http://www.dailymotion.com/video/x20rcp_wendy-tilby-stringscordes_shortfilms

Die Gleichung der Tangentialebene: Aufgaben

Gesucht ist die Tangentialebene der Funktion $f = f(x, y)$ im Punkt P

Aufgabe 9: $f(x, y) = \sqrt{x} + \sqrt{y}, \quad P(1, 1)$

Aufgabe 10: $f(x, y) = \sqrt{x^2 - 4} + \sqrt{4 - y^2}, \quad P(3, 1)$

Aufgabe 11: $f(x, y) = \sqrt{x y - 1}, \quad P(5, 1)$

Aufgabe 12: $f(x, y) = \dfrac{2}{1 + x^2 \cdot y^2}, \quad P(-1, 2)$

Aufgabe 13: $f(x, y) = \ln(x^2 + y), \quad P(-2, 1)$

Aufgabe 14: $f(x, y) = (x^2 - y^2)\sin y, \quad P\left(3, \dfrac{\pi}{2}\right)$

Aufgabe 15: $f(x, y) = x^3 + 2\cos y, \quad P\left(-3, \dfrac{\pi}{6}\right)$

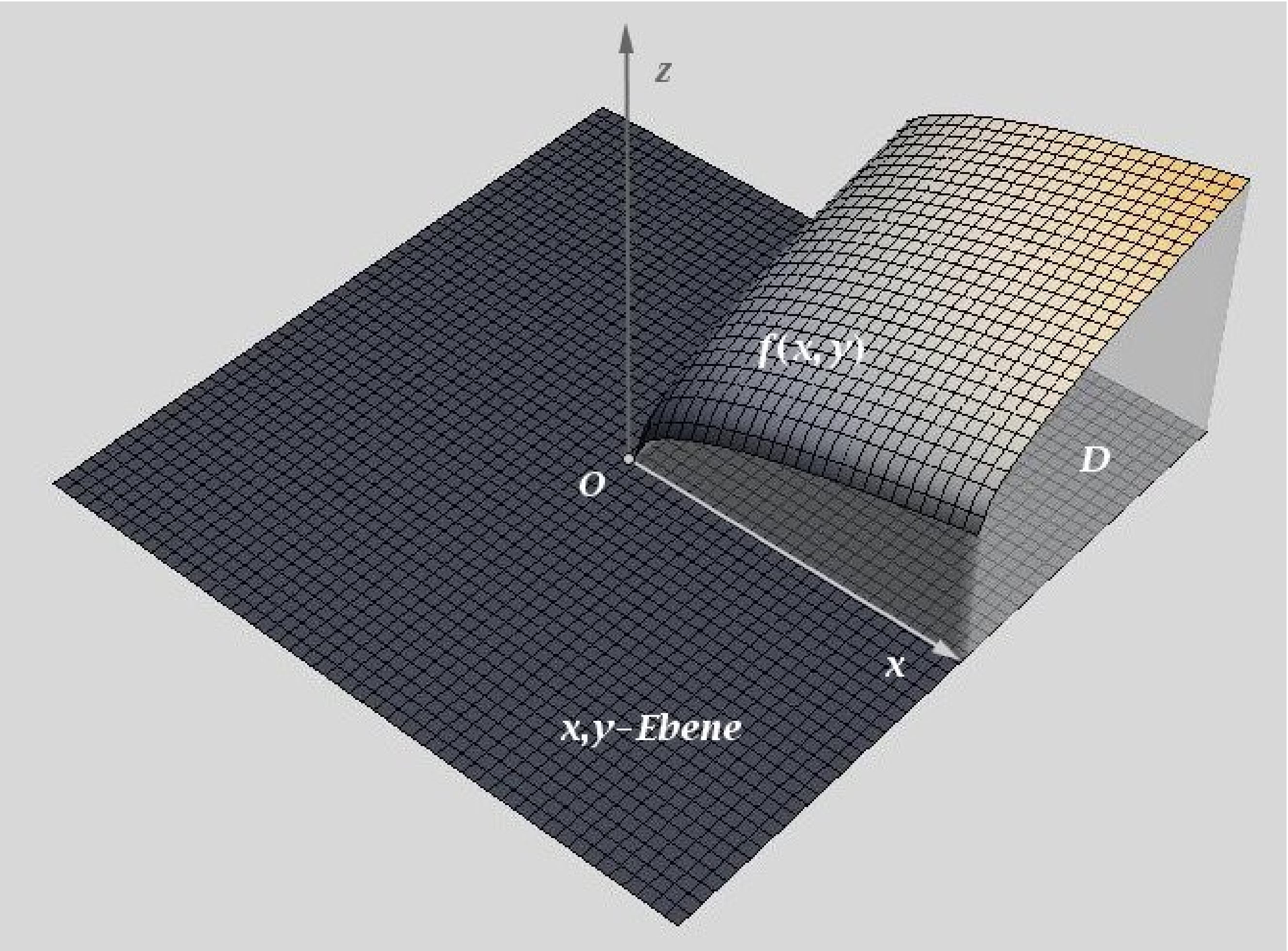

Abb. L9: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x, y) = \sqrt{x} + \sqrt{y}$$

Gleichung der Tangentialebene: Lösung 9

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f(x, y) = \sqrt{x} + \sqrt{y}, \quad P(1, 1), \quad x_0 = 1, \quad y_0 = 1$$

