

A handwritten mathematical derivation on a clipboard. The function $f(x, y)$ is given as $(x^2 - \sqrt{y}) \cdot e^{2x}$. The first derivative with respect to x is calculated as $2x \cdot e^{2x} + (x^2 - \sqrt{y}) \cdot 2e^{2x}$, which simplifies to $= 2e^{2x} \cdot (x + x^2 - \sqrt{y})$. The second derivative with respect to y is calculated as $-e^{2x} \cdot \frac{1}{2\sqrt{y}} = -\frac{e^{2x}}{2\sqrt{y}}$.

$$f(x, y) = (x^2 - \sqrt{y}) \cdot e^{2x}$$
$$= (x^2 - y^{\frac{1}{2}}) e^{2x}$$
$$\frac{\partial f}{\partial x} = 2x \cdot e^{2x} + (x^2 - \sqrt{y}) \cdot 2e^{2x}$$
$$= 2e^{2x} \cdot (x + x^2 - \sqrt{y})$$
$$\frac{\partial f}{\partial y} = -e^{2x} \cdot \frac{1}{2\sqrt{y}} = -\frac{e^{2x}}{2\sqrt{y}}$$

Partielle Ableitungen: Schriftliche Arbeit 2

Partielle Ableitungen



Partielle Ableitungen: Aufgabe

Bestimmen Sie die partiellen Ableitungen 1. Ordnung
der Funktion f

$$1) \ f(x, y) = (x^2 - \sqrt{y}) e^{2x}$$

$$2) \ f(x, y, z) = \ln\left(\frac{x}{y}\right) + e^{-z^2}$$

$$3) \ f(x, y, z) = \ln\left(\frac{x^2 \sqrt{y}}{z^3}\right)$$

$$4) \ f(x, y) = e^{\sin x} + \cos(xy)$$

$$5) \ f(x, y) = e^{x^2 + \ln y}$$

Partielle Ableitungen: Lösung 1

$$f(x, y) = (x^2 - \sqrt{y}) e^{2x}$$

$$\begin{aligned}\frac{\partial f}{\partial x} &= e^{2x} \frac{\partial}{\partial x} (x^2 - \sqrt{y}) + (x^2 - \sqrt{y}) \frac{\partial}{\partial x} e^{2x} = \\ &= 2x e^{2x} + 2(x^2 - \sqrt{y}) e^{2x} = 2(x^2 + x - \sqrt{y}) e^{2x}\end{aligned}$$

$$\frac{\partial f}{\partial y} = e^{2x} \frac{\partial}{\partial y} (x^2 - \sqrt{y}) = -e^{2x} \frac{\partial}{\partial y} (y^{1/2}) = -\frac{e^{2x}}{2\sqrt{y}}$$

Partielle Ableitungen: Aufgabe 1

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$$f(x,y) = (x^2 - y^{1/2}) \cdot e^{2x}$$

$$\frac{\partial}{\partial x} = 2x \cdot e^{2x} + (x^2 - y^{1/2}) \cdot 2e^{2x}$$

$$= \underline{2x e^{2x} + 2(x^2 - \sqrt{y}) e^{2x}}$$

$$\frac{\partial}{\partial y} = \left(-\frac{1}{2} y^{-1/2} \right) \cdot e^{2x} + (x^2 - y^{1/2}) \cdot e^{2x}$$

$$\frac{\partial}{\partial y} = -\frac{1}{2\sqrt{y}} \cdot e^{2x} + (x^2 - \sqrt{y}) \cdot e^{2x}$$

Partielle Ableitungen: Aufgabe 1

$$f(x,y) = (x^2 - y^{\frac{1}{2}}) \cdot e^{2x} = x^2 \cdot e^{2x} - y^{\frac{1}{2}} \cdot e^{2x} \quad \text{Test}$$

$$f_x = 2x \cdot e^{2x} + 2x^2 \cdot e^{2x} - \cancel{y^{\frac{1}{2}} \cdot 2\sqrt{y} \cdot e^{2x}} = e^{2x} \cdot (2x + 2x^2 - 2\sqrt{y})$$

$$= e^{2x} \cdot (2x \cdot (1+x) - 2\sqrt{y})$$

$$f_y = -\frac{1}{2} e^{2x} \cdot y^{-\frac{1}{2}} = \frac{-e^{2x}}{2\sqrt{y}}$$

Partielle Ableitungen: Aufgabe 1

$$1) f(x,y) = (x^2 - \sqrt{y}) e^{2x} = x^2 e^{2x} - \sqrt{y} e^{2x}$$

$$\frac{\partial f}{\partial x} = 2x e^{2x} + x^2 2e^{2x} - 2\sqrt{y} e^{2x} = (\cancel{x^2} + x - \cancel{1}) \underline{2e^{2x}}$$

$$\frac{\partial f}{\partial y} = -\frac{1}{2} e^{2x} y^{-\frac{1}{2}} = -\frac{e^{2x}}{2\sqrt{y}}$$

