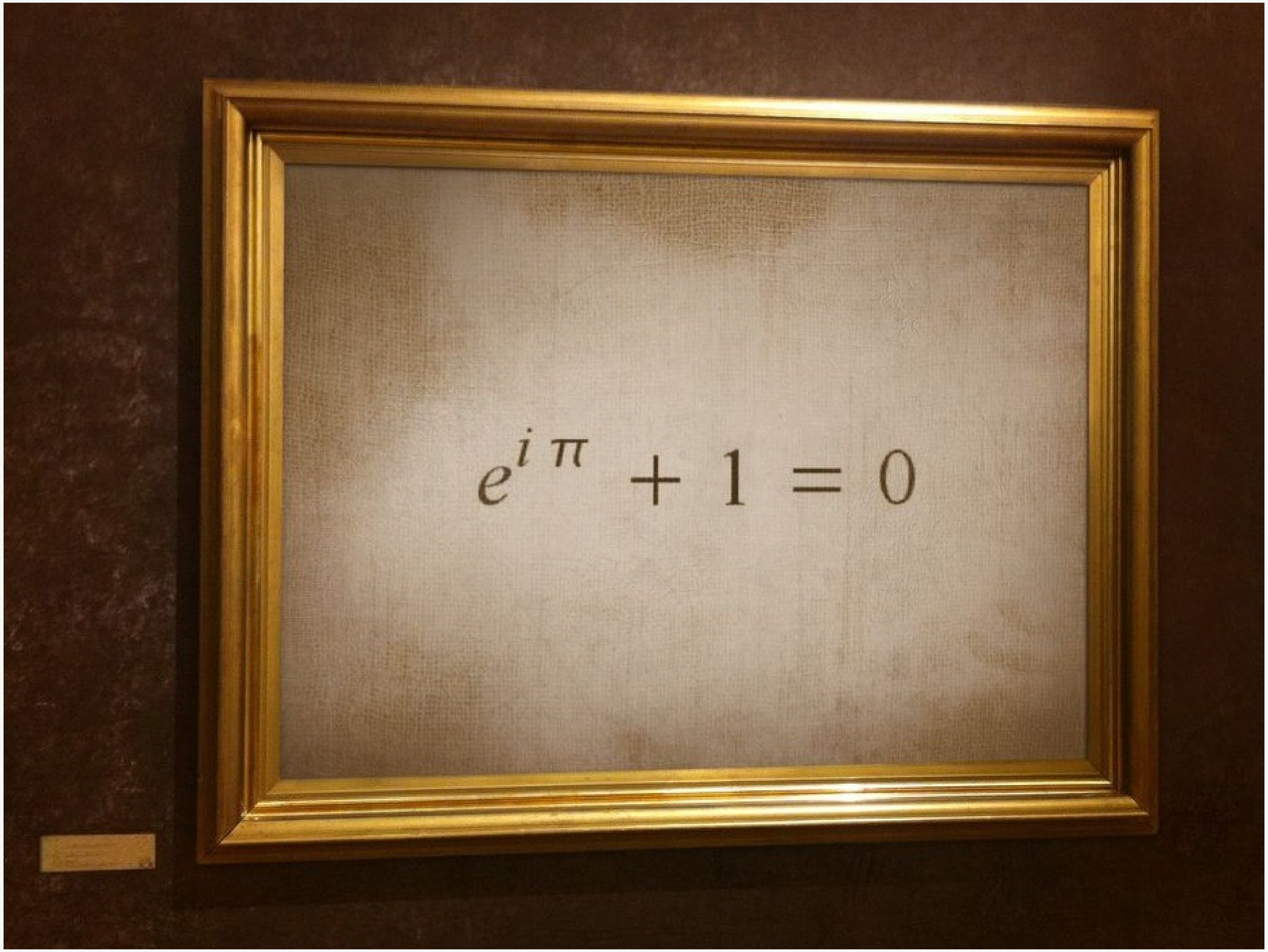


Exponential form of complex numbers

Euler's fomula

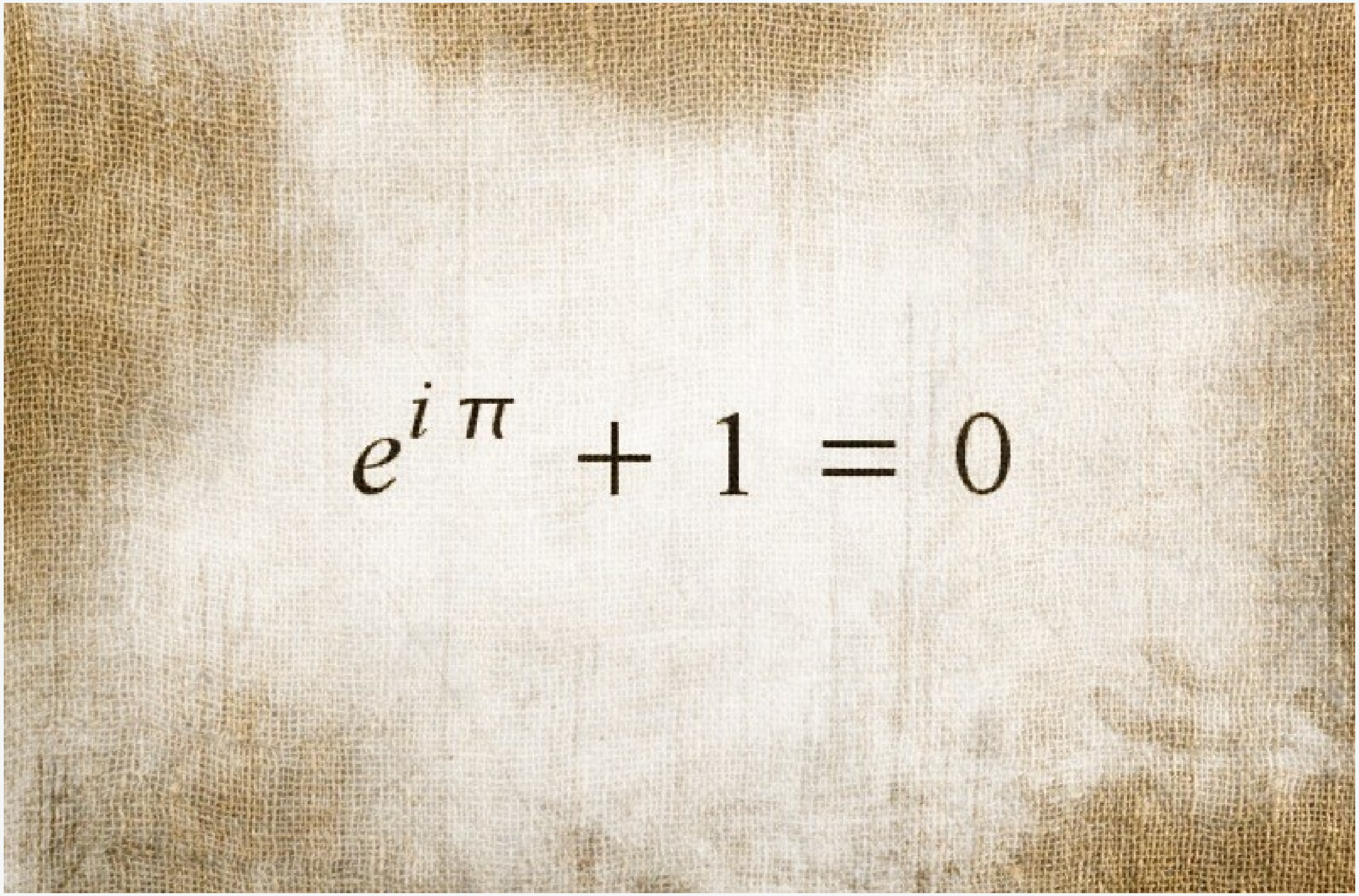
$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

connects trigonometric functions and complex numbers.

