Roots as powers with fractional exponents

We have used the following formula before:

$$(x^m)^n = x^{m \cdot n}$$

A power is exponentiated by multiplying the exponents. Therefore one expects

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}} \right)^m$$

$$x = x^1 = x^{\frac{n}{n}} = \left(x^{\frac{1}{n}} \right)^n$$

In other words, $x^{1/2}$ is the number, the square of which is x $(n = 2)$.

$x^{1/n}$ is the number, the n-th power of which is x.

Roots are written with the radical symbol $\sqrt{}$. Written like that, it means square root, which is also called root of degree 2. More generally $\sqrt[n]{}$ is the symbol for n-th root, also called root of degree n. The root symbol is an operator acting on the expression written beneath the root symbol. The index n is a natural number.

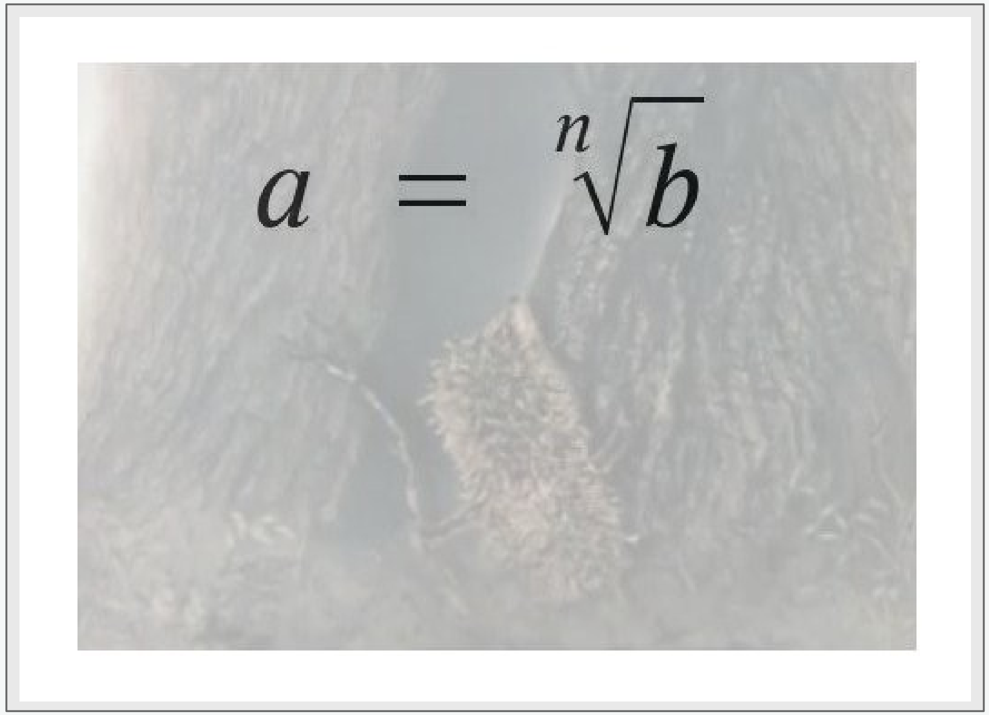
To extract the root means to find the base a in the equation

$$a^n = b \,,$$

where the exponent n and power value b are known,

$$n \in \mathbb{N} \setminus \{0\}, \qquad b \geqslant 0$$

We write: $$a = \sqrt[n]{b}$$



http://www.youtube.com/watch?v=JZS1fLK4DYM&feature=related

Here *a* is called the *n-th* root, *b* is called the radicand, *n* is the root index or degree of the root. A root of degree *2* is called a square root, a root of degree 3 is called a cube root.

The *n-th* root of $b \geq 0$ is the positive number *a*, whose *n-th* power has the value *b*, with:

$$a \in \mathbb{R}, \quad n \in \mathbb{N} \setminus \{ 0 \}$$

With this definition of the *n-th* root, with the requirement $b \geq 0$, one cannot extract a root of negative numbers (e.g. -2 is not the cube root of -8).

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}, \qquad m, n \in \mathbb{N}, \quad n \neq 0, \quad a \in \mathbb{R}, \quad a \geqslant 0$$

The calculation rules for powers with integer exponents are still valid for powers with non integer exponents.

