http://www.youtube.com/watch?v=qGpfhqLP6j0

Steps and Slopes

Objective

http://www.youtube.com/watch?v=qGpfhqLP6j0

We hope to get:

- a clear understanding of tangent lines to a curve
- a definition of the gradient of a curve
- a definition of the derivative of a function
- a geometrical interpretation of the derivative
- computations of derivatives

The differential calculus turned out to be a very important branch of mathematics. The reason is, that it connects rates of change and the slope of curves. In the following we want to explain this connection. First we will define the slope of a straight line and then define the slope of a curve at each of its points.

Imagine a particle moving in a plane from one point to another. We find the resulting change of its coordinates by subtracting the coordinates of the starting point from those of the end point.

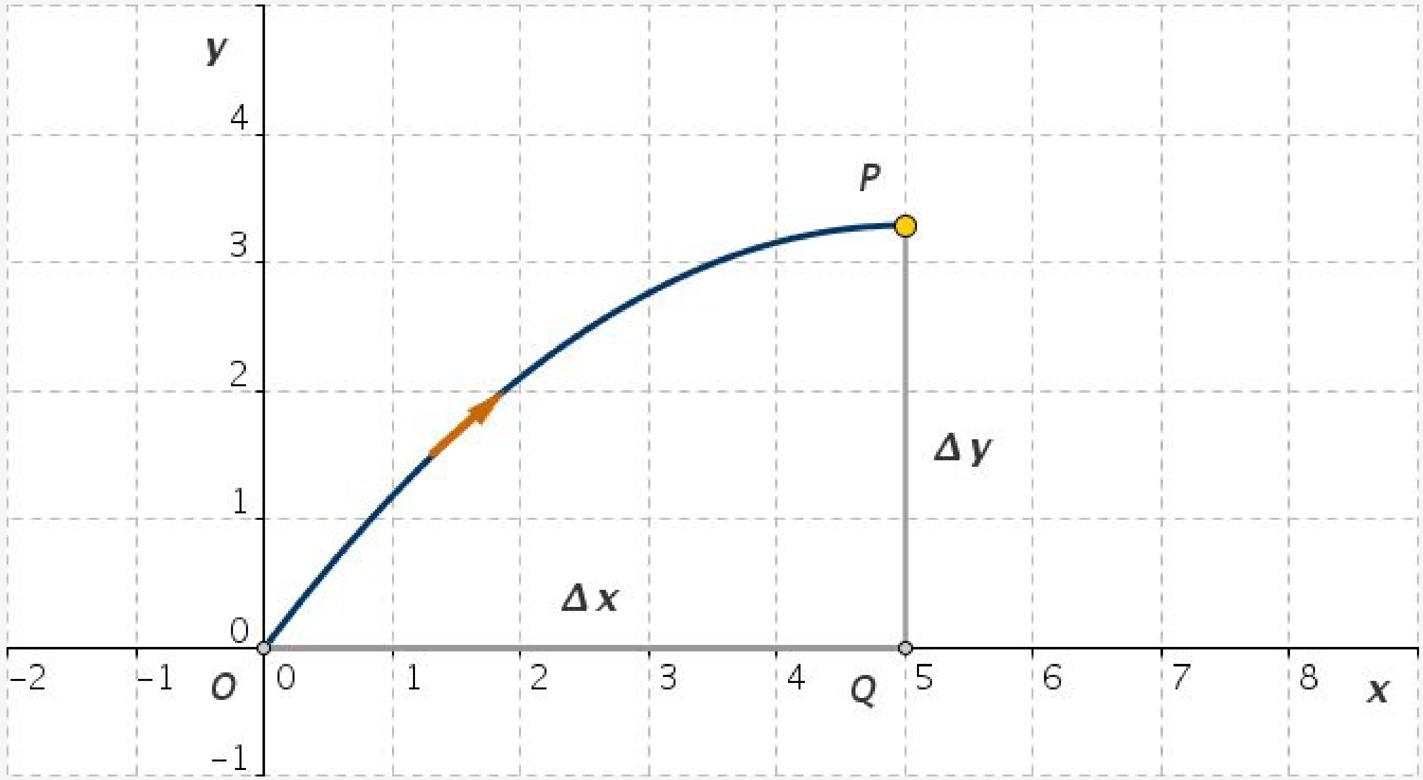

Fig. 1-1: The path of a point in cartesian coordinates

A particle is moving from the point *O (0, 0)* to the point *P (5, 3.3)* (see figure. 1-1). The x- and y-coordinates change by

$$\Delta x = 5 - 0 = 5, \qquad \Delta y = 3.3 - 0 = 3.3$$

Such changes are also called increments.

