Cartesian Form, Polar Form: Conversion

Convert the numbers given below into polar form:

Exercise 1: $z = \sqrt{2} + \sqrt{2}\, i$

Exercise 2: $z = -3 + 5\, i$

Exercise 3: $z = -\dfrac{3\sqrt{3}}{2} - \dfrac{3}{2}\, i$

Exercise 4: $z = 1 - i$

Exercise 5: $z = 2\, i$

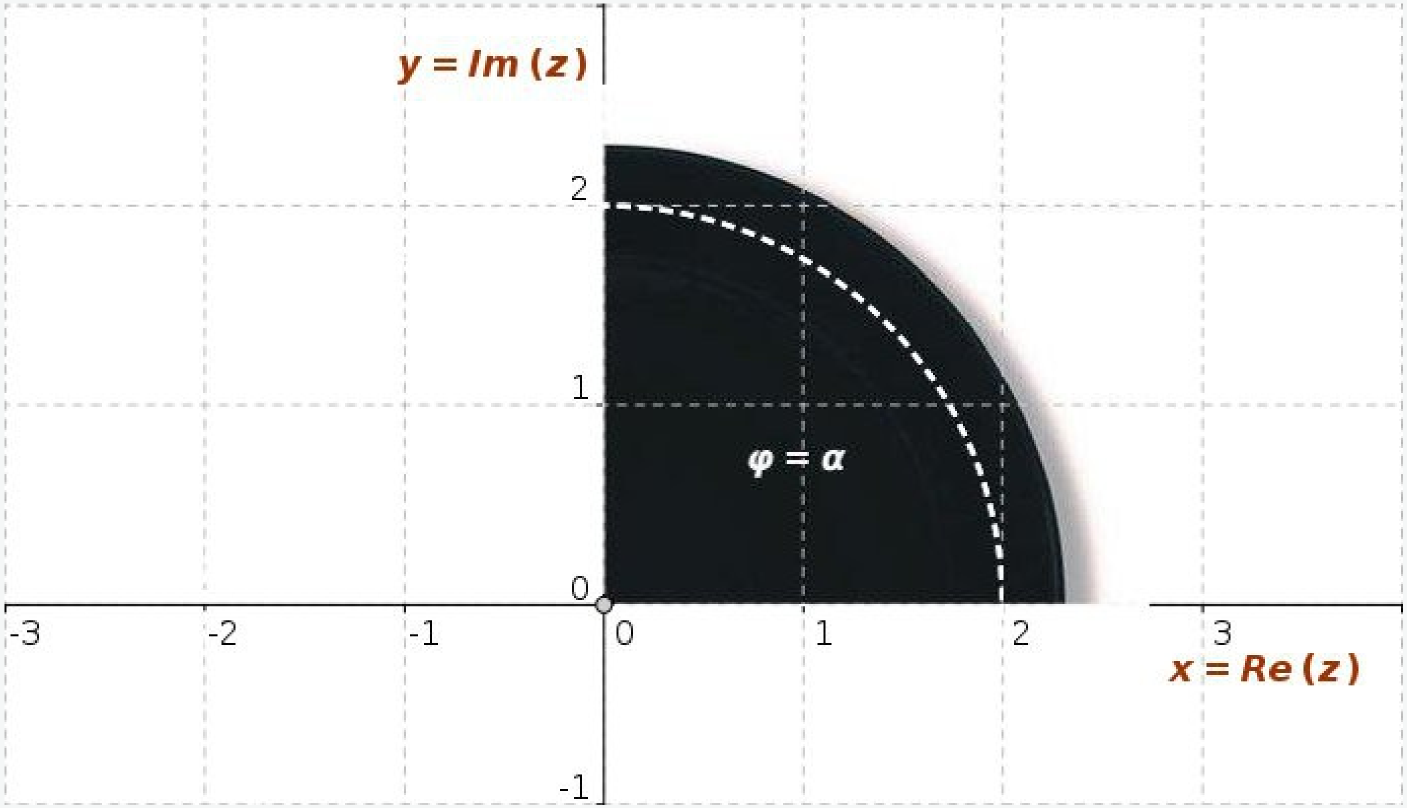

Fig. 5-1: Polar angle in quadrant 1

$$z = \sqrt{2} + \sqrt{2}\, i\,, \qquad r = |\, z\,| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2+2} = 2$$

