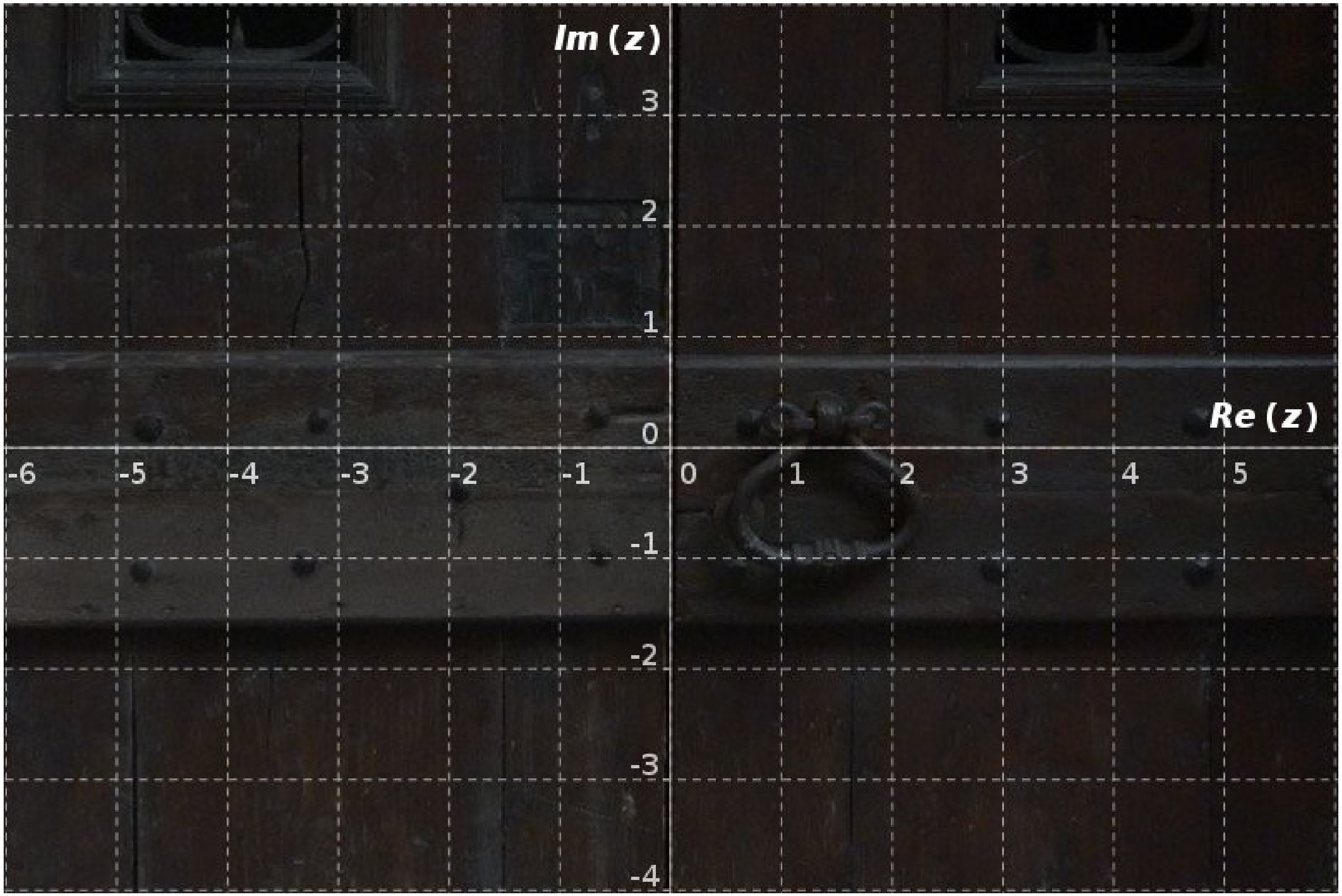
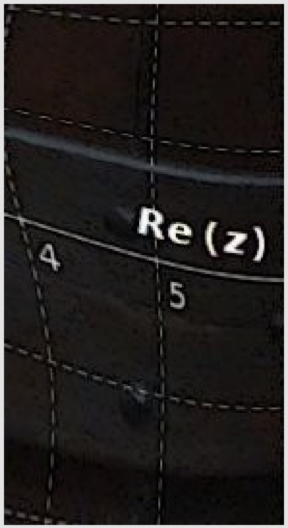


Sets of complex numbers on the complex plane

Exercises, section 1

Complex plane (Gauss plane): Exercise 5



Represent the following sets on the complex plane:

$$M_a = \{ z \in \mathbb{C} \mid \mathrm{Re}(z) \geqslant 1 \}$$

$$M_b = \{ z \in \mathbb{C} \mid \mathrm{Im}(z) \geqslant -3 \}$$

$$M_c = \{ z \in \mathbb{C} \mid \mathrm{Re}(z) \geqslant -1, \quad \mathrm{Im}(z) \geqslant 1 \}$$

$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant \mathrm{Re}(z) < 3 \}$$

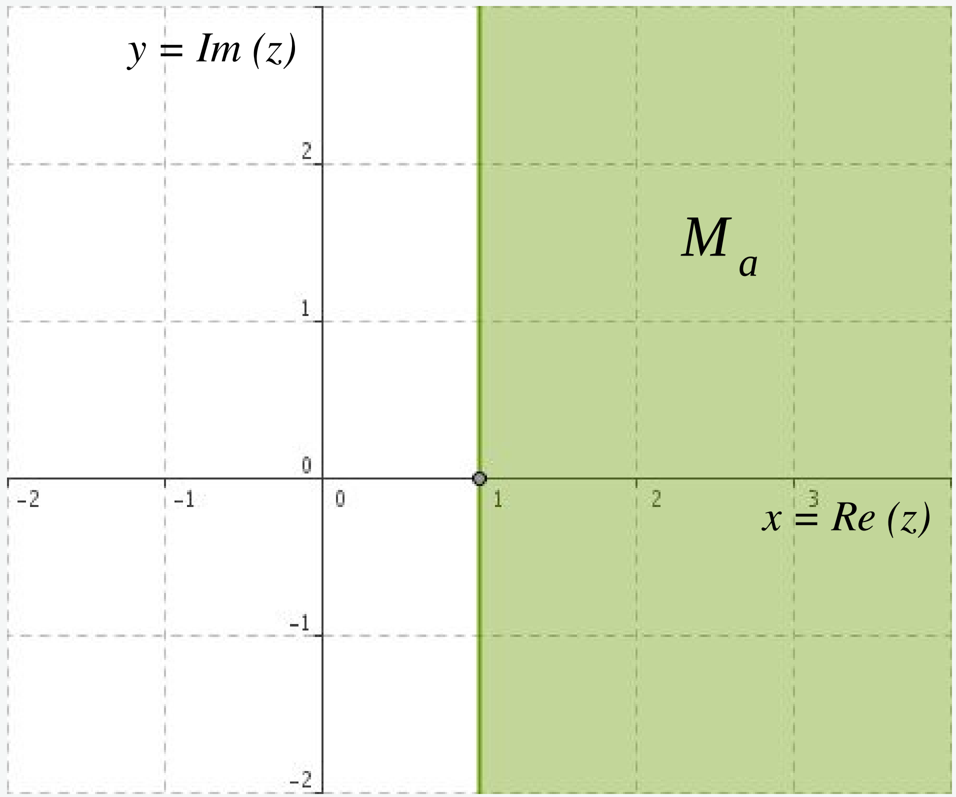
$$M_e = \{ z \in \mathbb{C} \mid -3 \leqslant \mathrm{Re}(z) \leqslant 2, \quad -1 < \mathrm{Im}(z) < 2 \}$$