For the angle π we get

$$e^{i\pi} = -1 \quad \Leftrightarrow \quad e^{i\pi} + 1 = 0$$

Euler's formula



This fomula provides a remarkable simple connection of 5 very important mathematical constants: Euler's number ***e***, the imaginary unit ***i*** of complex numbers, the number ***π***, the unit number **1** and zero, **0**.

EQUATION OF THE WEEK

Euler's Formula

In honour of the International Year of Physics, Ideas asked 10 Canadian scientists and mathematicians to share their favourite equations. This is the fifth instalment.

$$e^{i\pi} = -1$$

Who loves it: Marianne Douglas, a geology professor at the University of Toronto and researcher of microfossils in lake sediments of the Arctic and Antarctic.

What it all means: "In harmonics and so forth, one sometimes can't work with normal numbers, and one has to go into the imaginary numbers and these complex numbers. That's where i, the square root of minus 1, comes in," Douglas says. Beyond its practical scientific applications, Euler's formula is also hailed by mathematicians for its beauty, because when it is rewritten with 1 added to each side, it contains 0,1, i, e and π, the "five fundamental numbers."

Why it's her fave: Douglas first heard of the equation from her great aunt, an astrophysicist who taught at Queen's University from the 1930s to the 1950s. "She got to help design one of the residences (Adelaide Hall). She had that equation, which was very important to her kind of research, engraved in one of the rock sculptures over the front door of this residence at Queen's. And so now hundreds of students go under this, and I'm sure many of them never even see this or know that it's engraved up there."