$$\sin\ \alpha = \frac{|\, y\,|}{r} = \frac{\sqrt{2}}{2}, \qquad \alpha = 45^\circ = \frac{\pi}{4}, \qquad \varphi = \alpha \quad (x > 0, \quad y > 0)$$

$$\sqrt{2} + \sqrt{2}\, i = 2\ e^{i\frac{\pi}{4}} = 2\left[\cos\left(\frac{\pi}{4}\right) + i\ \sin\left(\frac{\pi}{4}\right)\right]$$

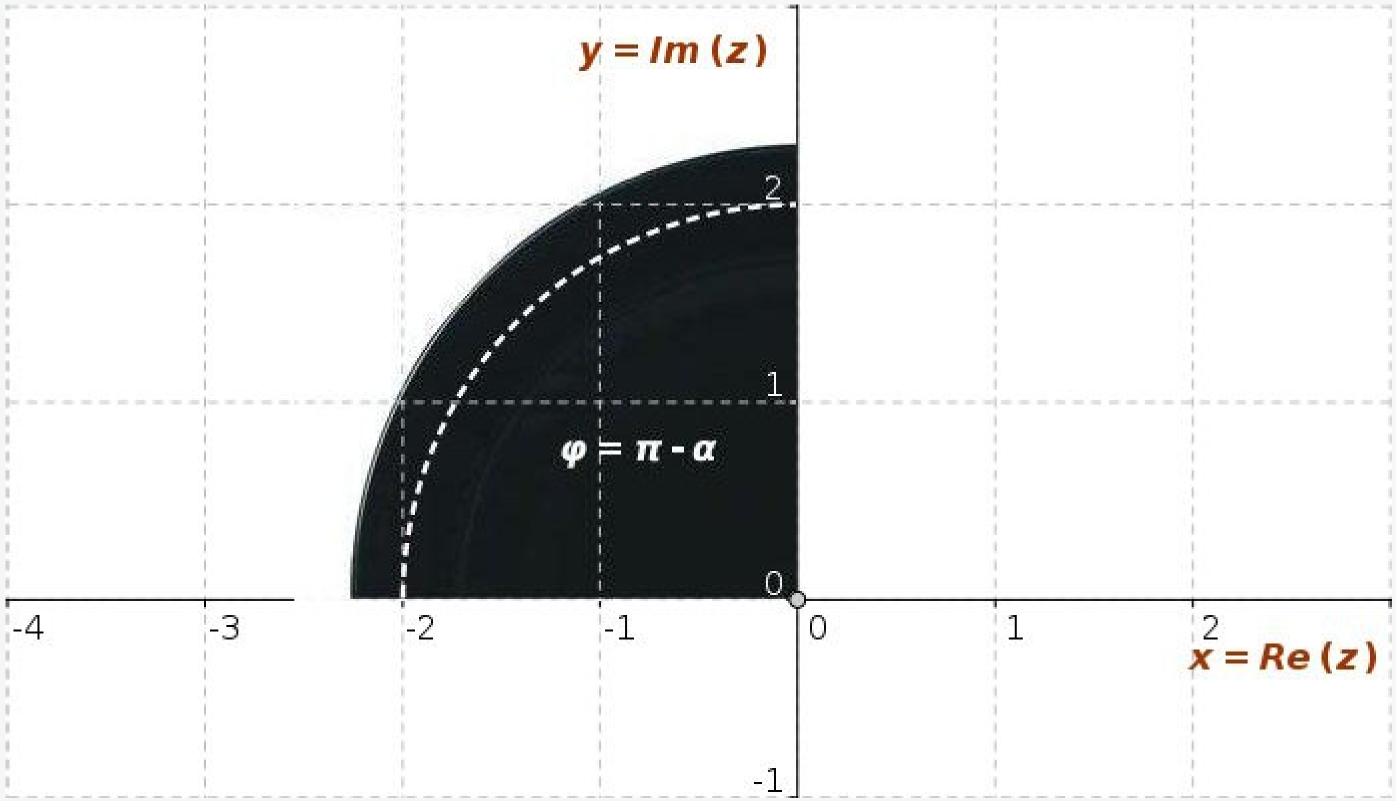

Fig. 5-2: Polar angle in quadrant 2

$$z = -3 + 5\,i\,, \qquad r = |\,z\,| = \sqrt{(-3)^2 + 5^2} = \sqrt{34} \simeq 5.831$$

$$\sin\ \alpha = \frac{|\,y\,|}{r} = \frac{5}{\sqrt{34}} \simeq 0.858\,, \qquad \alpha = 59.04$$

$$x < 0, \quad y > 0 \quad \Rightarrow \quad \varphi = \pi - \alpha = 180^\circ - 59.04 \simeq 120.96^\circ$$

$$z = 5.83\ e^{i\,120.96^\circ} = 5.83\ \left(\cos\left(120.96^\circ\right) + i\,\sin\left(120.96^\circ\right)\right)$$