$$M_f = \{ z \in \mathbb{C} \mid \mathrm{Im}(z) - \mathrm{Re}(z) \leqslant 2 \}$$

$$M_g = \{ z \in \mathbb{C} \mid \mathrm{Re}(z) + \mathrm{Im}(z) = 1 \}$$

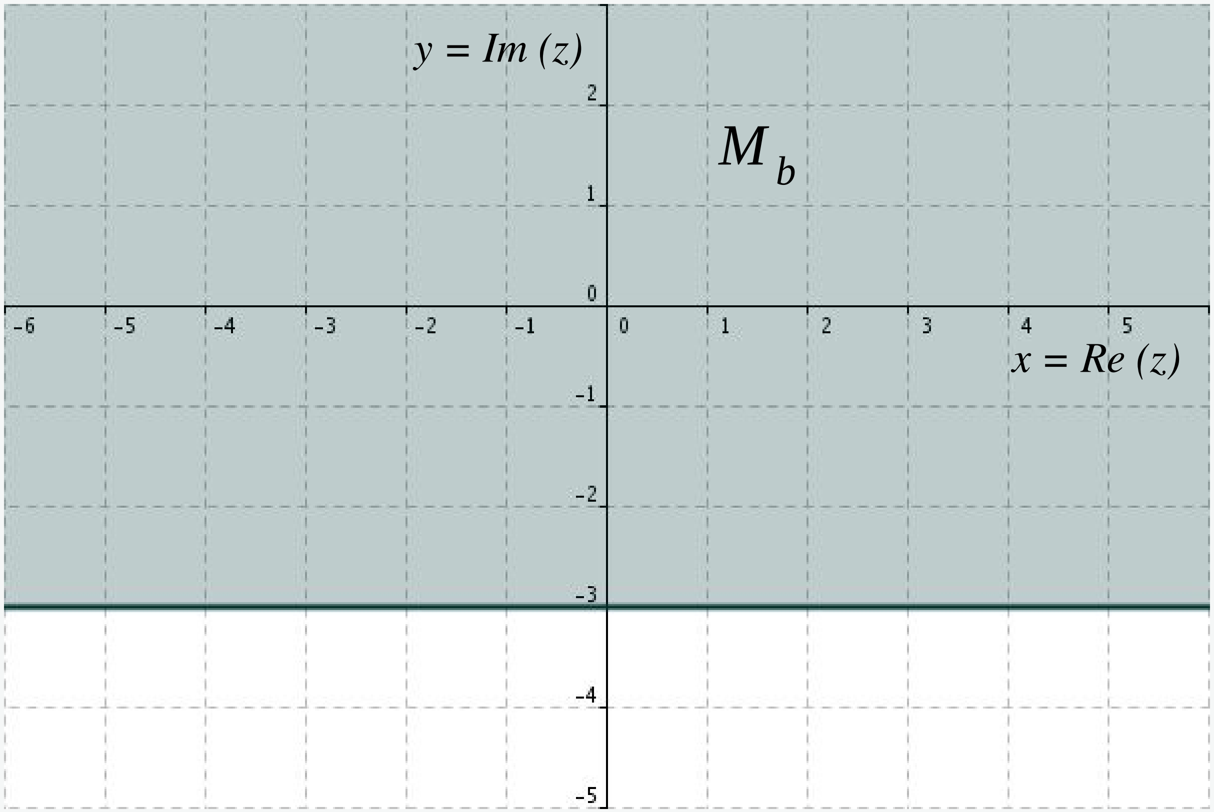
$$M_h = \{ z \in \mathbb{C} \mid |z| = 2 \}$$

$$M_i = \{ z \in \mathbb{C} \mid |z| < 2 \}$$



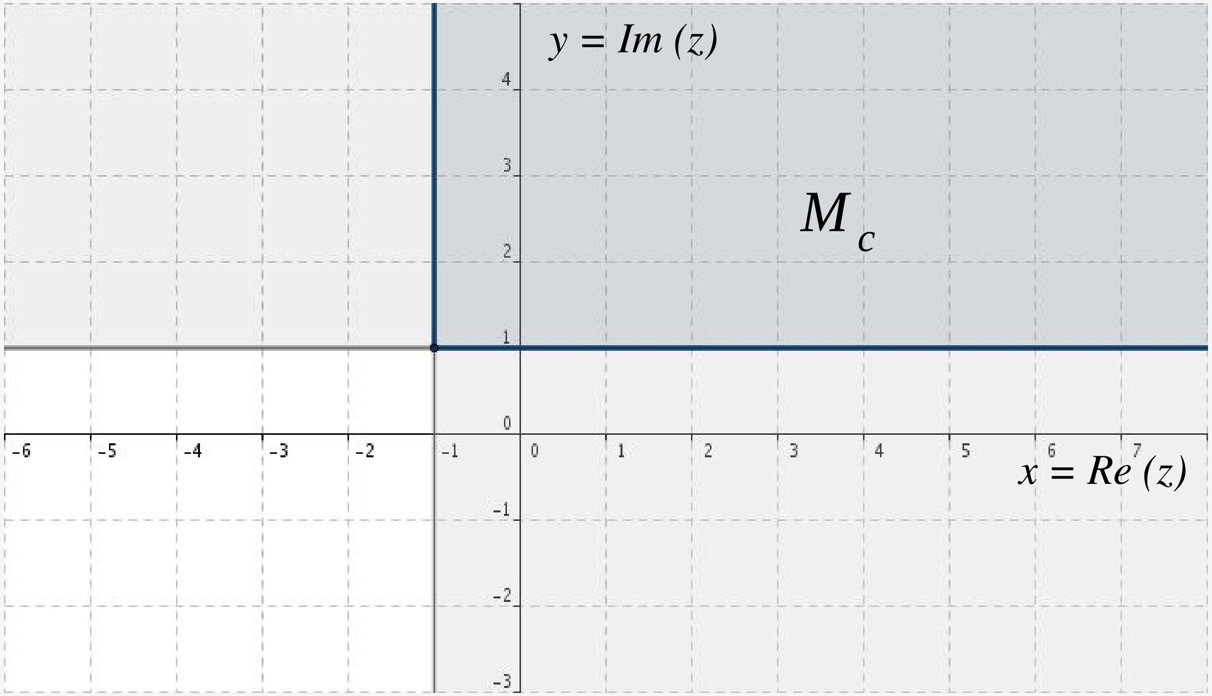
$$M_a = \{ z \in \mathbb{C} \mid \mathrm{Re}(z) \geqslant 1 \}$$