$$f_x(x, y) = \frac{1}{2\sqrt{x}}, \qquad f_x(1,1) = \frac{1}{2}$$

$$f_y(x, y) = \frac{1}{2\sqrt{y}}, \qquad f_y(1,1) = \frac{1}{2}$$

$$z_0 = f(x_0, y_0) = 2$$

$$z = \frac{x}{2} + \frac{y}{2} + 1$$

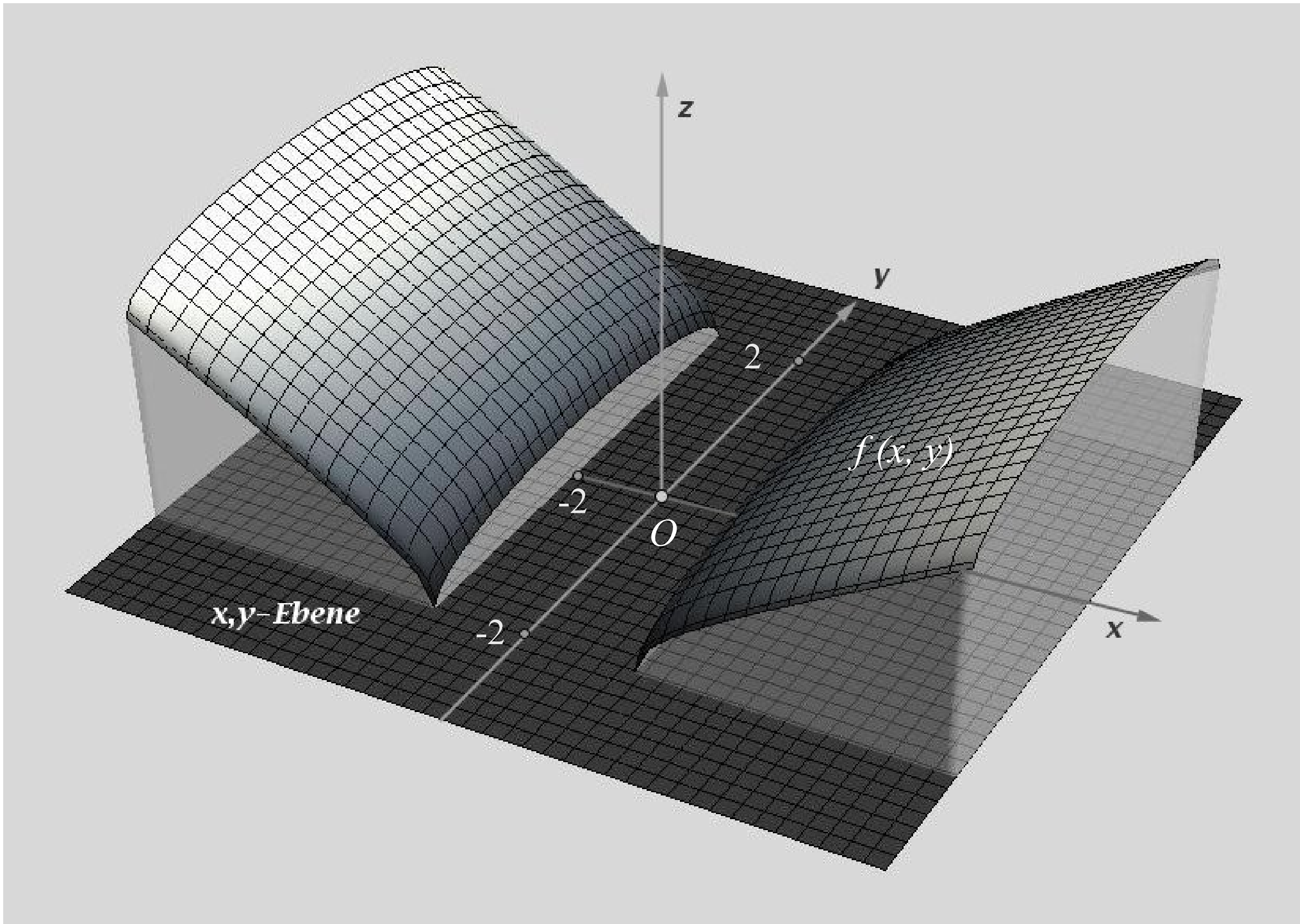

Abb. L10: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x, y) = \sqrt{x^2 - 4} + \sqrt{4 - y^2} ,$$

Gleichung der Tangentialebene: Lösung 10

$$f(x, y) = \sqrt{x^2 - 4} + \sqrt{4 - y^2}, \quad P(3, 1), \quad x_0 = 3, \quad y_0 = 1$$

$$f_x(x, y) = \frac{x}{\sqrt{x^2 - 4}}, \qquad f_x(3, 1) = \frac{3}{\sqrt{5}}$$

$$f_y(x, y) = -\frac{y}{\sqrt{4 - y^2}}, \qquad f_y(3, 1) = -\frac{1}{\sqrt{3}}$$

$$z_0 = f(x_0, y_0) = \sqrt{x_0^2 - 4} + \sqrt{4 - y_0^2} = \sqrt{5} + \sqrt{3}$$

$$z = \frac{3}{\sqrt{5}}(x - 3) - \frac{1}{\sqrt{3}}(y - 1) + \sqrt{5} + \sqrt{3} =$$

$$= \frac{3}{\sqrt{5}} x - \frac{1}{\sqrt{3}} y + 4\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{5}}\right)$$