1. Roots can be added only, if radicand and root index agree.

$$2\sqrt[5]{u} + 3\sqrt[6]{u} - 4\sqrt[5]{u} - 2\sqrt[6]{u} = \sqrt[6]{u} - 2\sqrt[5]{u}$$

2. Roots with same root index are multiplied by extracting the root with same index of the product of the radicands.

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

Calculation rules

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[m \cdot n]{a}$$

Exercise 1:

Simplify the expression as much as possible

$$2\sqrt{72} - 3\sqrt{75} - 4\sqrt{32} + 5\sqrt{27}$$

Exercise 2:

Simplify $\sqrt[20]{a^5}$

Exercise 3:

Write the expressions as powers with non integer exponents

$a)\ \sqrt{7},\quad b)\ \sqrt[3]{2},\quad c)\ \sqrt[5]{3^4},\quad d)\ \sqrt[4]{9^5}$

$e)\ \sqrt{a},\quad f)\ \sqrt[x]{b^y},\quad g)\ \sqrt{a+b}$

$h)\ \sqrt[4]{(x-y)^3},\quad i)\ \sqrt{\frac{1}{5}},\quad j)\ \frac{1}{\sqrt[4]{x^3}}$

$k)\ \frac{1}{\sqrt[5]{a+b}},\quad l)\ 5\sqrt[3]{5x^5y^3z^6}$

Solution 1: $2\sqrt{72} - 3\sqrt{75} - 4\sqrt{32} + 5\sqrt{27}$

At first one may think, that there is nothing to be simplified, because the square roots have all different radicands. However a closer look shows, that each radicand is a multiple of a square number.

$$2\sqrt{72} - 3\sqrt{75} - 4\sqrt{32} + 5\sqrt{27} =$$

$$= 2\sqrt{36 \cdot 2} - 3\sqrt{25 \cdot 3} - 4\sqrt{16 \cdot 2} + 5\sqrt{9 \cdot 3} =$$

$$= 2 \cdot 6\sqrt{2} - 3 \cdot 5\sqrt{3} - 4 \cdot 4\sqrt{2} + 5 \cdot 3\sqrt{3} = -4\sqrt{2}$$

Solution 2:

$$\sqrt[20]{a^5} = \left(a^5\right)^{\frac{1}{20}} = a^{\frac{5}{20}} = a^{\frac{1}{4}} = \sqrt[4]{a}$$

$a)\ \sqrt{7} = 7^{\frac{1}{2}}, \qquad b)\ \sqrt[3]{2} = 2^{\frac{1}{3}}, \qquad c)\ \sqrt[5]{3^4} = 3^{\frac{4}{5}}$

$d)\ \sqrt[4]{9^5} = \sqrt[4]{3^{10}} = 3^{\frac{10}{4}} = 3^{\frac{5}{2}}$

$e)\ \sqrt{a} = a^{\frac{1}{2}}, \qquad f)\ \sqrt[x]{b^y} = b^{\frac{y}{x}}$

$g)\ \sqrt{a+b} = (a+b)^{\frac{1}{2}}, \qquad h)\ \sqrt[4]{(x-y)^3} = (x-y)^{\frac{3}{4}}$

$i)\ \sqrt{\frac{1}{5}} = \sqrt{5^{-1}} = 5^{-\frac{1}{2}}, \qquad j)\ \frac{1}{\sqrt[4]{x^3}} = \frac{1}{x^{3/4}} = x^{-\frac{3}{4}}$

$k)\ \frac{1}{\sqrt[5]{a+b}} = (a+b)^{-\frac{1}{5}}, \qquad l)\ 5\ \sqrt[3]{5\,x^5\,y^3\,z^6} = 5^{\frac{4}{3}}\ x^{\frac{5}{3}}\ y\ z^2$

Calculations with roots: Exercise 4

Write the following expressions using the radical symbol:

$a)\ 2^{\frac{1}{2}}$, $\quad b)\ 3^{\frac{2}{3}}$, $\quad c)\ 7^{0.5}$, $\quad d)\ 11^{0.8}$

