e^{\cos x} \frac{\partial}{\partial x} \cos x = -\sin x e^{\cos x}$$

$$\frac{\partial f}{\partial y} = 2y$$

$$5) f(x, y) = \cos(x^2 + \sqrt{y})$$

$$\frac{\partial f}{\partial x} = -\sin(x^2 + \sqrt{y}) \frac{\partial}{\partial x} (x^2 + y^{\frac{1}{2}}) = -2x \sin(x^2 + \sqrt{y})$$

$$\frac{\partial f}{\partial y} = -\sin(x^2 + \sqrt{y}) \frac{\partial}{\partial y} (x^2 + y^{\frac{1}{2}}) = -\frac{1}{2\sqrt{y}} \sin(x^2 + \sqrt{y})$$

$f_x = \frac{1}{x}$ ✓
 $f_y = \frac{1}{y}$ ✓
 $f_z = 2xz e^{x^2}$ ✓
 3) $f(x, y, z) = \ln(\sqrt{x^2 - y^2} \cdot z^3 \cdot e^z) = \frac{1}{2}(\ln x + 3 \ln z)$
 $f_x = \frac{1}{2x}$
 $f_y = \frac{3}{y}$
 $f_z = \frac{2}{z}$
 $f_x = \cos x + y$
 $f_y = \cos x$
 $f_z = 2y$