Partielle Ableitungen: Aufgabe 1

A photograph of a handwritten mathematical derivation pinned to a corkboard. The derivation is written on a blue grid paper card. A yellow sticky note with a black smiley face is pinned next to it. Four clear pushpins hold the card in place.

1) $f(x, y) = (x^2 - \sqrt{y}) \cdot e^{2x}$
 $= (x^2 - y^{\frac{1}{2}}) e^{2x}$

$\frac{\partial f}{\partial x} = 2x \cdot e^{2x} + (x^2 - \sqrt{y}) \cdot 2e^{2x}$
 $= 2e^{2x} (x + x^2 - \sqrt{y})$

$\frac{\partial f}{\partial y} = -e^{2x} \cdot \frac{1}{2\sqrt{y}} = -\frac{e^{2x}}{2\sqrt{y}}$

Partielle Ableitungen: Lösung 2

$$f(x, y, z) = \ln\left(\frac{x}{y}\right) + e^{-z^2} = \ln x - \ln y + e^{-z^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial f}{\partial y} = -\frac{1}{y}, \quad \frac{\partial f}{\partial z} = \frac{\partial}{\partial z} e^{-z^2} = e^{-z^2} \frac{\partial}{\partial z} (-z^2) = -2z e^{-z^2}$$

Partielle Ableitungen: Lösung 3

3) $f(x, y, z) = \ln\left(\frac{x^2 \sqrt{y}}{z^3}\right) = \ln(x^2) + \cancel{\ln(z^3)} \ln(\sqrt{y})$

$$\frac{\partial f}{\partial x} = \frac{2}{x} ; \quad \frac{\partial f}{\partial y} = \frac{1}{2y} ;$$

$$\frac{\partial f}{\partial z} = -\frac{3x^2}{z^3} = -\frac{3}{z}$$

$$f(x, y, z) = \ln\left(\frac{x^2 \sqrt{y}}{z^3}\right) = 2 \ln x + \frac{1}{2} \ln y - 3 \ln z$$

$$\frac{\partial f}{\partial x} = \frac{2}{x}, \quad \frac{\partial f}{\partial y} = \frac{1}{2y}, \quad \frac{\partial f}{\partial z} = -\frac{3}{z}$$

Partielle Ableitungen: Lösung 3

$$f_x = \frac{2x}{x^2} = \cancel{\partial f / \partial x} \cdot \cancel{\partial x / \partial x}$$

$$f_y = \frac{1}{2xy} \cdot \frac{1}{\partial y / \partial y} = \frac{1}{2y} = \underline{\underline{\frac{1}{2y}}} ?$$