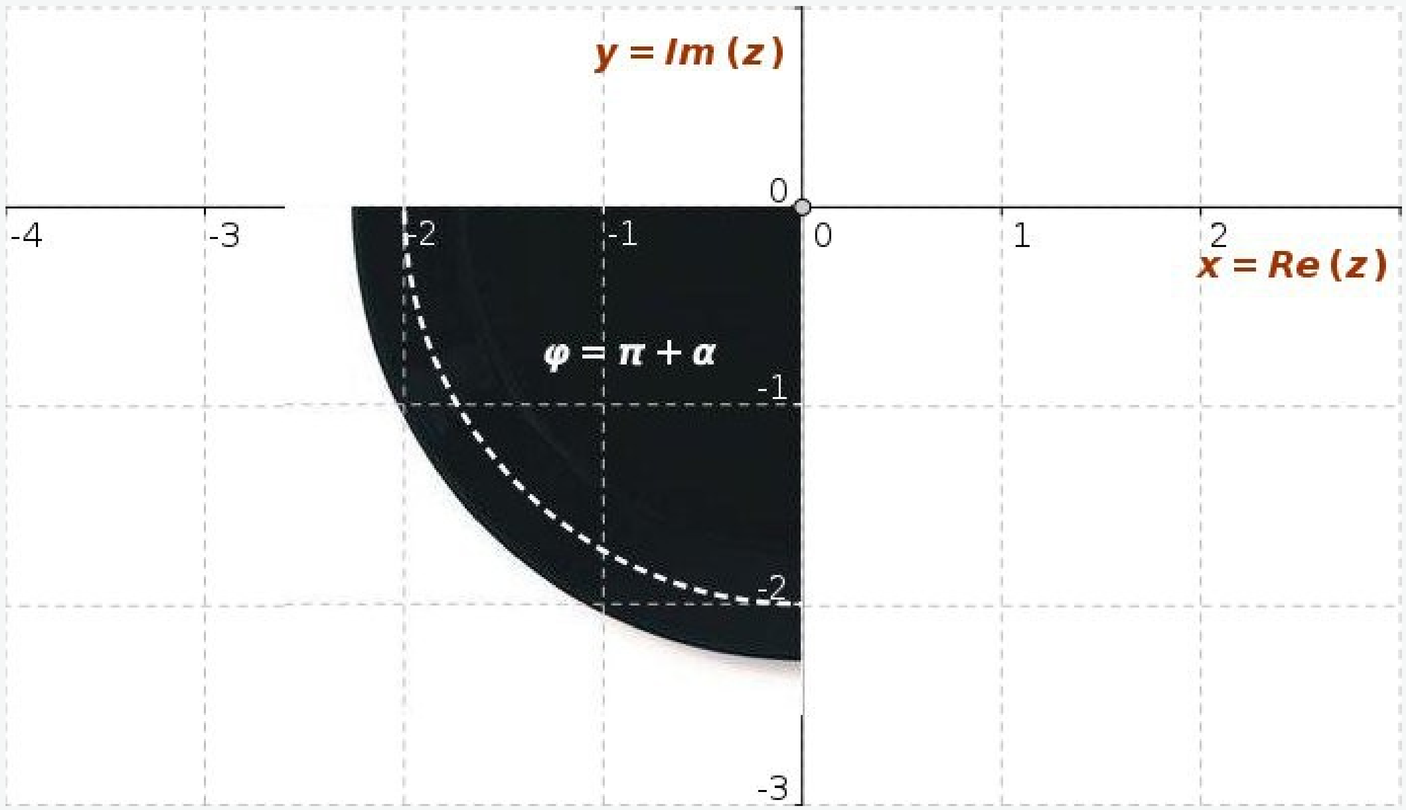

Fig. 5-3a: Polar angle in quadrant 3

$$z = -\frac{3\sqrt{3}}{2} - \frac{3}{2}\,i\,, \qquad r = |\,z\,| = \sqrt{\left(-\frac{3\sqrt{3}}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \frac{3}{2}\sqrt{3+1} = 3$$

$$\sin\alpha = \frac{|\,y\,|}{r} = \frac{1}{2}\,, \quad \alpha = 30^\circ = \frac{\pi}{6}\,, \quad \varphi = \pi + \alpha = \frac{7\pi}{6} \quad (x < 0,\;\; y < 0)$$

