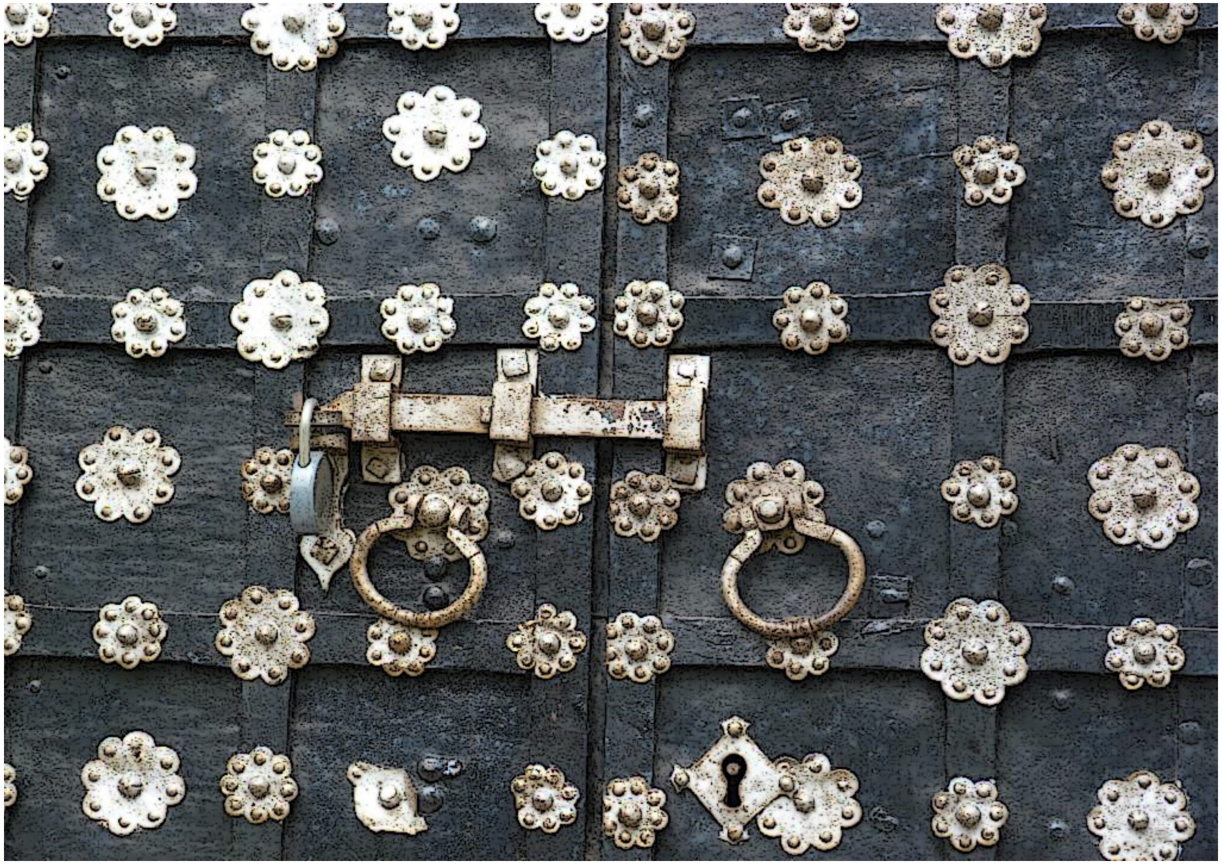


The set of complex numbers

Cartesian (algebraic) form of complex numbers:

$$z = x + i\,y$$

$x = Re\,(z)$ – real part of z

$y = Im\,(z)$ – imaginary part of z

Set of complex numbers:

$$\mathbb{C} = \{\, z \mid z = x + i\,y;\ \ x,\ y \in \mathbb{R} \,\}$$

Exercise:

Represent the sets of real, imaginary and complex numbers by an Euler-Venn-diagram.