$e)\ a^{\frac{2}{7}}$, $\quad f)\ x^{\frac{y}{x}}$, $\quad g)\ 5\,x^{\frac{3}{8}}$, $\quad h)\ (5\,x)^{\frac{3}{8}}$

$i)\ 3^{-\frac{1}{4}}$, $\quad j)\ 5^{-1.25\,a}$, $\quad k)\ \left(\frac{1}{5}\right)^{-\frac{2}{a}}$, $\quad l)\ (x+y)^{-\frac{2}{5}}$

a) $2^{\frac{1}{2}} = \sqrt{2}$, b) $3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$

c) $7^{0.5} = 7^{\frac{1}{2}} = \sqrt{7}$, d) $11^{0.8} = 11^{\frac{4}{5}} = \sqrt[5]{11^4}$

e) $a^{\frac{2}{7}} = \sqrt[7]{a^2}$, f) $x^{\frac{y}{x}} = \sqrt[x]{x^y}$

g) $5\ x^{\frac{3}{8}} = 5\ \sqrt[8]{x^3}$

h) $(5\ x)^{\frac{3}{8}} = \sqrt[8]{(5\ x)^3} = \sqrt[8]{5^3\ x^3} = \sqrt[8]{125\ x^3}$

i) $3^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{3}}$, j) $5^{-1.25\,a} = 5^{-\frac{5}{4}\,a} = \frac{1}{\sqrt[4]{5^{5\,a}}}$

k) $\left(\frac{1}{5}\right)^{-\frac{2}{a}} = 5^{\frac{2}{a}} = \sqrt[a]{5^2} = \sqrt[a]{25}$

l) $(x + y)^{-\frac{2}{5}} = \frac{1}{(x + y)^{\frac{2}{5}}} = \frac{1}{\sqrt[5]{(x + y)^2}}$