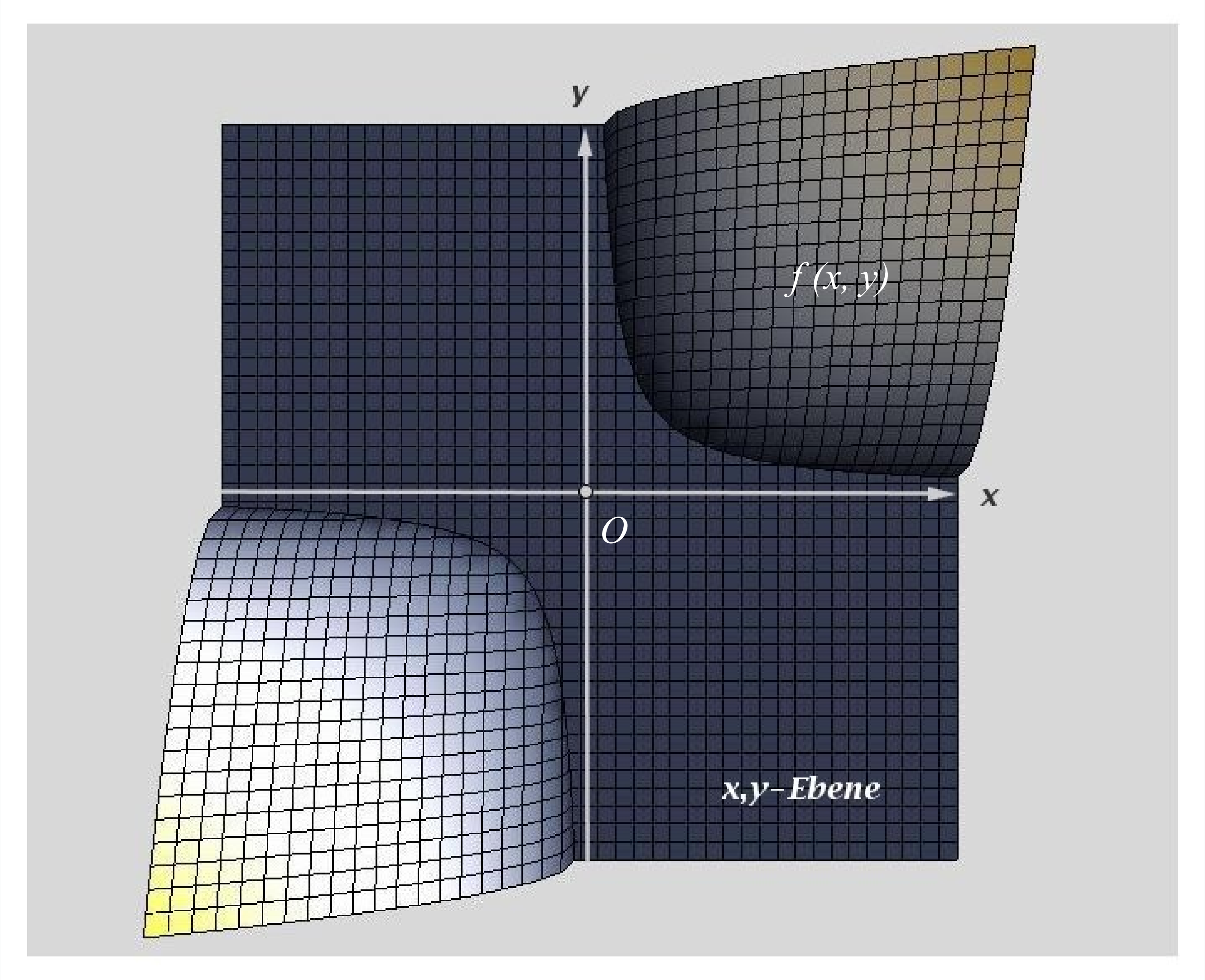

Abb. L11: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x,\ y) = \sqrt{x\,y - 1}$$