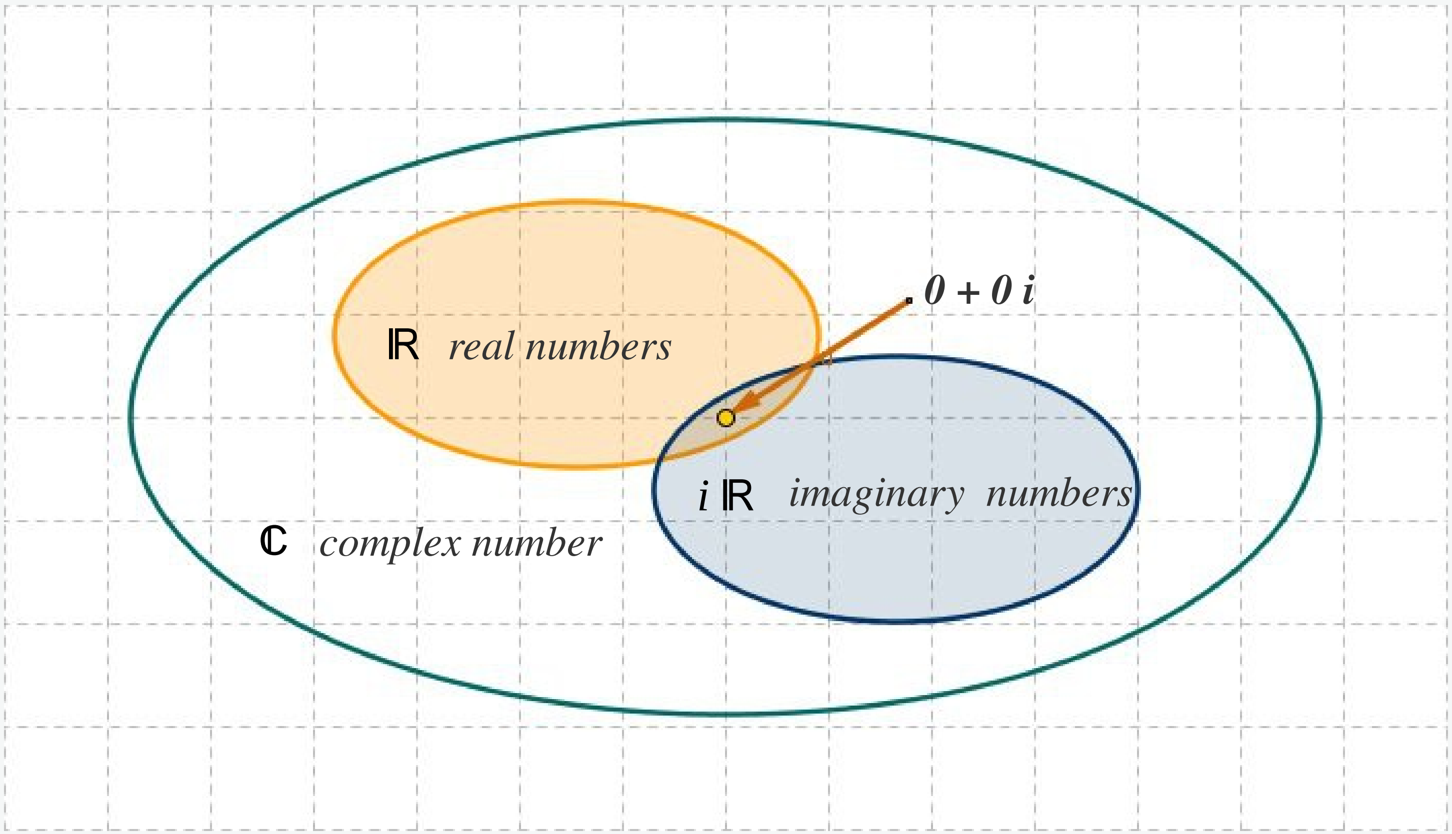
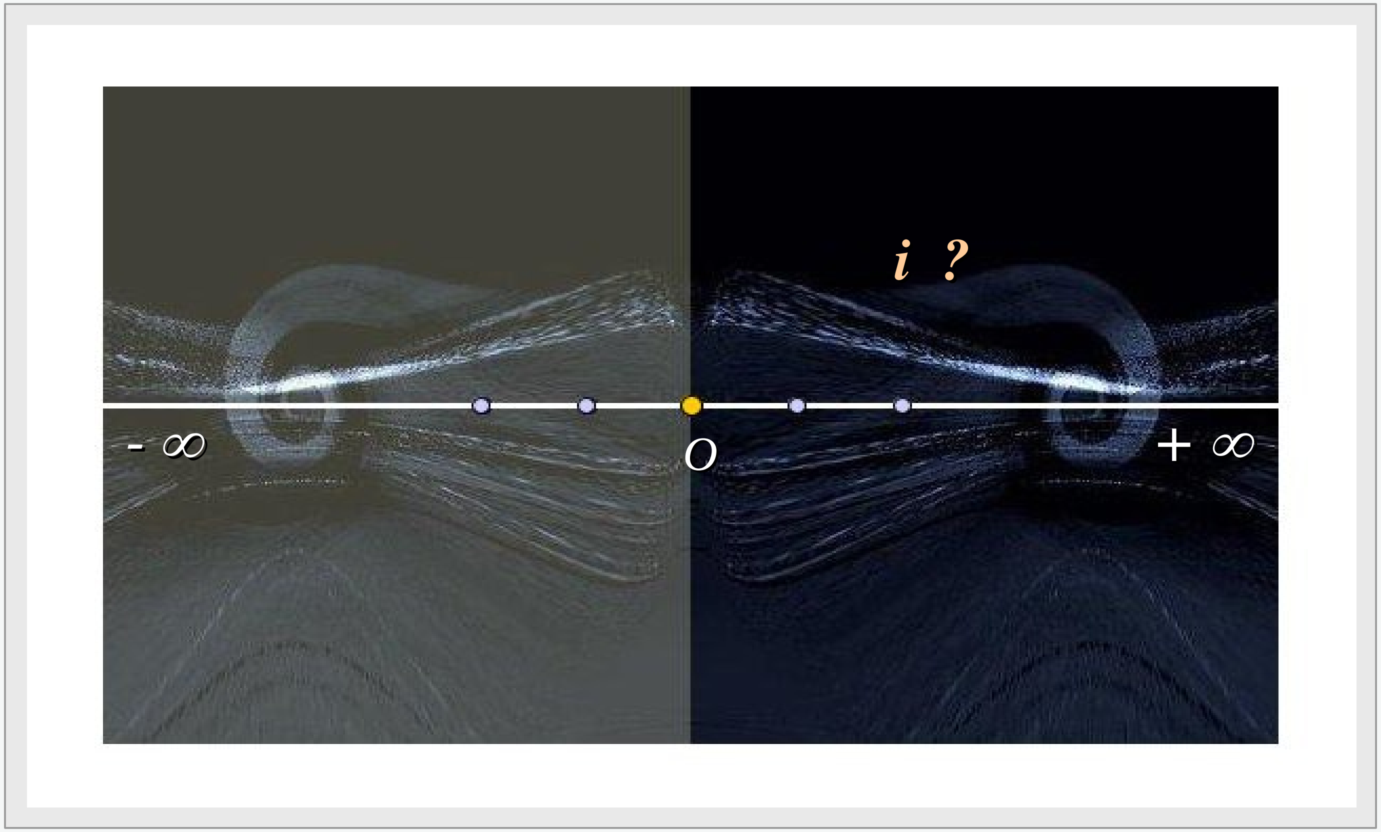