Simplify the radicand by partially extracting the root

$a)\ \sqrt{32}$, $b)\ \sqrt{48}$, $c)\ \sqrt{54}$, $d)\ \sqrt{88}$

$e)\ \sqrt{108}$, $f)\ \sqrt{128}$, $g)\ \sqrt{250}$, $h)\ \sqrt{375}$

$i)\ \sqrt[3]{32}$, $j)\ \sqrt[3]{48}$, $k)\ \sqrt[3]{54}$, $l)\ \sqrt[3]{88}$

Calculations with roots: Solution 5

$a)\ \sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

$b)\ \sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

$c)\ \sqrt{54} = \sqrt{3^3 \cdot 2} = 3\sqrt{6}$

$d)\ \sqrt{88} = \sqrt{22 \cdot 4} = 2\sqrt{22}$

$e)\ \sqrt{108} = \sqrt{27 \cdot 4} = \sqrt{3^3 \cdot 2^2} = 6\sqrt{3}$

$f)\ \sqrt{128} = \sqrt{2^7} = 2^3\sqrt{2} = 8\sqrt{2}$

$g)\ \sqrt{250} = 5\sqrt{10}$

$h)\ \sqrt{375} = 5\sqrt{15}$

$i)\ \sqrt[3]{32} = \sqrt[3]{2^5} = \sqrt[3]{2^3 \cdot 2^2} = 2\sqrt[3]{4}$

$j)\ \sqrt[3]{48} = \sqrt[3]{16 \cdot 3} = \sqrt[3]{2^4 \cdot 3} = 2\sqrt[3]{6}$

$k)\ \sqrt[3]{54} = \sqrt[3]{3^3 \cdot 2} = 3\sqrt[3]{2}$

$l)\ \sqrt[3]{88} = \sqrt[3]{2^3 \cdot 11} = 2\sqrt[3]{11}$

Exercise 6: Simplify the radicand by partially extracting the root

1) $\sqrt{a^5}$, 2) $\sqrt[4]{x^7\ y}$, 3) $\sqrt[5]{4\ x^6\ y^5}$

4) $\sqrt{(a+b)^3}$, 5) $\sqrt[3]{24\ x^3\ y}$, 6) $\sqrt[4]{32\ x^5\ y}$

7) $\sqrt[5]{32\ x^5\ y^7}$, 8) $\sqrt[3]{(x^2-4x+4)^4}$

Exercise 7: Bring the factor in front of the root beneath the root

1) $2\sqrt{2}$, 2) $4\sqrt{3}$, 3) $\frac{3}{2}\sqrt{8}$

4) $4\sqrt{0.25}$, 5) $3\sqrt{\frac{1}{3}}$, 6) $5\sqrt{0.04}$

7) $x\sqrt{x}$, 8) $x\ y\sqrt{z}$, 9) $(x+y)\sqrt{z}$

10) $\frac{x^3}{y}\sqrt{z}$, 11) $x\sqrt{\frac{1}{x}}$, 12) $\frac{a}{b}\sqrt{\frac{b}{a}}$

1) $\sqrt{a^5} = a^2 \sqrt{a}$, 2) $\sqrt[4]{x^7\ y} = x\ \sqrt[4]{x^3\ y}$

3) $\sqrt[5]{4\ x^6\ y^5} = x\ y\ \sqrt[5]{4\ x}$

4) $\sqrt{(a+b)^3} = (a+b)\sqrt{a+b}$

5) $\sqrt[3]{24\ x^3\ y} = \sqrt[3]{2^3 \cdot 3\ x^3\ y} = 2\,x\,\sqrt[3]{3\ y}$

6) $\sqrt[4]{32\ x^5\ y} = \sqrt[4]{2 \cdot 2^4\ x^5\ y} = 2\,x\ \sqrt[4]{2\ x\ y}$

7) $\sqrt[5]{32\ x^5\ y^7} = \sqrt[5]{2^5\ x^5\ y^7} = 2\,x\,y\,\sqrt[5]{y^2}$

8) $\sqrt[3]{(x^2 - 4\,x + 4)^4} = (x^2 - 4\,x + 4)\ \sqrt[3]{x^2 - 4\,x + 4}$

1) $2\sqrt{2} = \sqrt{2^3} = \sqrt{8}$, 2) $4\sqrt{3} = \sqrt{4^2 \cdot 3} = \sqrt{48}$

3) $\frac{3}{2}\sqrt{8} = \sqrt{18}$, 4) $4\sqrt{0.25} = \sqrt{4} = 2$

5) $3\sqrt{\frac{1}{3}} = \sqrt{3}$, 6) $5\sqrt{0.04} = \sqrt{1} = 1$

7) $x\sqrt{x} = \sqrt{x^3}$, 8) $x\, y\sqrt{z} = \sqrt{x^2\, y^2\, z}$

9) $(x + y)\sqrt{z} = \sqrt{z\,(x + y)^2}$

10) $\frac{x^3}{y}\sqrt{z} = \sqrt{\frac{x^6\, z}{y^2}}$, 11) $x\sqrt{\frac{1}{x}} = \sqrt{x}$

12) $\frac{a}{b}\sqrt{\frac{b}{a}} = \sqrt{\frac{a}{b}}$

Calculations with roots: Exercise 8

1) $\frac{x^2}{2}\sqrt[3]{\frac{16}{x^5}}$

2) $\frac{a^3\,b^2}{c^2}\sqrt[3]{\frac{c^7}{a^8\,b^5}}$

3) $(a-b)\sqrt[4]{\frac{a+b}{(a-b)^3}}$

4) $(u+v)\sqrt[3]{1-\frac{3\,u\,v}{(u+v)^2}}$

5) $(\sqrt{2}-1)\sqrt{\sqrt{2}+1}$

6) $(\sqrt{5}-2)\sqrt{2+\sqrt{5}}$

Calculations with roots: Solution 8

1) $\dfrac{x^2}{2}\sqrt[3]{\dfrac{16}{x^5}} = \sqrt[3]{2x}$

2) $\dfrac{a^3 b^2}{c^2}\sqrt[3]{\dfrac{c^7}{a^8 b^5}} = \sqrt[3]{a b c}$

3) $(a - b)\sqrt[4]{\dfrac{a + b}{(a - b)^3}} = \sqrt[4]{a^2 - b^2}$

4) $(u + v)\sqrt[3]{1 - \dfrac{3 u v}{(u + v)^2}} = \sqrt[3]{(u + v)^3 - 3 u v (u + v)} = \sqrt[3]{u^3 + v^3}$