$$z = x + i\, y\,, \qquad \mathrm{Re}(z) = x \geqslant 1$$



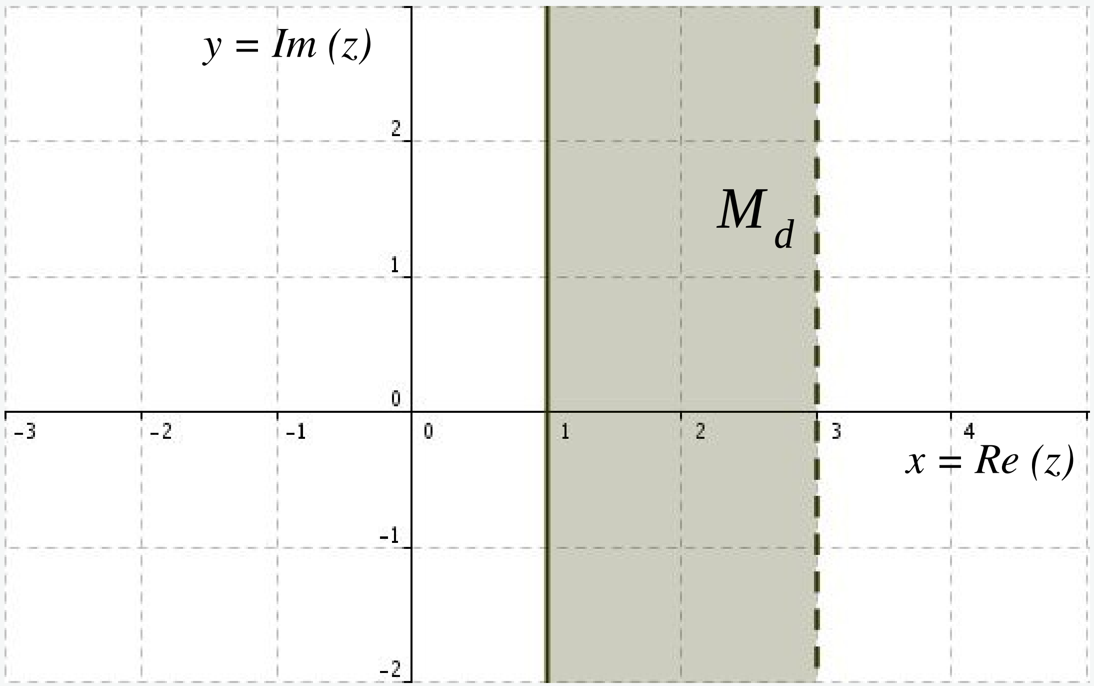
$$M_b = \{ z \in \mathbb{C} \mid \mathrm{Im}(z) \geqslant -3 \}$$

$$z = x + i\,y, \quad \mathrm{Im}(z) = y \geqslant -3$$



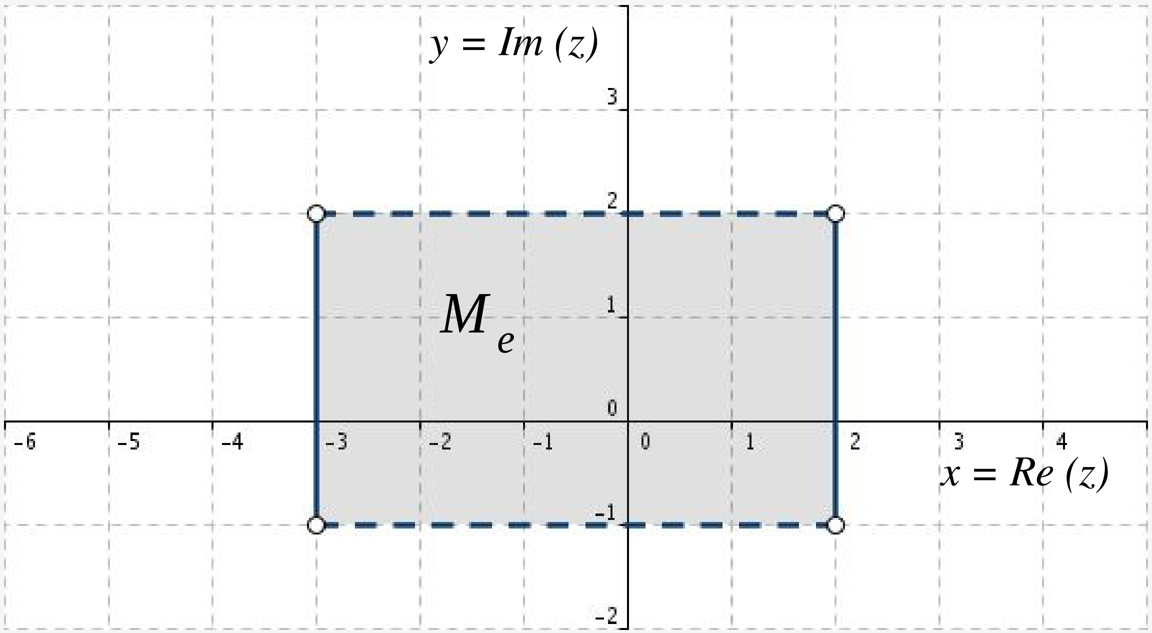
$$M_c = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) \geqslant -1 , \quad \operatorname{Im}(z) \geqslant 1 \}$$

$$z = x + i\, y , \quad \operatorname{Re}(z) = x \geqslant -1, \quad \operatorname{Im}(z) = y \geqslant 1$$