Fig. 3: Euler-Venn-diagram of the sets of real, imaginary and complex numbers

The real and imaginary numbers are subsets of the complex numbers. Zero is the only complex number which is both, real and imaginary.

How can we represent a complex number geometrically?



http://www.flickr.com/photos/24117329@N06/3597678605/in/pool-the_infinite

Fig. 4: A representation of the number line

The idea of a number line is useful for real numbers, but not sufficient to represent complex numbers.

To better understand, that the number line does not help to represent complex numbers, we first ask again, how can we find the square root of a negative number, for example

$$\sqrt{-1} \ ,$$

or what is the meaning of the equation

$$x^2 = -1 \ .$$

Here x is a number, which leads to a negative number, if multiplied by itself. Such a number is neither positive, because a positive number times a positive number is a positive number, nor negative, because a negative number times a negative number is a positive number again. Therefore, there is no number on the number line of real numbers which is the square root of a negative number.

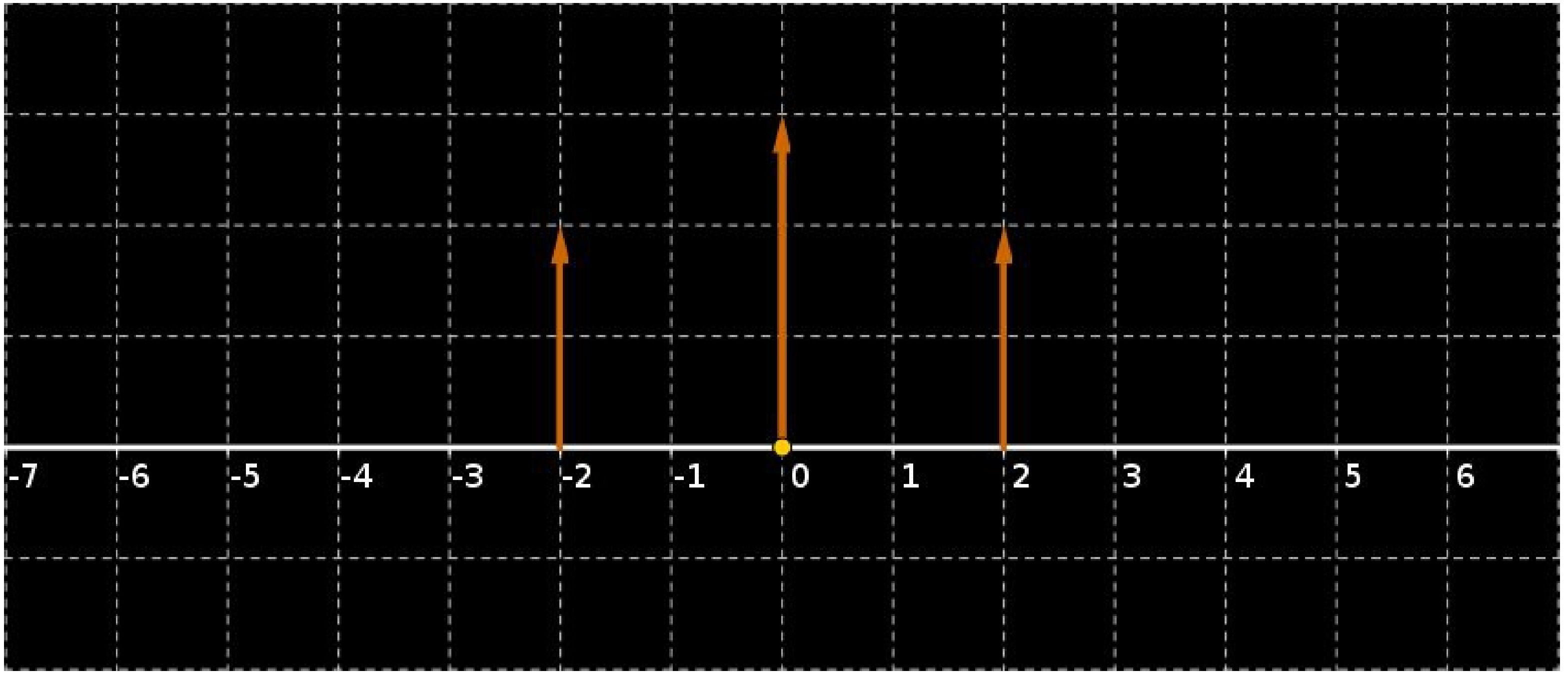


Fig. 5-1: Towards a geometrical representation of complex numbers

Another approach is more useful: one has to abandon the idea, that numbers are sitting on a line. Because the real and imaginary part of complex numbers are independent, we can take them as Cartesian coordinates of a plane, and we get the complex plane or Gauss plane.