$$f_z = \frac{3z^2}{z^3} =$$

Partielle Ableitungen: Lösung 3

$$\partial z = x \ln x + y \ln y - 2z e^{-z}$$

3) $f(xyz) = \ln\left(\frac{x^2\sqrt{y}}{z^3}\right)$

$$= \ln(x^2\sqrt{y}) - \ln(z^3)$$

$\frac{\partial f}{\partial x}$

$$= x^2 \ln x \cdot \frac{1}{2\sqrt{y}} - \ln z^3$$

$\frac{\partial f}{\partial y} ?$

$$= x^2 \ln x \cdot \frac{1}{2} \frac{1}{\sqrt{y}} - \ln z^3$$

$\frac{\partial f}{\partial z} ?$

$$= x^2 \ln x \cdot \frac{1}{2} \frac{1}{y} - z^3 \ln z^3$$

4) $f(x,y) = e^{\sin x} + \cos(xy)$

Partielle Ableitungen: Lösungen

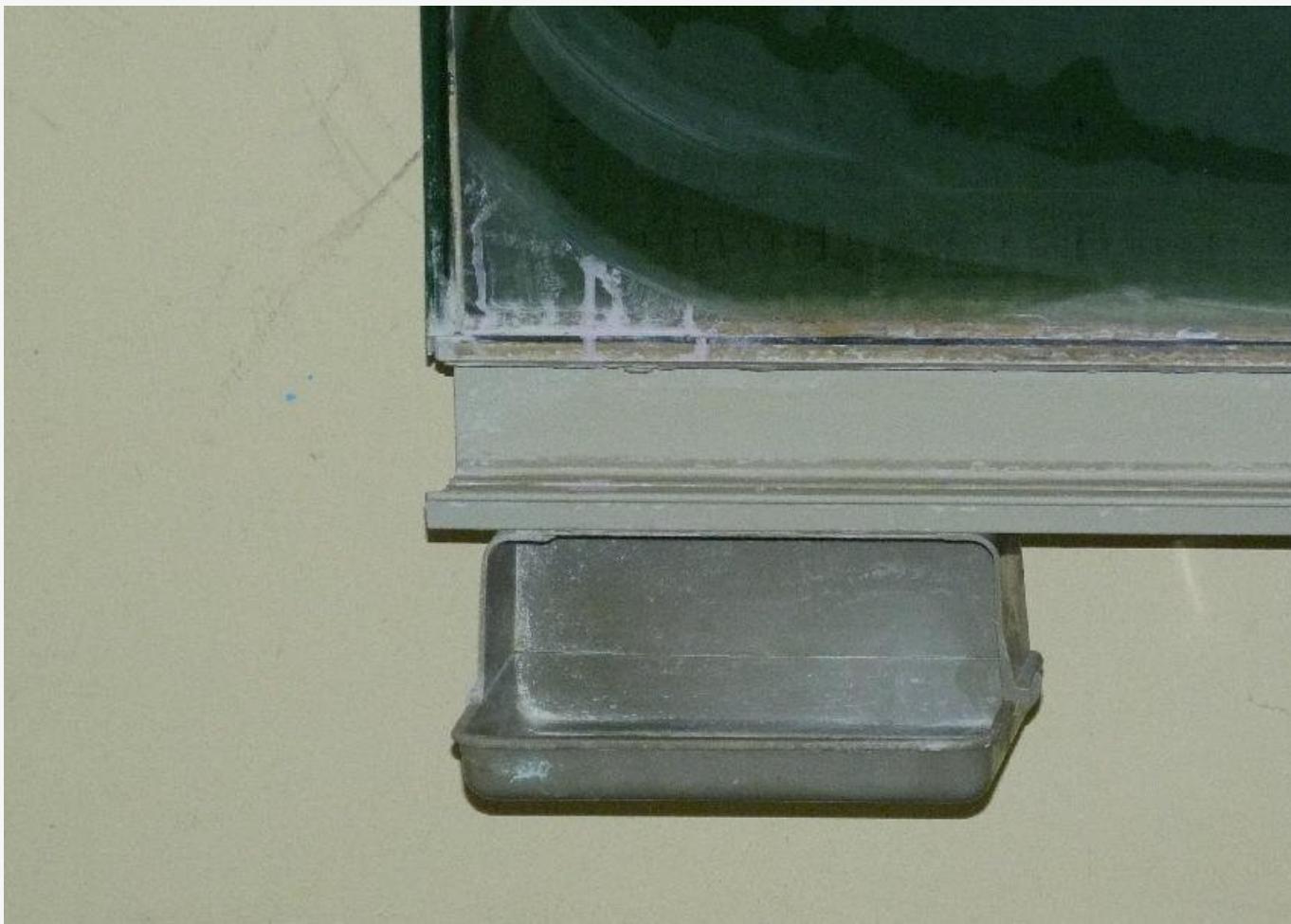
$$4) \ f(x, y) = e^{\sin x} + \cos(xy)$$

$$\frac{\partial f}{\partial x} = e^{\sin x} \frac{\partial}{\partial x} \sin x + \frac{\partial}{\partial x} \cos(xy) = \cos x \ e^{\sin x} - y \sin(xy)$$

$$\frac{\partial f}{\partial y} = -x \sin(xy)$$

$$5) \ f(x, y) = e^{x^2 + \ln y} = y e^{x^2}$$

$$\frac{\partial f}{\partial x} = y \frac{\partial}{\partial x} e^{x^2} = 2x y e^{x^2}, \quad \frac{\partial f}{\partial y} = e^{x^2}$$



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Ma 2 – Lubov Vassilevskaya