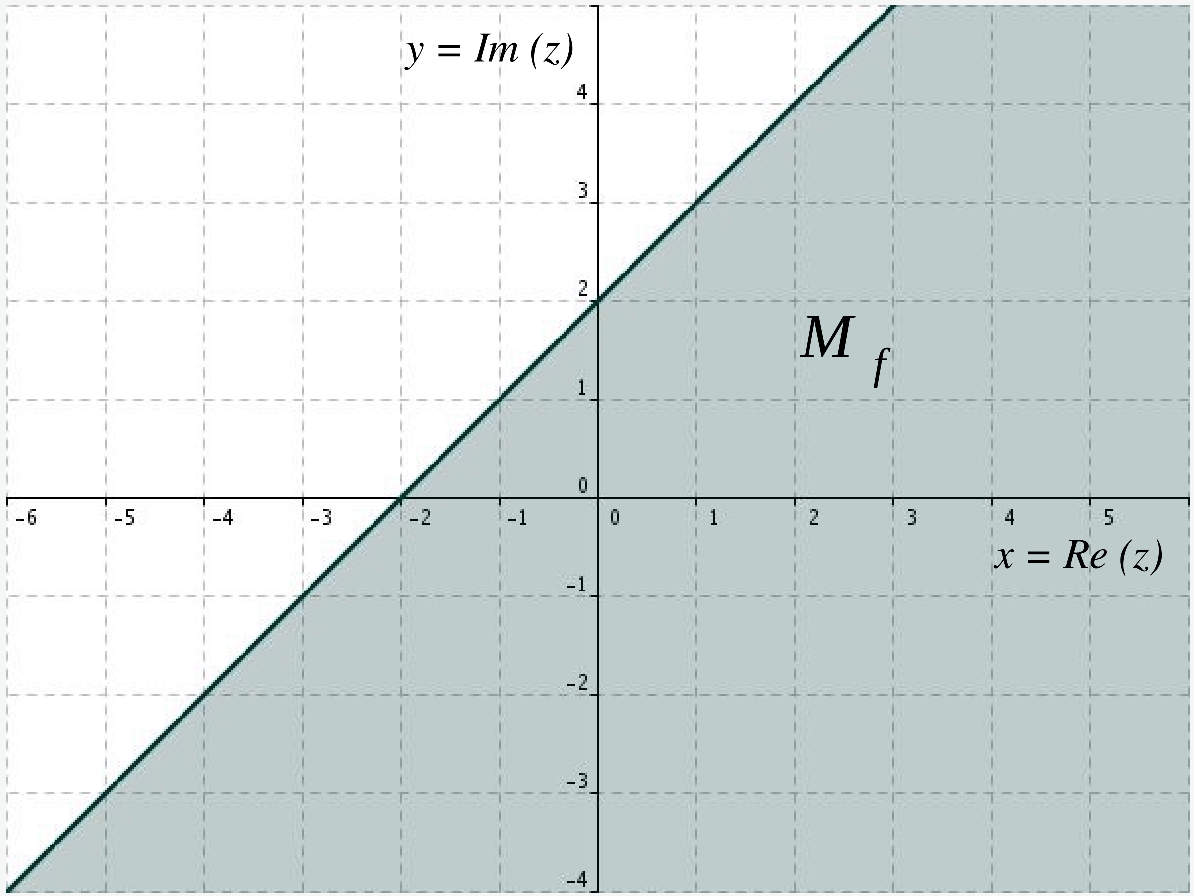


$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant \mathrm{Re}(z) < 3 \}$$

$$z = x + i\,y, \quad 1 \leqslant \mathrm{Re}(z) = x < 3$$

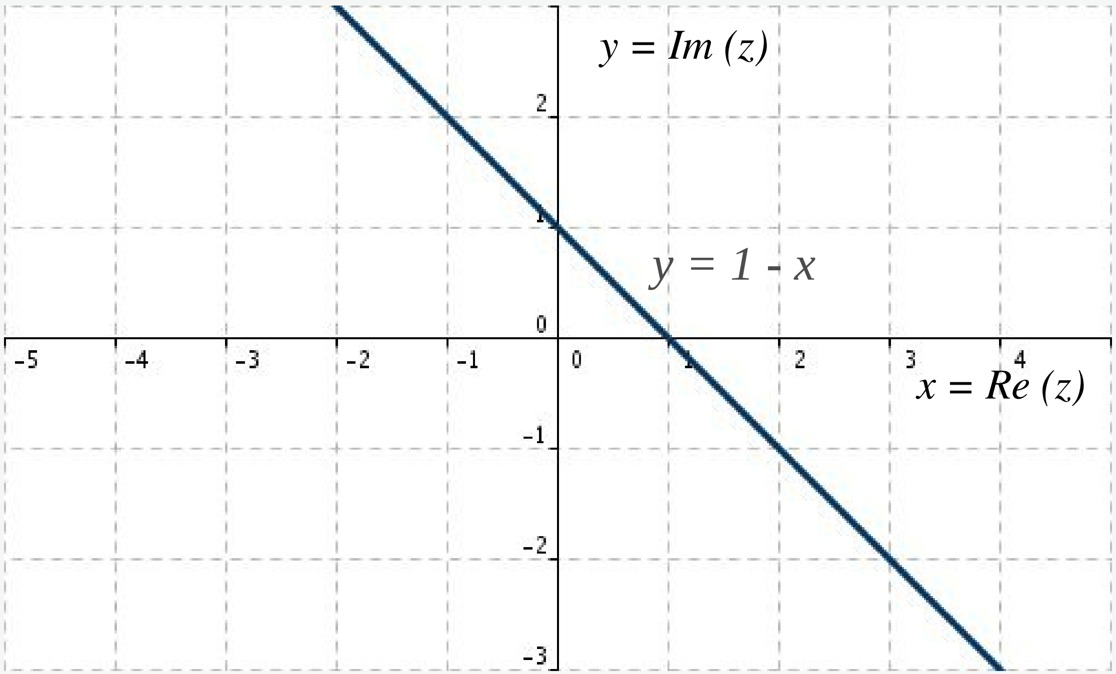


$$M_e = \{\, z \in \mathbb{C} \mid -3 \leqslant \operatorname{Re}(z) = x \leqslant 2, \quad -1 < \operatorname{Im}(z) = y < 2 \,\}$$