5) $(\sqrt{2} - 1)\sqrt{\sqrt{2} + 1} = \sqrt{\sqrt{2} - 1}$

6) $(\sqrt{5} - 2)\sqrt{2 + \sqrt{5}} = \sqrt{\sqrt{5} - 2}$

Simplify the expressions as much as possible:

1) $\sqrt{3} + \sqrt{12} + \sqrt{75}$

2) $\sqrt[3]{2} + \sqrt[3]{16} + \sqrt[3]{432}$

3) $2\sqrt{2} + 3\sqrt{8} + 4\sqrt{50}$

4) $4\sqrt[3]{3} + 5\sqrt[3]{24} - 2\sqrt[3]{81}$

5) $3\sqrt{5} + 2\sqrt{45} + 4\sqrt{20}$

1) $\sqrt{3} + \sqrt{12} + \sqrt{75} = 8\sqrt{3}$

2) $\sqrt[3]{2} + \sqrt[3]{16} + \sqrt[3]{432} = 9\sqrt[3]{2}$

3) $2\sqrt{2} + 3\sqrt{8} + 4\sqrt{50} = 28\sqrt{2}$

4) $4\sqrt[3]{3} + 5\sqrt[3]{24} - 2\sqrt[3]{81} = 8\sqrt[3]{3}$

5) $3\sqrt{5} + 2\sqrt{45} + 4\sqrt{20} = 17\sqrt{5}$

Simplifying Radicals:

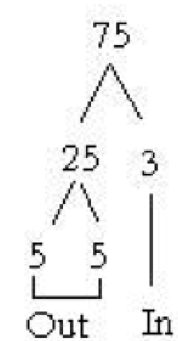
First, work with the numbers and variables separately. For the numbers, factor them into prime numbers, and then write them under the radical so that the same numbers are next to each other. You will then group the like numbers together, to determine how many like numbers go in each group you will look for the root (this is the number inside the check of the radical). For each group, you can take one of the numbers in the group out of the radical. What does not fit in a group stays inside the radical. With the variables, divide their power (exponent) by the root. The number of times the root goes into the power is the power of the variable outside of the radical. The remainder is the power of the variable that stays inside.

Example 11: $\sqrt{75\,m^5}$

•Note: the group size for this radical is two because we are using a square root.

Step 1: find the prime factors of 75 and rewrite them in the radical.

$$\sqrt{5 \times 5 \times 3 \times m^5}$$



Step 2: Pull out the group of two 5s.

$$5\sqrt{3 \times m^5}$$

Step 3: Divide the power of m (5) by the value of the root (2). Remember, the whole number is the power of the variable that comes out of the radical and the remainder is the power of the variable that remains inside the radical.

5÷2 = 2 R 1

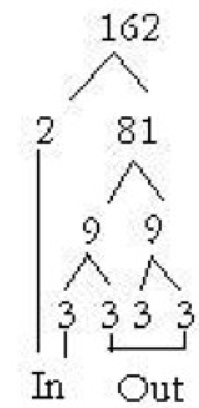
$$5m^2\sqrt{3m^1}$$

Example 12: $\sqrt[3]{162x^2y^{16}z^3}$

•Note: the group size for this radical is three because we are using a cubed root.

Step 1: Factor 162 into prime factors and write them in order under the radical.

$$\sqrt[3]{2 \times 3 \times 3 \times 3 \times 3 \times x^2y^{16}z^3}$$



Step 2: Pull out the group of three 3s.

$$3\sqrt[3]{2 \times 3 \times x^2y^{16}z^3}$$

Step 3: Divide the exponents by the root.

x: 2 ÷3= 0 R 2

y: 16 ÷ 3 = 5 R 1

z: 3 ÷ 3 = 1 R 0

$$3y^5z^1\sqrt[3]{6x^2y^1}$$

Adding or Subtracting Radicals:

First, simplify each radical. Then add or subtract like terms. "Like terms" are those terms that have the same radicals and variables. This means that only radicals with the same root, the same variables and numbers on the inside of the radical, and the same variables on the outside of the radical may be added or subtracted together.