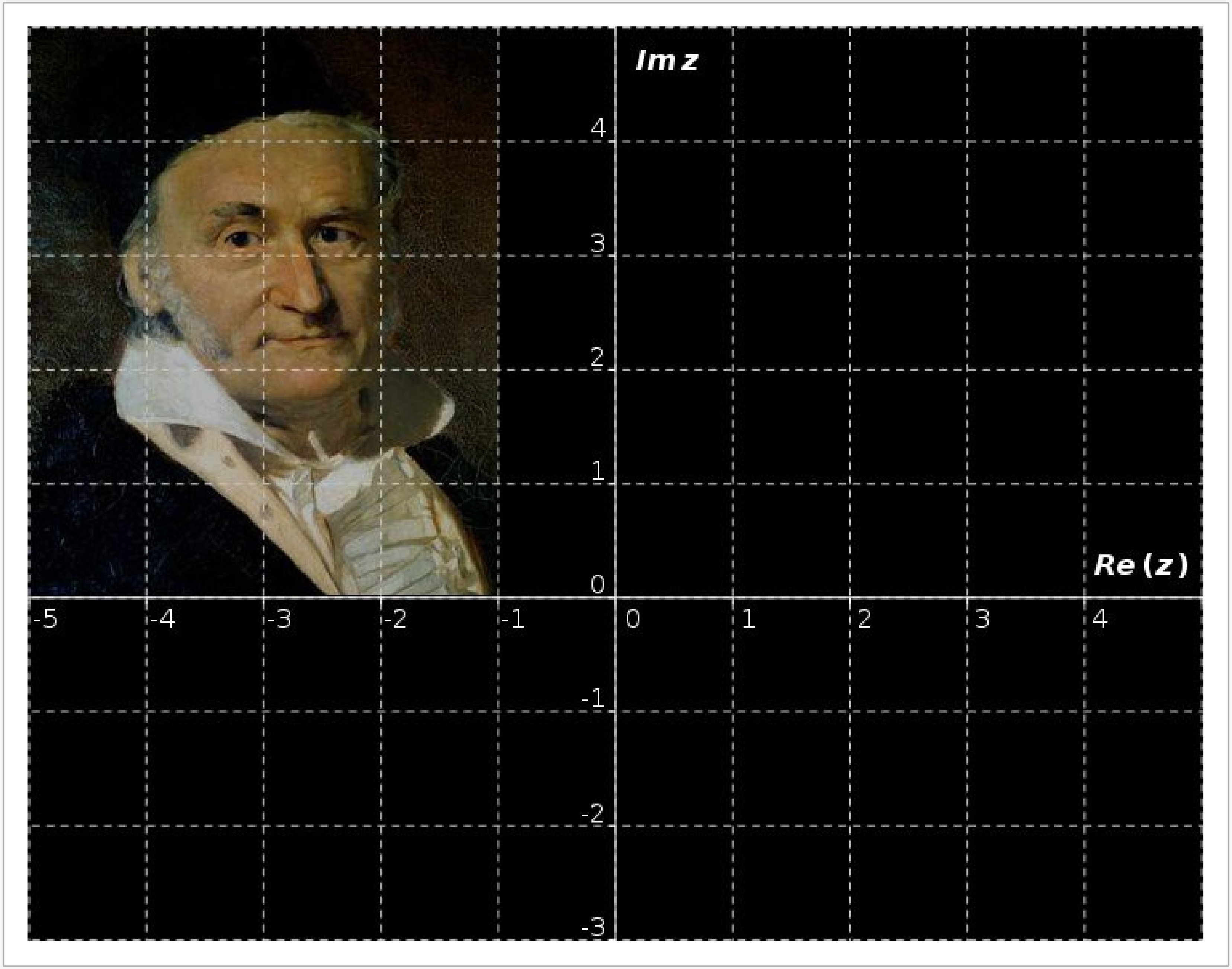


Fig. 5-2: Gauss plane

Carl Friedrich Gauß
(1777-1855)

The work of Carl Friedrich Gauss was a key to further progress of the theory of complex numbers in early 19th century. He introduced their geometrical interpretation as points in a plane, the complex plane or Gauss plane.