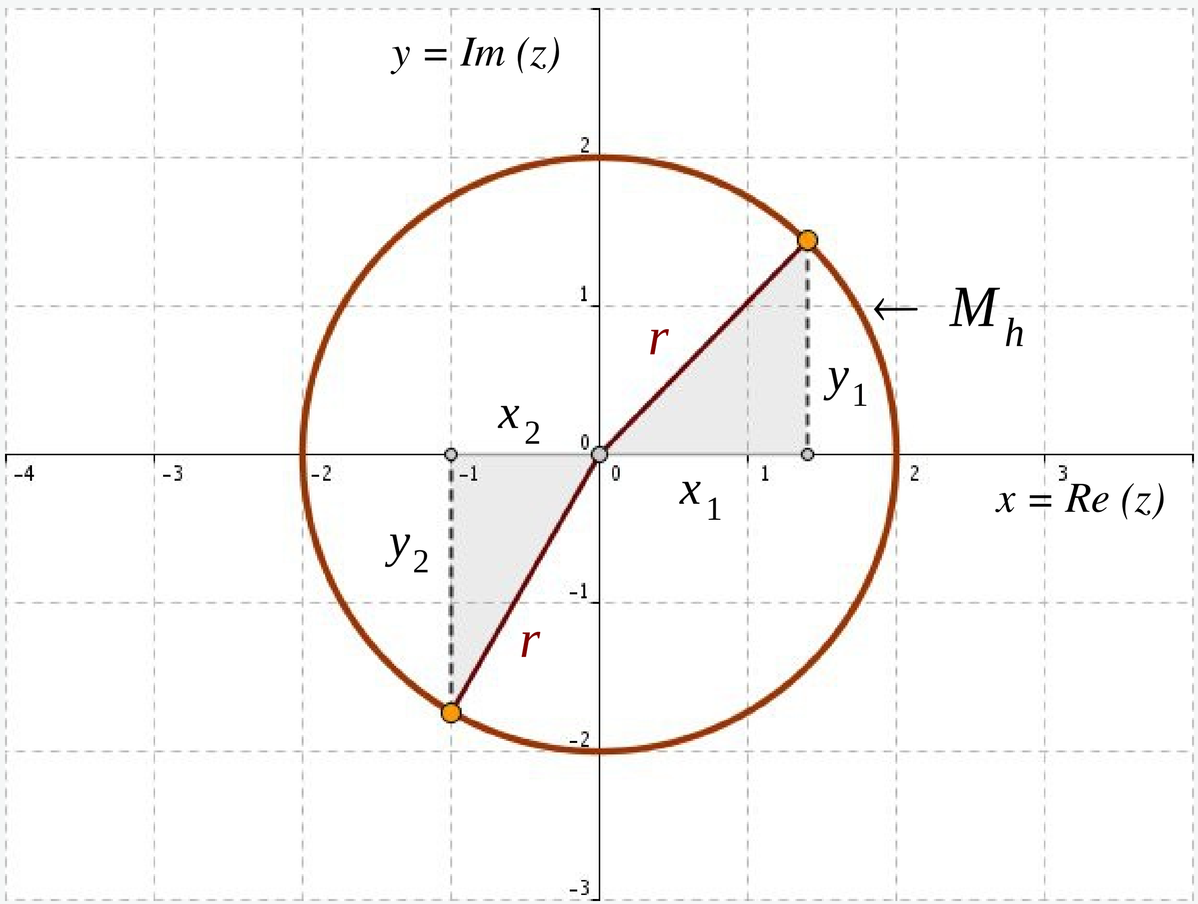
$$M_f = \{ z \in \mathbb{C} \mid \operatorname{Im}(z) - \operatorname{Re}(z) \leqslant 2 \}$$

$$\operatorname{Im}(z) - \operatorname{Re}(z) = y - x \leqslant 2$$



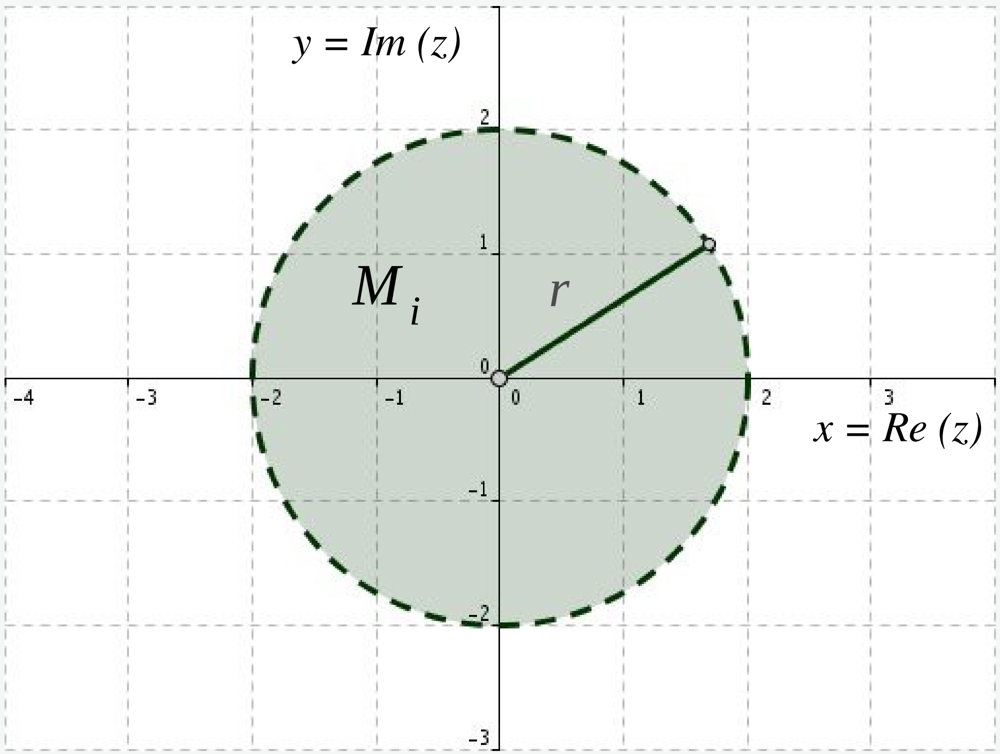
$$M_g = \{\, z \in \mathbb{C} \mid \operatorname{Re}(z) + \operatorname{Im}(z) = 1 \,\}$$

$$\operatorname{Re}(z) + \operatorname{Im}(z) = x + y = 1 \quad \Leftrightarrow \quad y = 1 - x$$