Next week: The science of origami

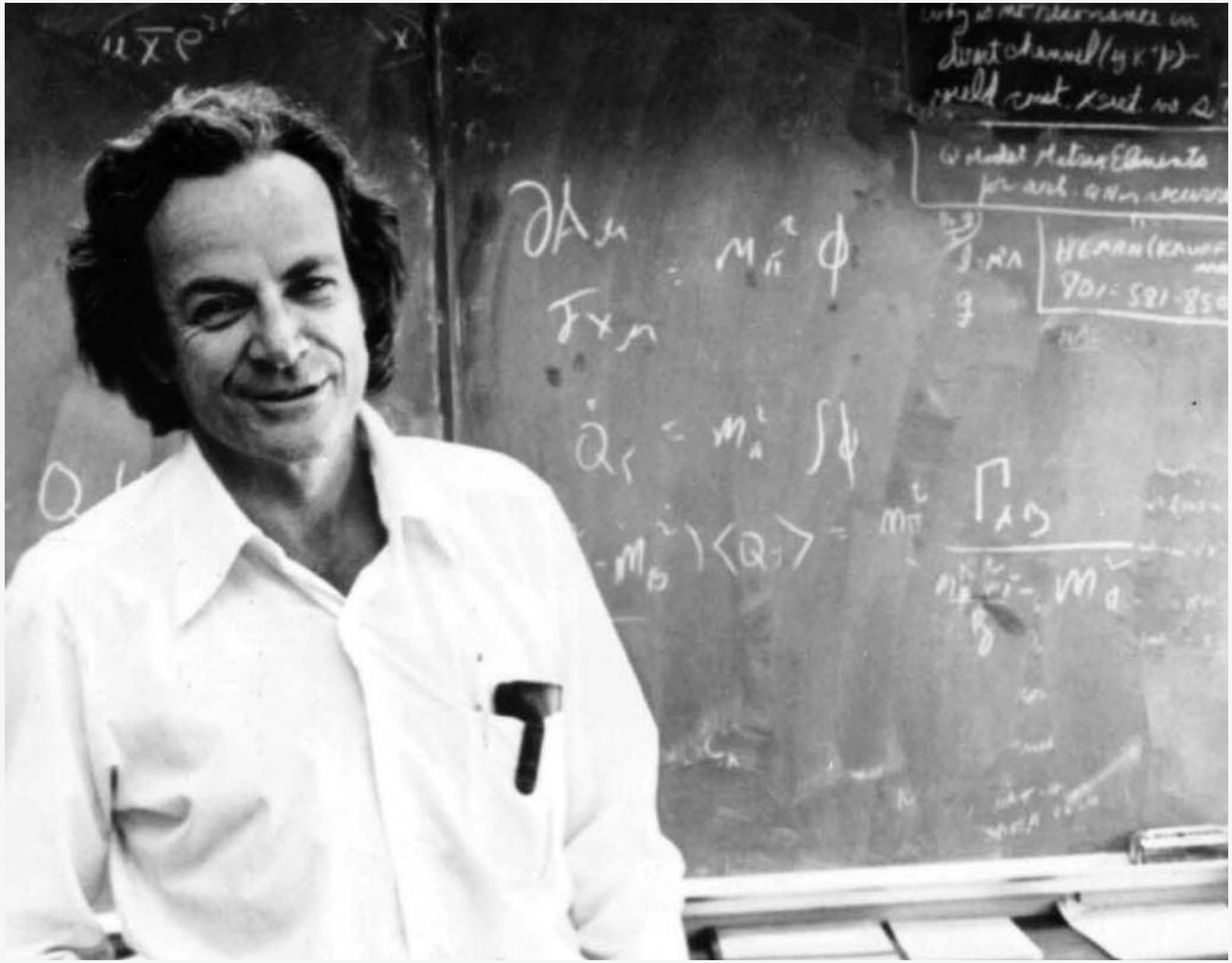
Written by Emily Chung

TORONTO STAR GRAPHIC

http://www.cap.ca/wyp/mediaPhysics.asp

Richard Feynman, american physicist and Nobel laureate in *1965,* called Euler's formula one of the most remarkable formulas in all of mathematics.

Most remarkable formula



Richard Feynman (1918-1988)

Richard Feynman, american physicist and Nobel laureate in *1965,* called Euler's formula one of the most remarkable formulas in all of mathematics.

Exponential form

trigonometric form

$$z = r\,(\cos\varphi + i\,\sin\varphi)$$

↓

$$e^{i\varphi} = \cos\varphi + i\,\sin\varphi$$

↓

exponential form

$$z = r\,e^{i\varphi}$$

Representation of complex numbers: Summary

pair of real numbers: (x, y): $\text{Re}(z) = x$, $\text{Im}(z) = y$

$$z = x + i\,y$$ algebraic (Cartesian) form

polar form: (r, φ): r – absolute value of z, φ – argument of z

$$z = r\,(\cos\varphi + i\,\sin\varphi)$$ trigonometric form

$$z = r\,e^{i\varphi}$$ exponential form

complex conjugate: $\text{Im}(z^*) = -\,\text{Im}(z)$

Transform the complex numbers into Cartesian form:

$a)\quad z = 2e^{i\frac{\pi}{6}}$

$b)\quad z = 2\sqrt{3}\,e^{i\frac{\pi}{3}}$

$c)\quad z = 4e^{3\pi i}$

$d)\quad z = 4e^{i\frac{\pi}{2}}$

$e)\quad z = \sqrt{2}\,e^{i\frac{3\pi}{4}}$

$f)\quad z = 2\sqrt{3}\,e^{i\frac{2\pi}{3}}$

$g)\quad z = \sqrt{3}\,e^{i\frac{13\pi}{6}}$

$$a)\ z:\ r = 2, \quad \varphi = \frac{\pi}{6}, \qquad z = \sqrt{3} + i$$

$$b)\ z:\ r = 2\sqrt{3}, \quad \varphi = \frac{\pi}{3}, \qquad z = \sqrt{3} + 3i$$