Gleichung der Tangentialebene: Lösung 11

$$f(x, y) = \sqrt{x y - 1}, \qquad P(5, 1), \quad x_0 = 5, \quad y_0 = 1$$

$$f_x(x, y) = \frac{y}{2\sqrt{x y - 1}}, \qquad f_x(5, 1) = \frac{1}{4}$$

$$f_y(x, y) = \frac{x}{2\sqrt{x y - 1}}, \qquad f_y(5, 1) = \frac{5}{4}$$

$$z_0 = f(x_0, y_0) = \sqrt{x_0 y_0 - 1} = 2$$

$$z = \frac{x}{4} + \frac{5}{4} y - \frac{1}{2}$$

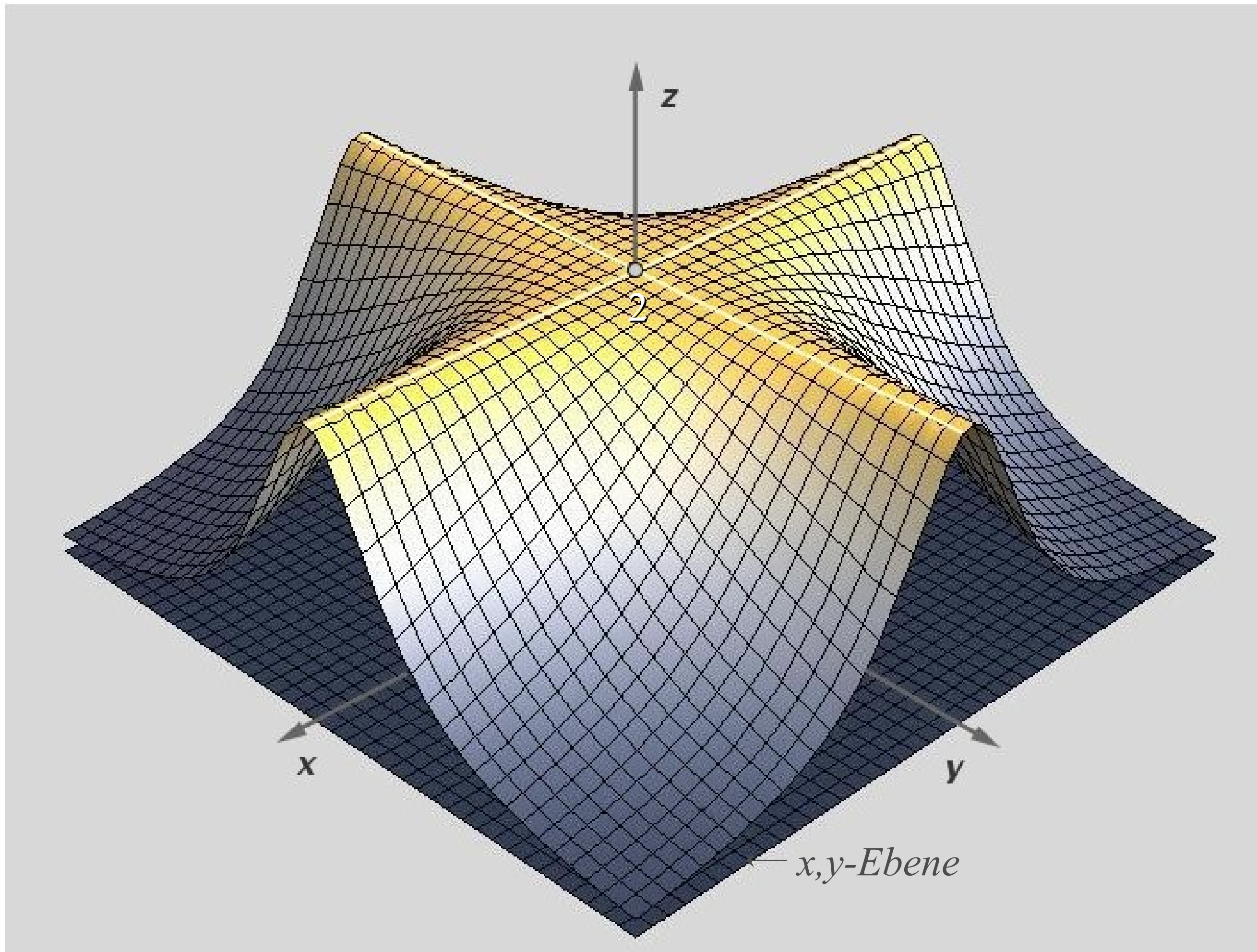

Abb. L12: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x, y) = \frac{2}{1 + x^2 \cdot y^2}$$

$$f(x, y) = \frac{2}{1 + x^2 \cdot y^2}, \qquad P(-1, 2), \qquad x_0 = -1, \qquad y_0 = 2$$

$$f_x(x, y) = -\frac{4xy^2}{(1 + x^2 y^2)^2}, \qquad f_x(-1, 2) = \frac{16}{25}$$

$$f_y(x, y) = -\frac{4x^2 y}{(1 + x^2 y^2)^2}, \qquad f_y(-1, 2) = -\frac{8}{25}$$