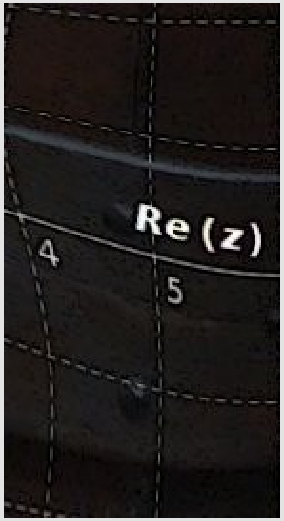
$$M_h = \{ z \in \mathbb{C} \mid |z| = 2 \}$$

$$|z| = \sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2} = r = 2$$



$$M_i = \{ z \in \mathbb{C} \mid |z| < 2 \}$$

$$|z| = \sqrt{x^2 + y^2} < 2$$



Represent the following sets on the complex plane:

$$M_a = \{ z \in \mathbb{C} \mid 1 \leqslant |\mathrm{Re}(z)| \leqslant 3 , \quad 2 \leqslant \mathrm{Im}(z) \leqslant 4 \}$$

$$M_b = \{ z \in \mathbb{C} \mid |z| \leqslant 3, \quad \mathrm{Re}(z) \geqslant 0 , \quad \mathrm{Im}(z) \geqslant 0 \}$$

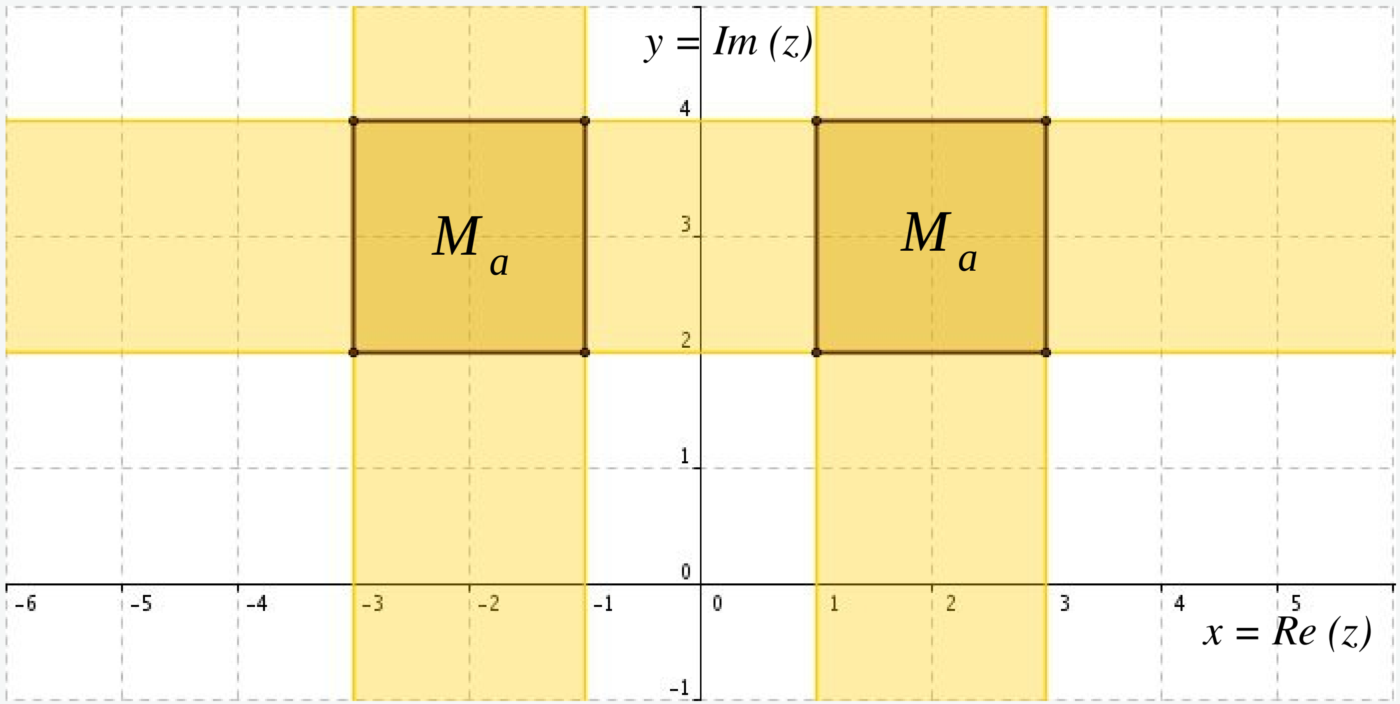
$$M_c = \{ z \in \mathbb{C} \mid A \cap B \cap C \}$$