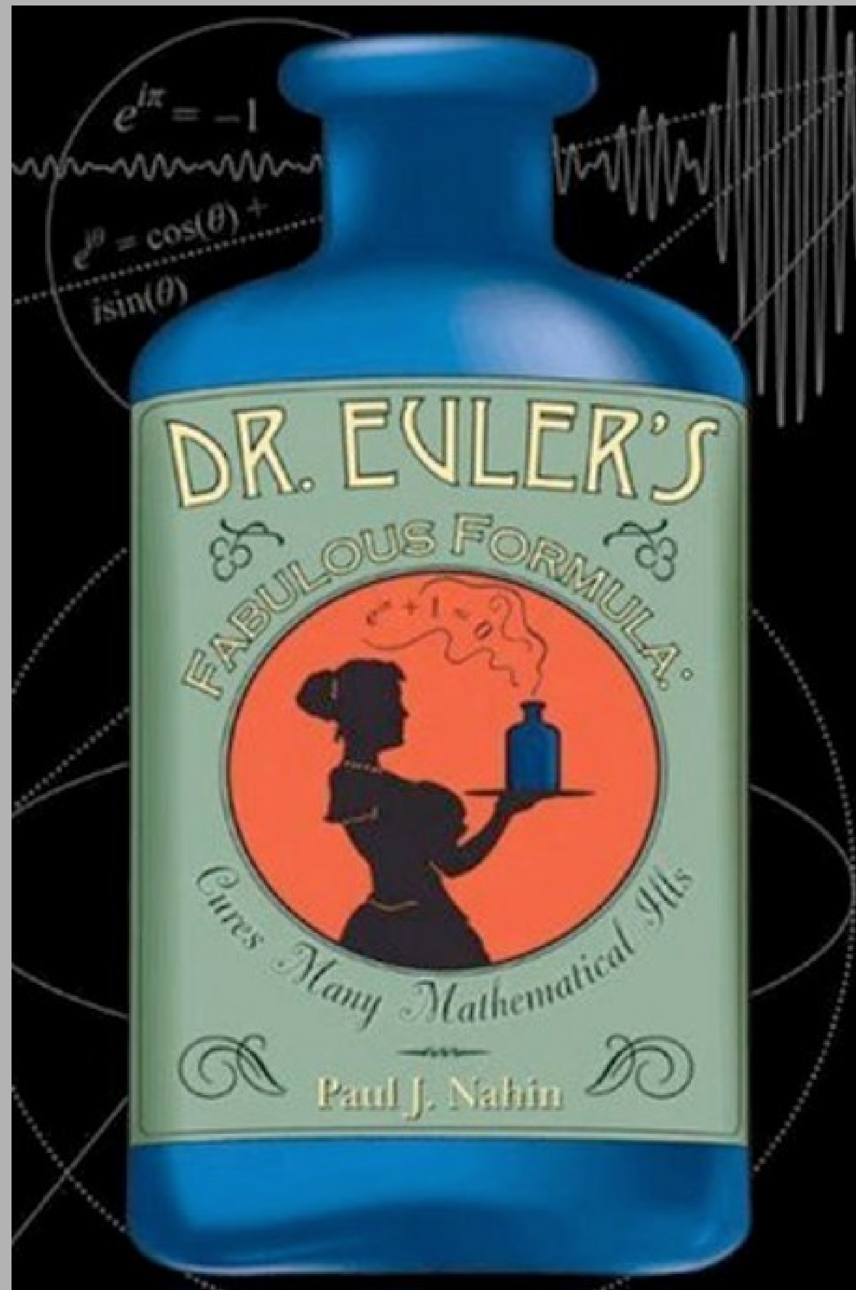
$$c)\ z:\ r = 4, \quad \varphi = 3\pi, \qquad z = -4$$

$$d)\ z:\ r = 4, \quad \varphi = \frac{\pi}{2}, \qquad z = 4i$$

$$e)\ z:\ r = \sqrt{2}, \quad \varphi = \frac{3\pi}{4}, \qquad z = -1 + i$$

$$f)\ z:\ r = 2\sqrt{3}, \quad \varphi = \frac{2\pi}{3}, \qquad z = -\sqrt{3} + 3i$$

$$g)\ z:\ r = \sqrt{3}, \quad \varphi = \frac{13\pi}{6}, \qquad z = \frac{3}{2} + \frac{\sqrt{3}}{2}\, i$$



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