$$z_0 = f(x_0, y_0) = \frac{2}{1 + x_0^2 \cdot y_0^2} = \frac{2}{5}$$

$$z = \frac{16}{25} x - \frac{8}{25} y + \frac{42}{25}$$

Gleichung der Tangentialebene: Lösung 13

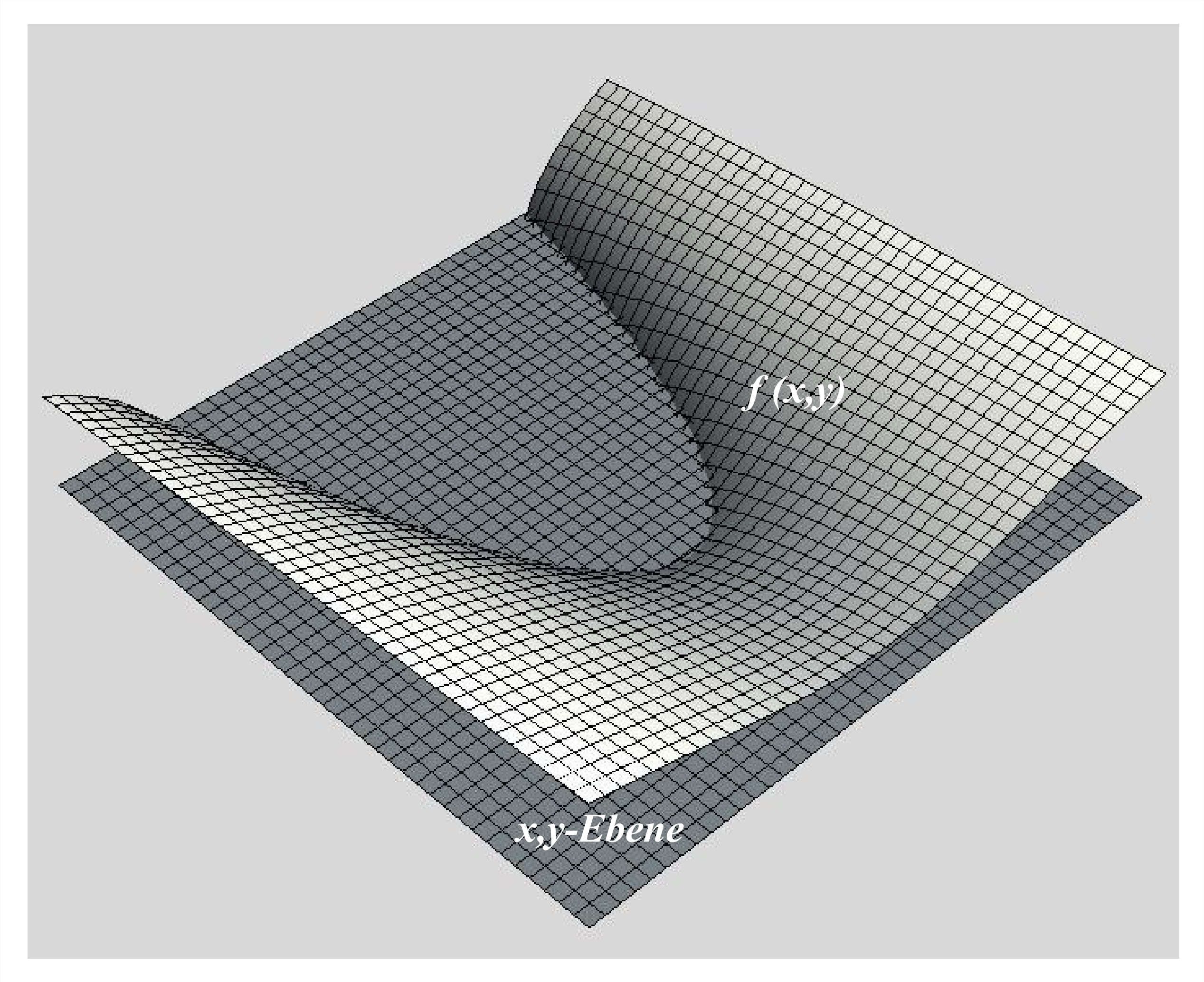

Abb. L13: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x, y) = \ln(x^2 + y)$$