Example 13: •Note that you cannot add square roots and cube roots.

$$y\sqrt{75x} + 3\sqrt{3xy^2} - y\sqrt[3]{81x}$$
$$5y\sqrt{3x} + 3y\sqrt{3x} - 3y\sqrt[3]{3x}$$
$$(5y + 3y)\sqrt{3x} - 3y\sqrt[3]{3x}$$
$$8y\sqrt{3x} - 3y\sqrt[3]{3x}$$

```
     75
     /\
   25 × 3
  /\    \
 5 × 5 × 3
```

```
      81
     /  \
   9  ×  9
  /\     /\
 3 × 3 × 3 × 3
```

Multiplying Radicals:

First, check to see if the two radicals have the same root. If so, multiply the radicals together. Multiply outside terms by outside terms and inside terms times by inside terms. Then simplify. If the roots are not the same you cannot multiple them together.

Example 14: $(2\sqrt{8})(3\sqrt{2})$

$6\sqrt{16}$

6×4

24

Dividing Radicals:

Radicals are not allowed in the denominator of a fraction. Therefore we must remove them. To do this we use a process called "Rationalizing the Denominator." In other words, we will multiply both the numerator and the denominator by a radical with the same index in order to remove the radical from the denominator.

Example 15: $\frac{3x}{\sqrt{5}}$

$$\frac{3x}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3x\sqrt{5}}{5}$$

Example 16: $\frac{2}{\sqrt[3]{4x}}$

$$\frac{2}{\sqrt[3]{4x}} \times \frac{\sqrt[3]{16x^2}}{\sqrt[3]{16x^2}} = \frac{2\sqrt[3]{2^4x^2}}{\sqrt[3]{2^6x^3}} = \frac{4\sqrt[3]{2x^2}}{4x} = \frac{\sqrt[3]{2x^2}}{x}$$

To simplify (or divide) a fraction with a square root radical and another term in the denominator, you must multiply both the numerator and the denominator by the conjugate of the denominator. Then simplify.

Example 17: $\frac{2}{\sqrt{5}+1}$

$$\frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{2(\sqrt{5}-1)}{5-1} = \frac{2(\sqrt{5}-1)}{4} = \frac{\cancel{2}(\sqrt{5}-1)}{\cancel{4}\,2} = \frac{\sqrt{5}-1}{2}$$

Sample Problems:

Simplify each of the following completely.

1. $(-1)^6$
2. $(-5)^0$
3. $\left(\frac{1}{3}\right)^4$
4. -2^4
5. $-2(-5)^2$
6. $(10^2)^3$
7. $2^2 2^3$
8. $a^5 a^4 a^0$
9. $(4a^2)^{\frac{3}{2}}$
10. $\left(\frac{x^2}{y^3}\right)^2$
11. $\sqrt{\frac{4}{81}}$
12. $\sqrt{a^2}$
13. $\sqrt{64x^4w^6}$
14. $\sqrt[3]{y^3}$
15. $\sqrt[3]{-0.008}$
16. 5^{-2}
17. $16^{\frac{3}{2}}$
22. $\frac{2a^{-1}}{3b^{-2}}$
23. $\frac{2^{-2}-3^{-2}}{2^{-2}3^{-2}}$
24. $\frac{c^{-1}-d^{-1}}{c^{-2}-d^{-2}}$, $c \neq \pm d$, $c \neq 0$, $d \neq 0$
25. $(a^{-2}y^{-3})^{-1}$, $a \neq 0$, $y \neq 0$
26. $(4y)^{-2} \cdot \frac{y^4}{y^{\frac{9}{2}}}$, $y > 0$
27. $(8a^{-3}y^6)^{\frac{2}{3}}$
28. $\sqrt{18x^3y^4}$, $x \geq 0$
29. $\sqrt[5]{-x^6y^7}$
30. $\frac{\sqrt{15}}{\sqrt{3}}$
31. $\frac{\sqrt{2a}}{\sqrt{3a}}$, $a \neq 0$
32. $\sqrt{5x} \cdot \sqrt{20x}$, $x \geq 0$
33. $\frac{1}{\sqrt{10}}$
34. $\sqrt[3]{\frac{3}{25}}$
35. $\frac{1-\sqrt{2}}{2+\sqrt{3}}$
37. $\left(\sqrt[3]{5}\right)^4$
38. $\sqrt{\sqrt[4]{a}}$, $a \geq 0$
39. $\sqrt[3]{x} \cdot \sqrt{x}$, $x \geq 0$
40. $\sqrt{3} \cdot \sqrt[4]{27}$
41. $\left(4^{1/2} + 2^{-1}\right)^{-2}$
42. $(a^{-2} + b^{-2})^{-1}$, $a \neq 0$, $b \neq 0$
43. $(a^{-2} + b^{-2})a^{-2}b^{-2}$,