$$A : \mathrm{Re}(z) + \mathrm{Im}(z) \geqslant 1$$

$$B : -\mathrm{Re}(z) + \mathrm{Im}(z) \geqslant -3$$

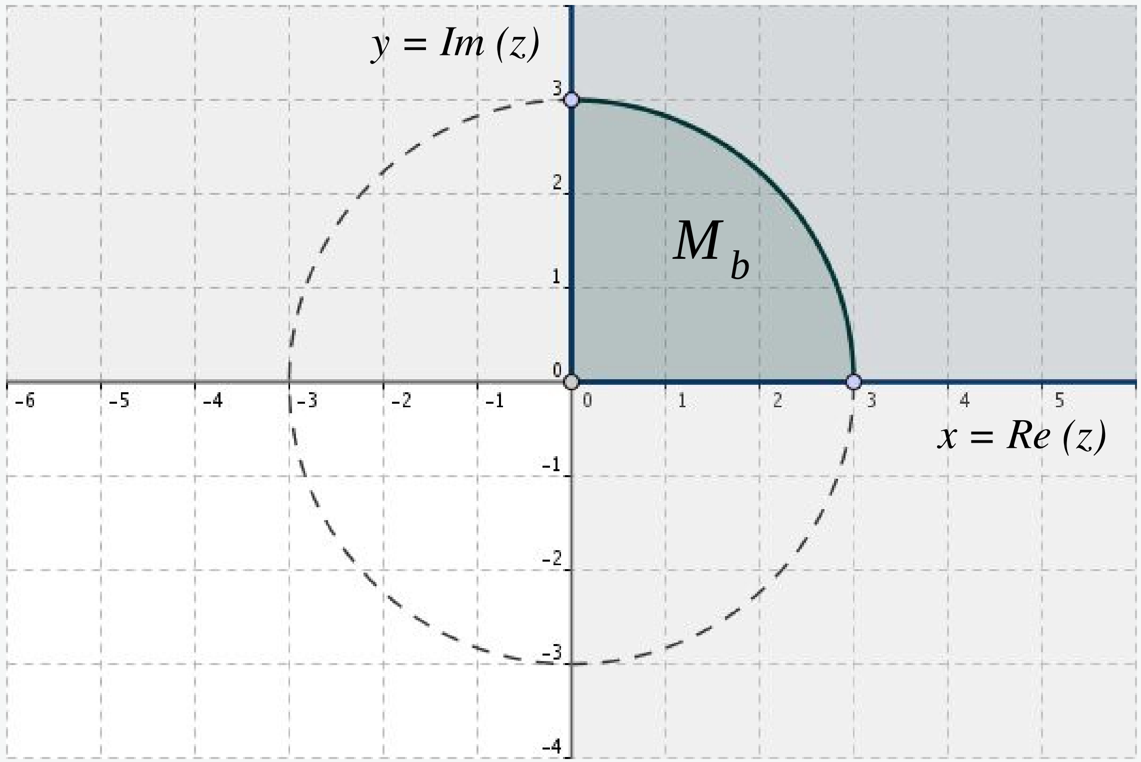
$$C : \mathrm{Im}(z) \leqslant 4$$

$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant |z + 2 + i| \leqslant 2 \}$$

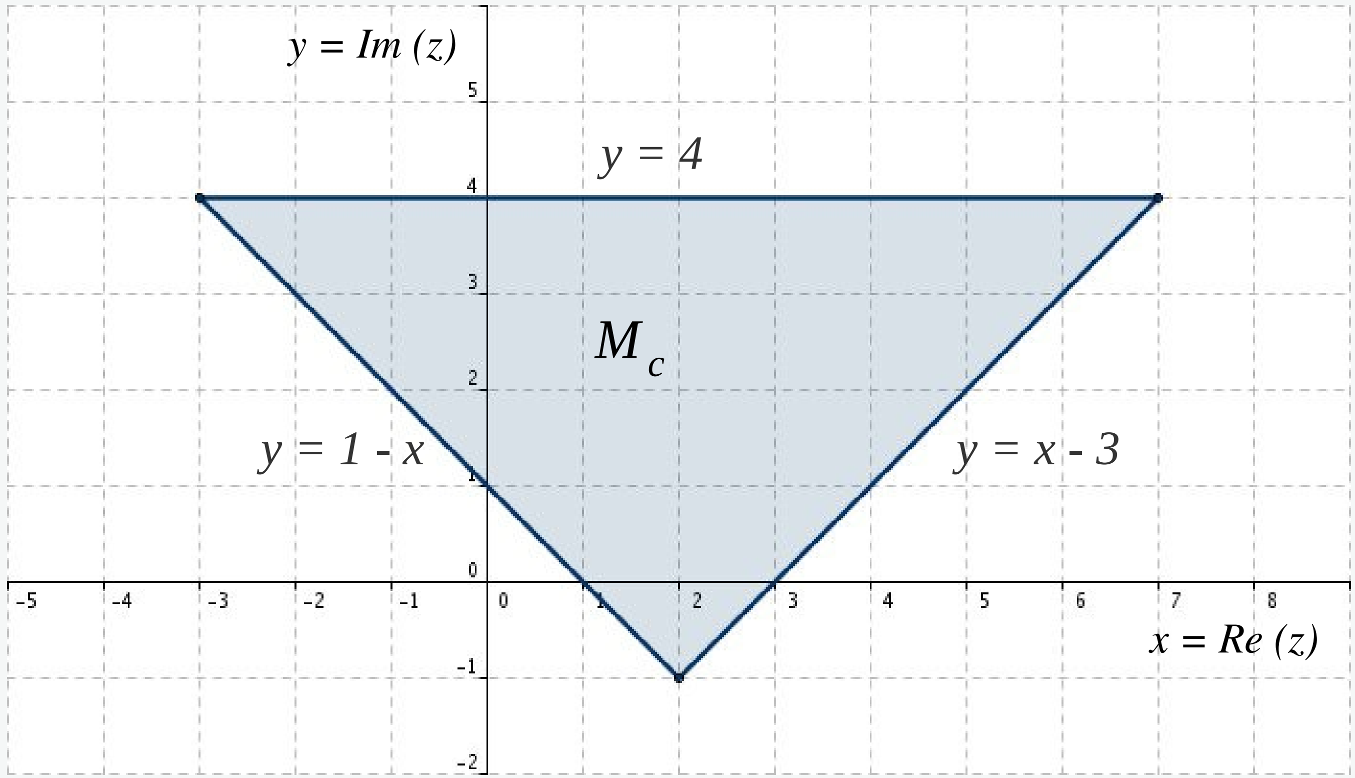


$$M_a = \{ z \in \mathbb{C} \mid 1 \leqslant |\operatorname{Re}(z)| \leqslant 3, \quad 2 \leqslant \operatorname{Im}(z) \leqslant 4 \}$$

$$x \in [-3, -1] \cup [1, 3], \qquad y \in [2, 4]$$

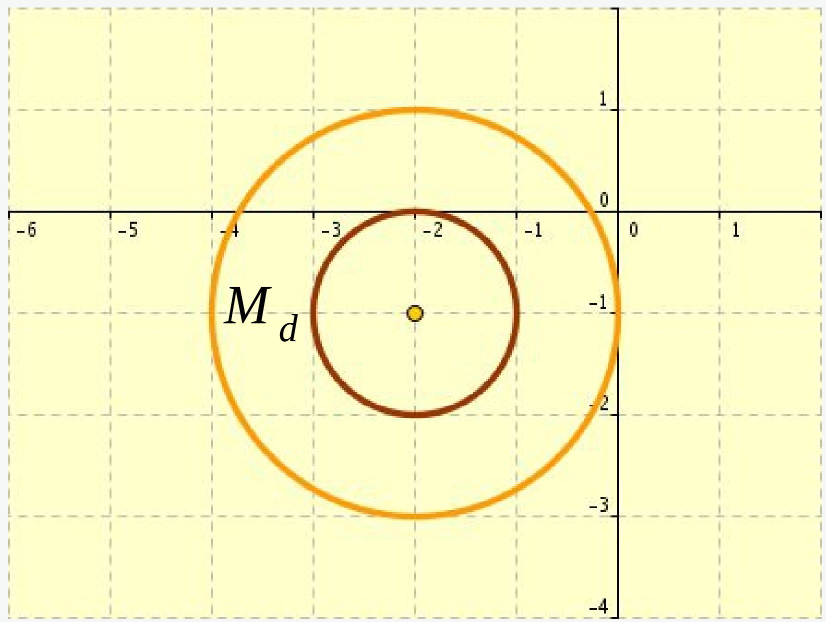


$$M_b = \{ z \in \mathbb{C} \mid |z| \leqslant 3, \quad \mathrm{Re}(z) \geqslant 0, \quad \mathrm{Im}(z) \geqslant 0 \}$$