$$f(x, y) = \ln(x^2 + y), \quad P(-2, 1), \quad x_0 = -2, \quad y_0 = 1$$

$$f_x(x, y) = \frac{2x}{x^2 + y}, \qquad f_x(-2, 1) = -\frac{4}{5}$$

$$f_y(x, y) = \frac{1}{x^2 + y}, \qquad f_y(-2, 1) = \frac{1}{5}$$

$$z_0 = f(x_0, y_0) = \ln(x_0^2 + y_0) = \ln 5$$

$$z = -\frac{4}{5}x + \frac{1}{5}y - \frac{9}{5} + \ln 5$$

$$= -0.8x + 0.2y - 0.191$$

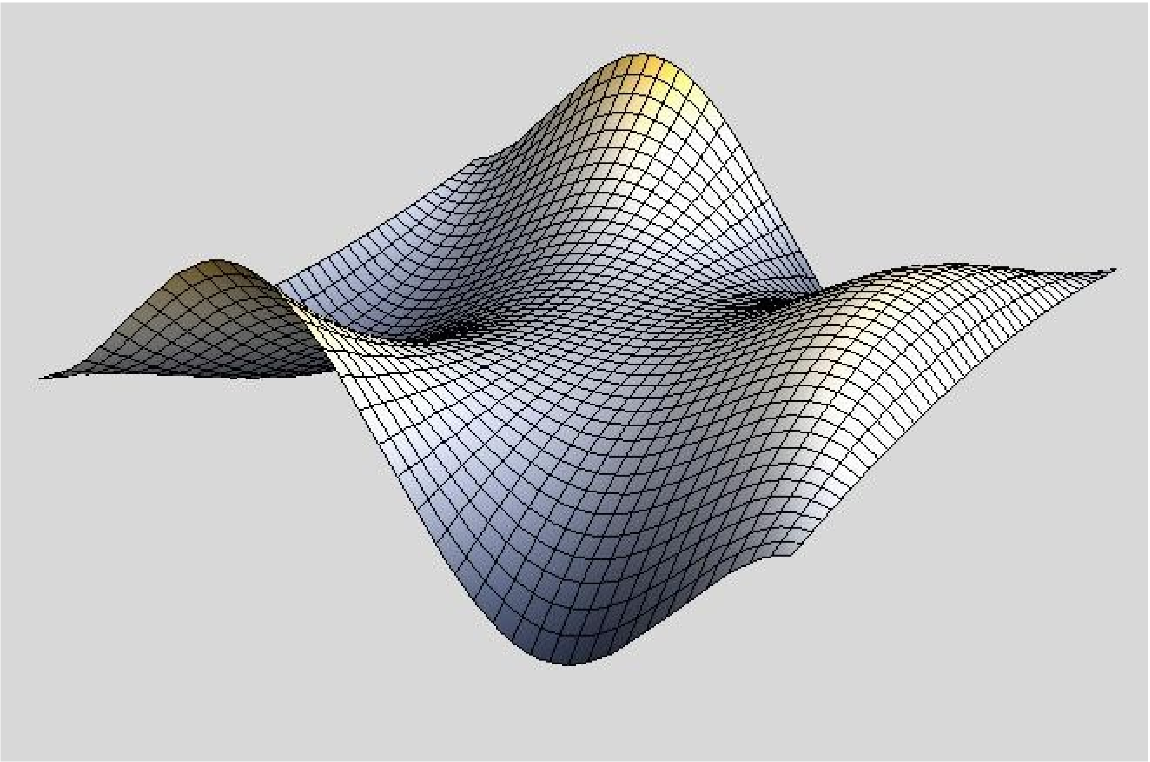

Abb. L14: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x, y) = (x^2 - y^2)\sin y$$

Gleichung der Tangentialebene: Lösung 14

$$f(x, y) = (x^2 - y^2)\sin y$$

$$P\left(3, \frac{\pi}{2}\right), \qquad x_0 = 3, \qquad y_0 = \frac{\pi}{2}$$

$$f_x(x, y) = 2x\sin y, \qquad f_x\left(3, \frac{\pi}{2}\right) = 6$$

$$f_y(x, y) = -2y\sin y + (x^2 - y^2)\cos y, \qquad f_y\left(3, \frac{\pi}{2}\right) = -\pi$$