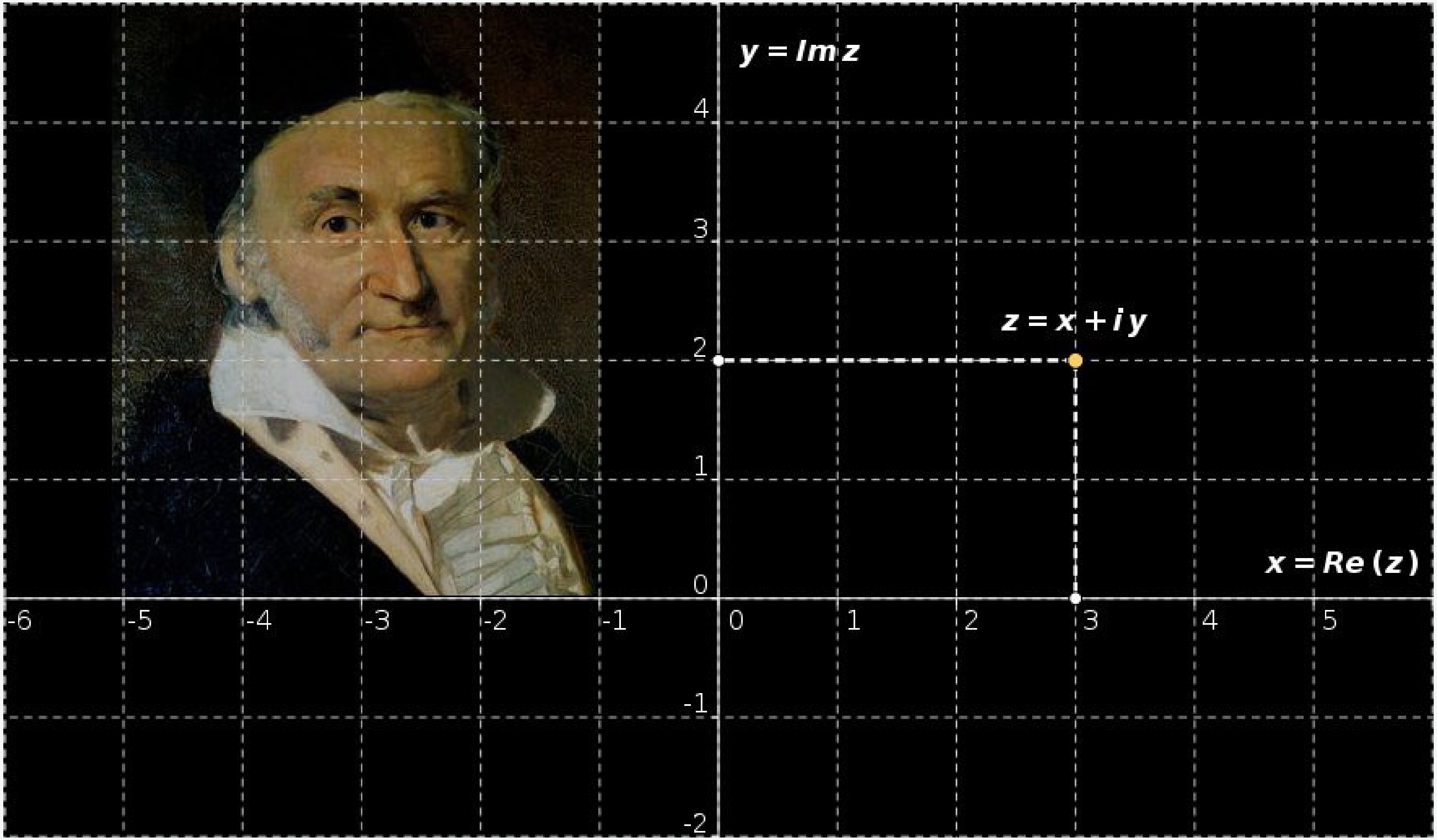


Fig. 5-3: Representation of a complex number $z = x + i\,y$ in the Gauss plane

Re (z), Im (z) – Cartesian coordinates of a point of the *x,y*-Ebene.

real numbers	$z = x + i \cdot 0 = x$	$\mathrm{Im}(z) = 0$	real axis
imaginary numbers	$z = 0 + i \cdot y = i\,y$	$\mathrm{Re}(z) = 0$	imaginary axis