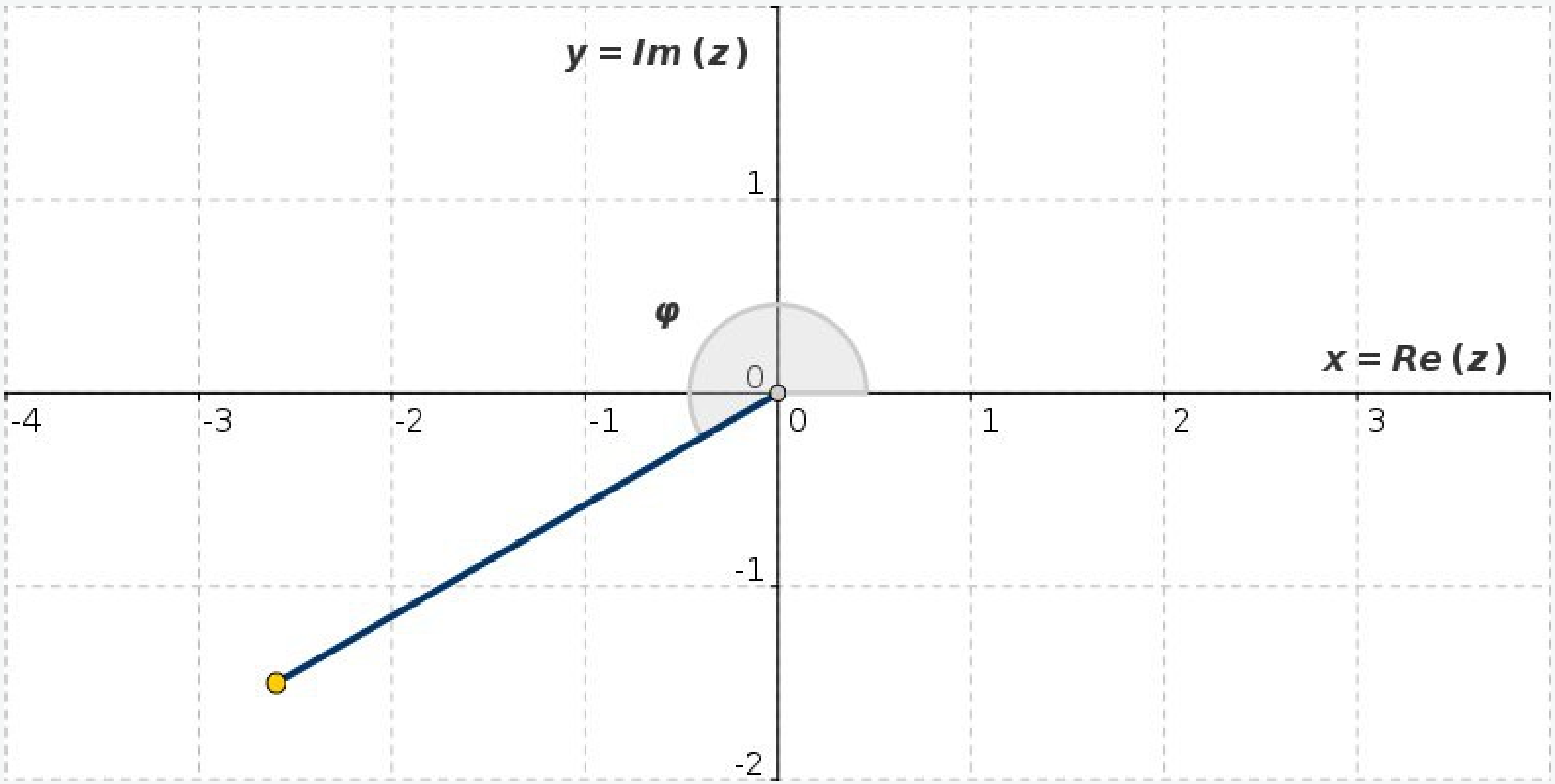

Fig. 5-3b: Representation of the complex number on the complex plane

$$-\frac{3\sqrt{3}}{2}-\frac{3}{2}\,i = 3\,e^{i\,\frac{7\pi}{6}} = 3\,e^{i\,\pi}\,e^{i\,\frac{\pi}{6}} = -3\left[\cos\left(\frac{\pi}{6}\right)+i\,\sin\left(\frac{\pi}{6}\right)\right]$$