$$z_0 = f(x_0, y_0) = (x_0^2 - y_0^2)\sin y_0 = 9 - \frac{\pi^2}{4}$$

$$z = 6x - \pi y - 9 + \frac{\pi^2}{4} \simeq 6x - 3.14y - 6.53$$

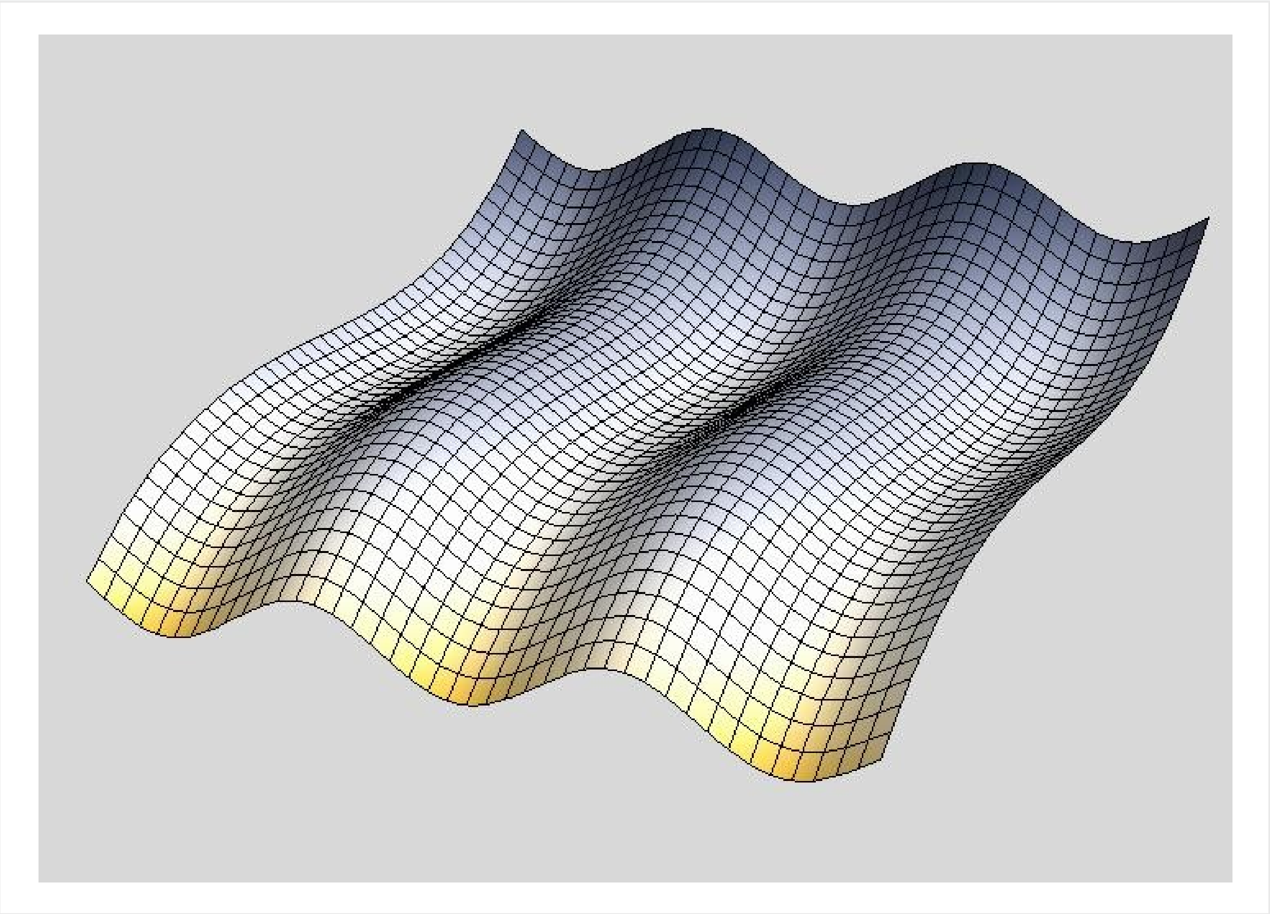

Abb. L15: Graphische Darstellung der Funktion $z = f(x, y)$

$$f(x, y) = x^3 + 2\cos y$$

$$f(x, y) = x^3 + 2\cos y, \quad P\left(-3, \frac{\pi}{6}\right), \quad x_0 = -3, \quad y_0 = \frac{\pi}{6}$$

$$f_x(x, y) = 3x^2, \qquad f_x\left(-3, \frac{\pi}{6}\right) = 27$$

$$f_y(x, y) = -2\sin y, \qquad f_y\left(-3, \frac{\pi}{6}\right) = -1$$

$$z_0 = f(x_0, y_0) = x_0^3 + 2\cos y_0 = -27 + \sqrt{3}$$

$$z = 27x - y + 54 + \frac{\pi}{6} + \sqrt{3} \simeq 27x - y + 56.26$$