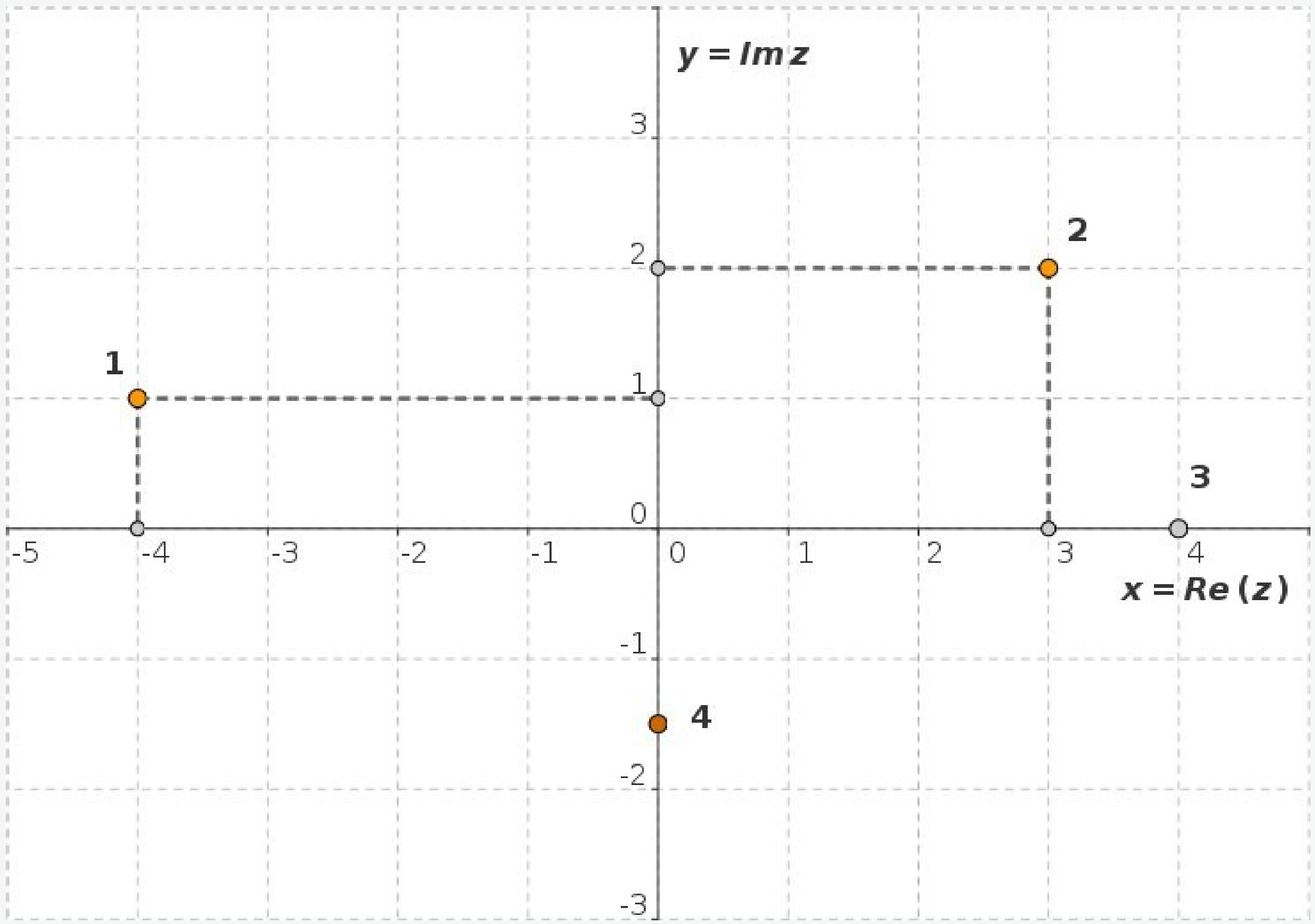


Fig. 6-1: Representation of complex numbers by points of the complex plane (Gauss plane)

1 : $z_1 = -4 + i$, 2 : $z_2 = 3 + 2i$, 3 : $z_3 = 4 + 0i$, 4 : $z_4 = 0 - 1.5i$

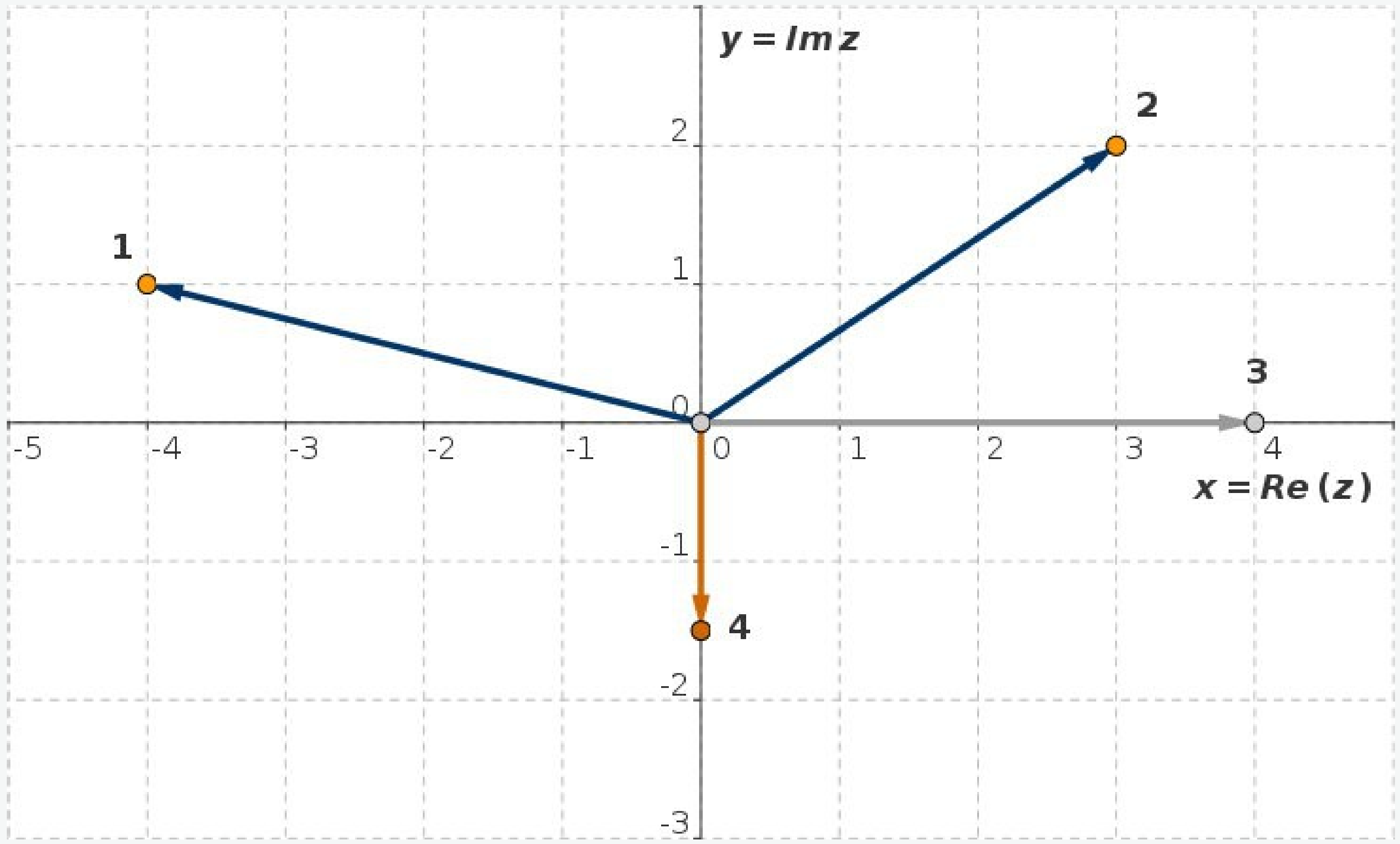


Fig. 6-2: Representation of complex numbers by position vectors

Complex numbers can also be represented by position vectors which are drawn as arrows from the origin to the position of the number in the complex plane (figure 6-2).