    $a \neq 0$, $b \neq 0$
44. $(x^2 - y^2)^{\frac{1}{2}}$
45. $(a^2 + b^2)^{\frac{1}{2}}$
46. $\sqrt{32} + \sqrt{18} - \sqrt{8} - \sqrt{3}$
47. $\sqrt{63} + \sqrt{24} - 2\sqrt{6} - 2\sqrt{7}$
48. $3\sqrt{50} - 2\sqrt{18}$
49. $2\sqrt{45} + 3\sqrt{20}$

Write in scientific notation

50. 5231554.156
51. 0.000031501

18. $(-8)^{\frac{1}{3}}+(-8)^{\frac{2}{3}}$

19. $9^{\frac{-1}{2}}$

20. $\left(\frac{1}{5}\right)^{-1}$

21. $x^{0}x^{\frac{5}{3}}x^{\frac{1}{6}}$, $x \geq 0$

36. $\frac{\sqrt{3}+2\sqrt{2}}{3\sqrt{2}+2\sqrt{3}}$

Write in decimal notation

52. $3.22215 * 10^{1}$
53. $1.20501 * 10^{-2}$

Solutions

1. 1
2. 1
3. $\frac{1}{81}$
4. -16
5. -50
6. $10^6 = 1{,}000{,}000$
7. $2^5 = 32$
8. a^9
9. $8a^3$
10. $\frac{x^4}{y^6}$
11. $\frac{2}{9}$
12. a
13. $8x^2w^3$
14. y
15. -.2
16. $\frac{1}{25}$
17. 64
18. 2
19. $\frac{1}{3}$
21. $x^{11/6}$
22. $\frac{2b^2}{3a}$
23. 5
24. $\frac{cd}{c+d}$
25. a^2y^3
26. $\frac{1}{16y^{5/2}}$
27. $\frac{4y^4}{a^2}$
28. $3xy^2\sqrt{2x}$
29. $-xy\sqrt[5]{xy^2}$
30. $\sqrt{5}$
31. $\frac{\sqrt{6}}{3}$
32. 10x
33. $\frac{\sqrt{10}}{10}$
34. $\frac{\sqrt[3]{15}}{5}$
35. $2 - 2\sqrt{2} - \sqrt{3} + \sqrt{6}$
37. $5^{\frac{4}{3}} = 5\sqrt[3]{5}$
38. $a^{\frac{1}{8}} = \sqrt[8]{a}$
39. $x^{\frac{5}{6}}$
40. $3^{\frac{5}{4}} = 3\sqrt[4]{3}$
41. $\frac{4}{25}$
42. $\frac{a^2b^2}{a^2+b^2}$
43. $\frac{a^2+b^2}{a^4b^4}$
44. does not simplify
45. does not simplify
46. $5\sqrt{2} - \sqrt{3}$
47. $\sqrt{7}$
48. $9\sqrt{2}$
49. $12\sqrt{5}$
50. 5.231554156×10^6
51. 3.1501×10^{-5}
52. 32.2215
53. 0.0120501

18. 2

19. $\frac{1}{3}$

20. 5

34. $\frac{\sqrt[3]{15}}{5}$

35. $2 - 2\sqrt{2} - \sqrt{3} + \sqrt{6}$

36. $\frac{-\sqrt{6} + 6}{6}$

51. $3.1501 \times 10^{\text{[illegible]}}$

52. 32.2215

53. 0.0120501