$$e^{i\,\pi} = \cos\pi + i\,\sin\pi = -1$$

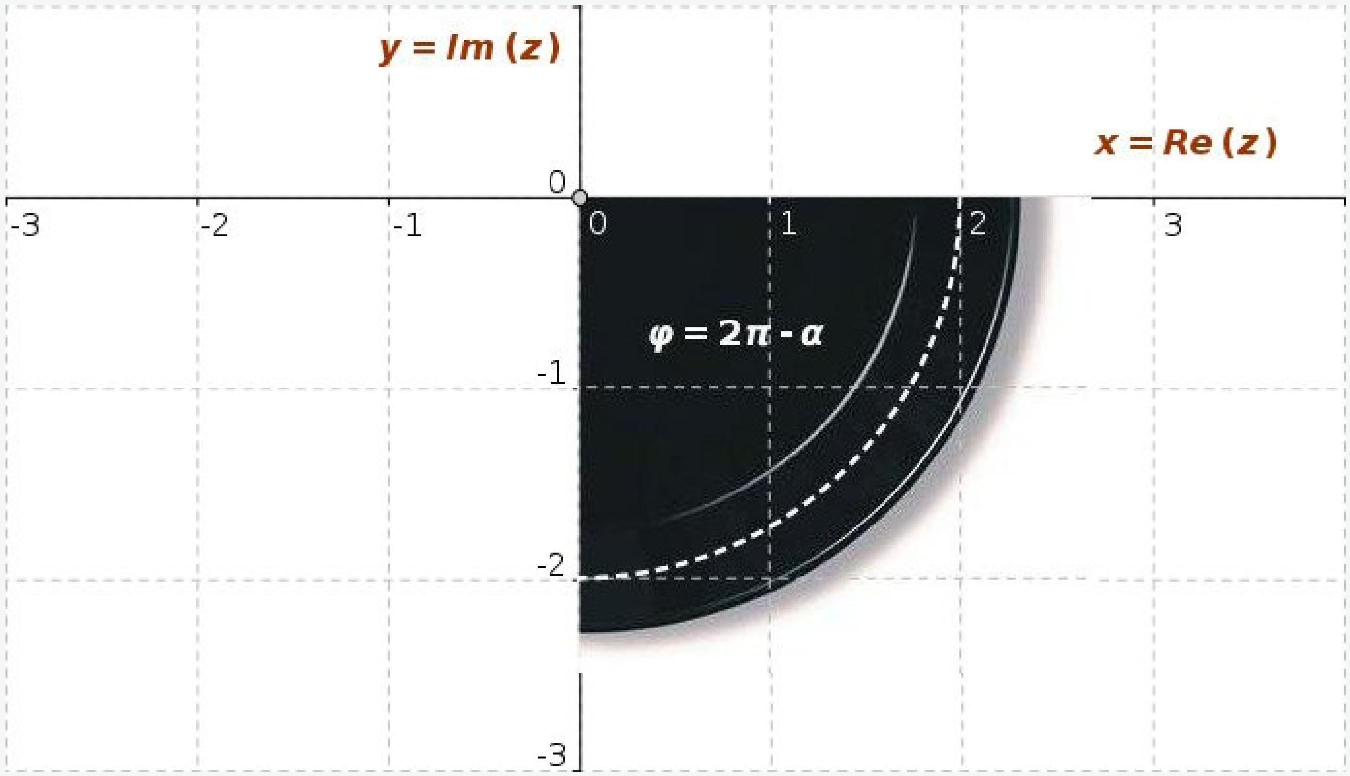

Fig. 5-4a: Polar angle in quadrant 4

$$z = 1 - i\,, \qquad r = |\,z\,| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin\ \alpha = \frac{|\,y\,|}{r} = \frac{1}{\sqrt{2}}\,, \qquad \alpha = 45^\circ = \frac{\pi}{4}\,, \qquad \varphi = 2\,\pi - \alpha \quad (x > 0, \quad y < 0)$$