$$M_c = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) + \operatorname{Im}(z) \geqslant 1 \cap -\operatorname{Re}(z) + \operatorname{Im}(z) \geqslant -3 \cap \operatorname{Im}(z) \leqslant 4 \}$$

Complex plane: Solution 6d

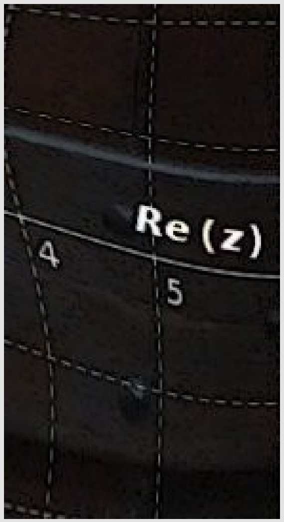


$$M_d = \{ z \in \mathbb{C} \mid 1 \leqslant |z + 2 + i| \leqslant 2 \}$$

$z + 2 + i = (x + 2) + i\,(y + 1)$, $\quad |z + 2 + i| = \sqrt{(x + 2)^2 + (y + 1)^2}$

$(x + 2)^2 + (y + 1)^2 = 1$ – circle with centre $P = (-2, -1)$ and radius $R = 1$

$(x + 2)^2 + (y + 1)^2 = 4$ – circle with centre $P = (-2, -1)$ and radius $R = 2$



Exercise 7:

Determine the geometric meaning of the equation below:

$$|z| = \mathrm{Re}(z) + 1$$

Exercise 8:

Determine the geometric meaning of the following inequality:

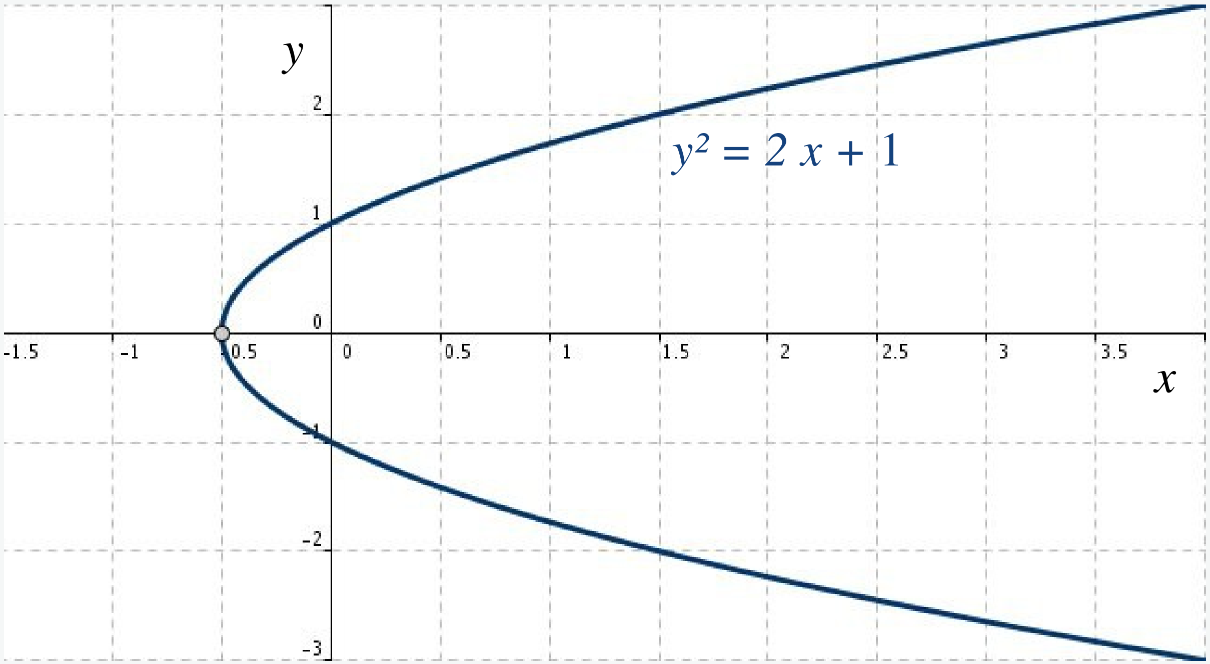
$$|z-1| \geqslant 2\,|z-i|$$

$$|z| = \mathrm{Re}\,(z) + 1$$

$$z = x + i\,y\,, \quad |z| = \sqrt{x^2 + y^2}\,, \quad \mathrm{Re}\,(z) + 1 = x + 1$$

$$\Rightarrow \quad \sqrt{x^2 + y^2} = x + 1\,, \quad x^2 + y^2 = (x + 1)^2 = x^2 + 2\,x + 1 \quad \Rightarrow$$

$y^2 = 2\,x + 1$ – equation of a parabola



$$|z-1| \geqslant 2\,|z-i|$$

$$|z-1| \geqslant 2\;|z-i| \quad \Leftrightarrow \quad |x-1+i\,y| \geqslant 2\;|x+i\,(y-1)| \quad \Rightarrow$$

$$\sqrt{(x-1)^2+y^2} \;\geqslant\; 2\sqrt{x^2+(y-1)^2}$$

$$(x-1)^2+y^2 \;\geqslant\; 4\left(x^2+(y-1)^2\right)$$

$$\left(x+\frac{1}{3}\right)^2+\left(y-\frac{4}{3}\right)^2 \leqslant \frac{8}{9}$$

$$R=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}\,, \qquad M=\left(-\frac{1}{3}\,,\;\frac{4}{3}\right)$$

The set of points, which obey the inequality, are all points inside a circle with radius R and centre point M.