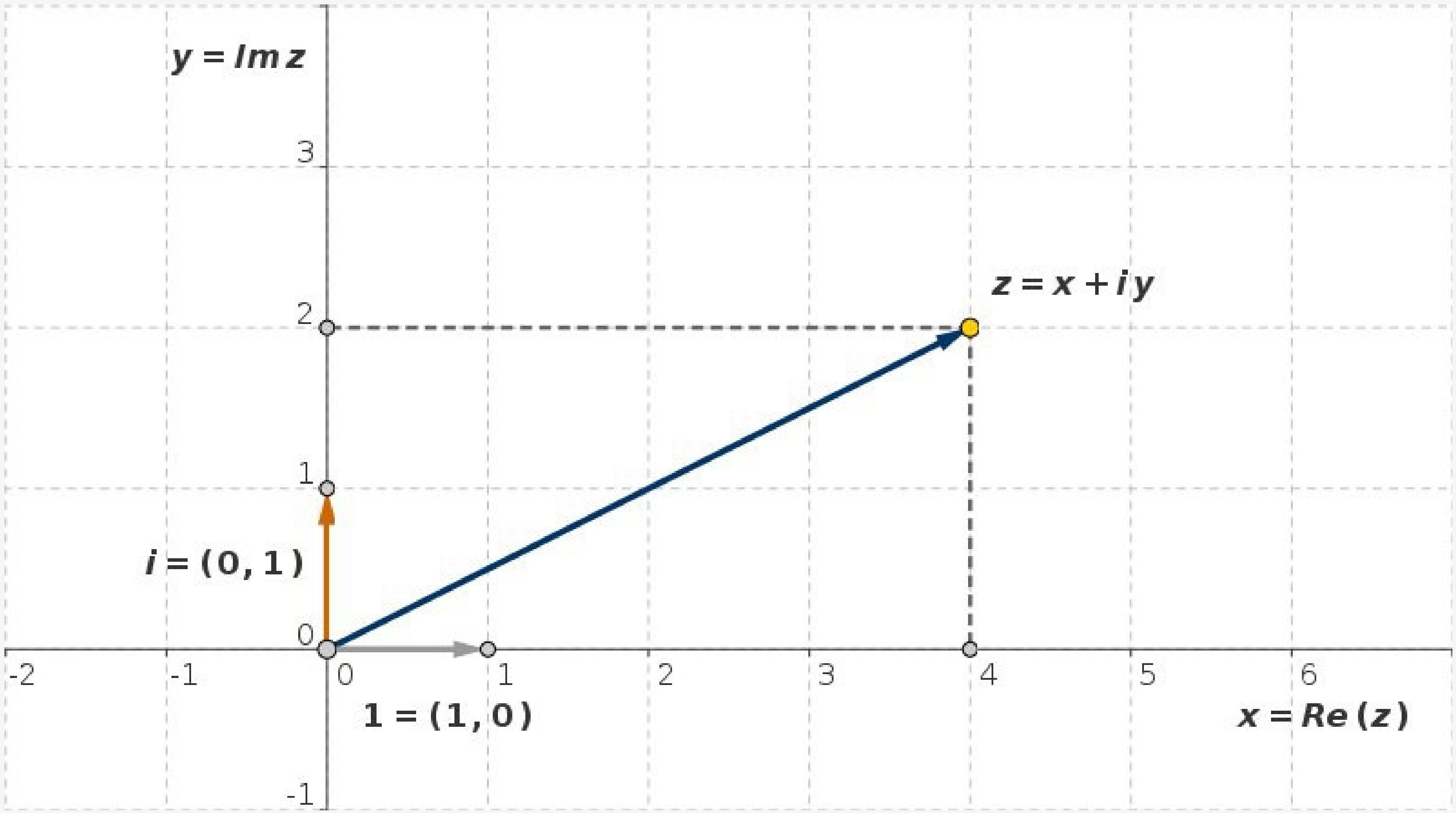


Fig. 6-3: Representation of a complex number by a position vector in the complex plane