Cartesian form $\rightarrow$ *polar form:* *Solution 4*

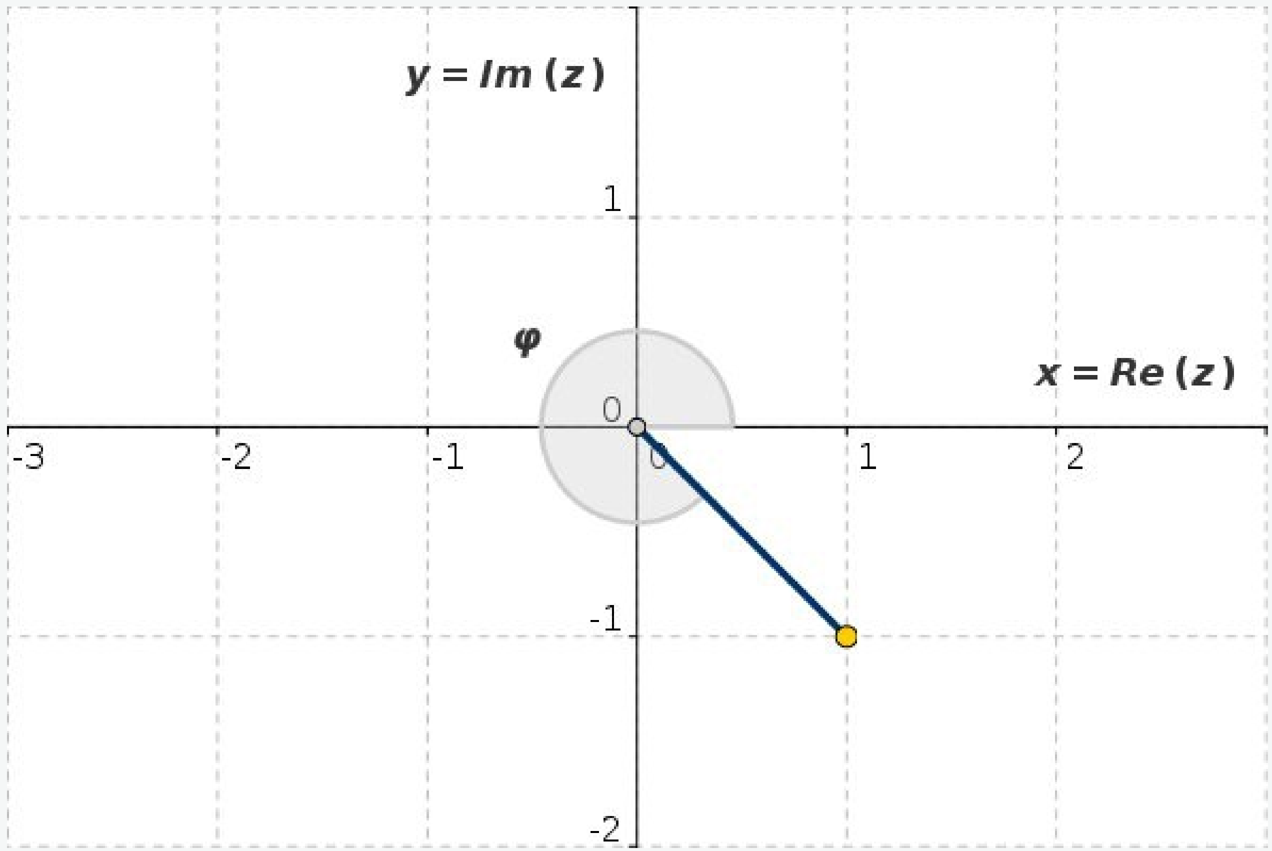

Fig. 5-4b: Representation of the complex number on the complex plane

$$1 - i = \sqrt{2}\ e^{i\,\left(2\pi - \frac{\pi}{4}\right)} = \sqrt{2}\ e^{2\pi i}\ e^{-i\,\frac{\pi}{4}} = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) - i\ \sin\left(\frac{\pi}{4}\right)\right]$$

$$e^{2\pi i} = \cos(2\pi) + i\ \sin(2\pi) = 1$$

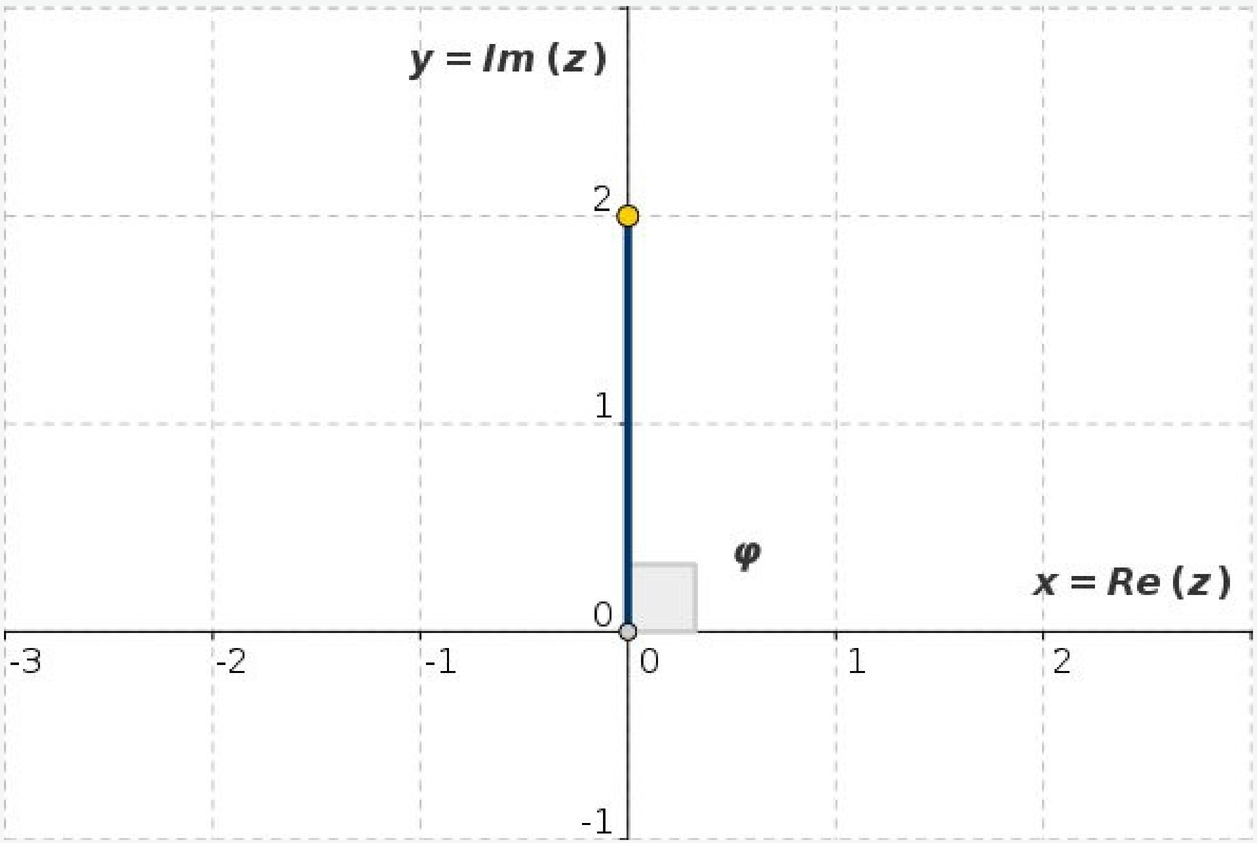

Fig. 5-5: Representation of the complex number on the complex plane

$$z = 2\,i\,, \qquad r = |\,z\,| = \sqrt{0^2 + 2^2} = 2$$

$$\sin\varphi = \frac{2}{2} = 1, \quad x = 0, \quad y > 0, \quad \varphi = \frac{\pi}{2} = 90^\circ, \quad z = 2\,e^{i\,90^\circ}$$

Convert the numbers given below into polar form:

Exercise 6: $z = -5 + 4i$

Exercise 7: $z = 2 + \sqrt{7}\, i$

Exercise 8: $z = -\sqrt{3} - \sqrt{8}\, i$

Exercise 9: $z = \sqrt{7} - i$

Solution 6: $z = -5 + 4\,i$

$$r = |\,z\,| = \sqrt{41}\,, \quad \varphi = \pi - \arctan\left(\frac{4}{5}\right), \quad \varphi \simeq 141.34^{\circ}$$

Solution 7: $z = 2 + \sqrt{7}\,i$

$$r = |\,z\,| = \sqrt{11}\,, \quad \varphi = \arctan\left(\frac{\sqrt{7}}{2}\right), \quad \varphi \simeq 52.91^{\circ}$$

Solution 8: $z = -\sqrt{3} - \sqrt{8}\,i$

$$r = |\,z\,| = \sqrt{11}\,, \quad \varphi = \arctan\left(\frac{2\sqrt{2}}{\sqrt{3}}\right) - \pi\,, \quad \varphi \simeq -121.48^{\circ}$$

Solution 9: $z = \sqrt{7} - i$

$$r = |\,z\,| = 2\sqrt{2}\,, \quad \varphi = -\arctan\left(\frac{1}{\sqrt{7}}\right), \quad \varphi \simeq -20.71^{\circ}$$