Definition:

An increment is a resulting change:
When a particle moves from P to Q

$$P = (x_1\,,\; y_1)\,, \qquad Q = (x_2\,,\; y_2)\;,$$

the coordinates change by, or in other words, the increments are

$$\Delta\, x = x_2 - x_1\,, \qquad \Delta\, y = y_2 - y_1$$

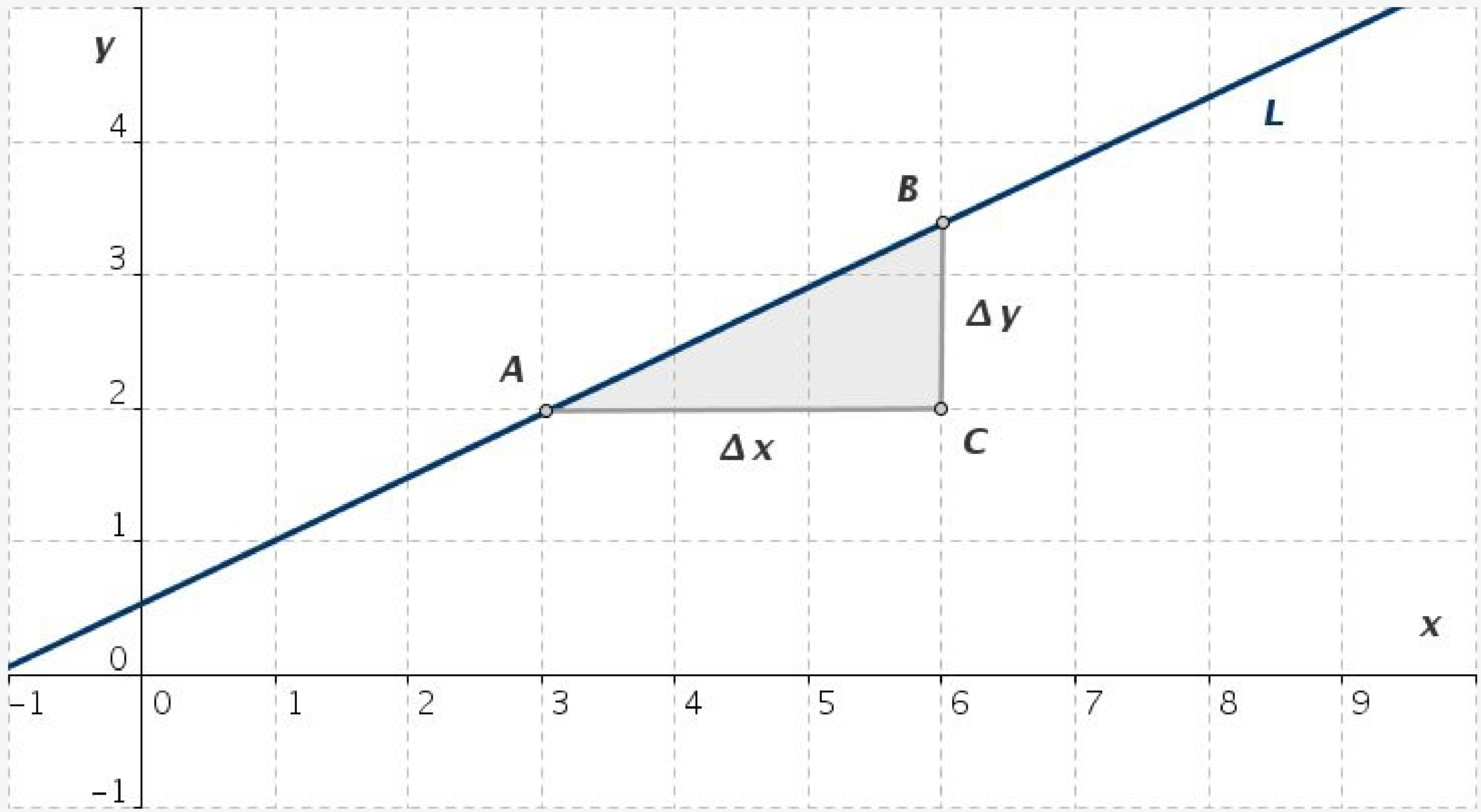

Fig. 1-2: Slope of the straight line

Assume L is a nonvertical straight line, and A and B are points on that line. The coordinates of a particle moving from point A to point B along L change as follows (Fig. 1-2):

$$A = (x_A\,,\ y_A)\,, \qquad B = (x_B\,,\ y_B)\,, \qquad C = (x_B\,,\ y_A)$$

$$\Delta y = y_B - y_A\,, \qquad \Delta x = x_B - x_A$$

We now can define slopes for all nonvertical straight lines:

Definition: The slope (also called gradient) of a nonvertical straight line is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_B - y_A}{x_B - x_A}$$

Assume, we now take different points to determine the slope, e.g. *D*, *E* and *F* in Fig. *1-3*

$$m' = \frac{\Delta y'}{\Delta x'} = \frac{y_E - y_D}{x_E - x_D}$$

Do we get the same slope? Yes, because m' and m are ratios of corresponding sides of similar triangles. The slope of a straight line depends only on how steeply the line rises or falls and stays the same, whatever points we choose to calculate it.

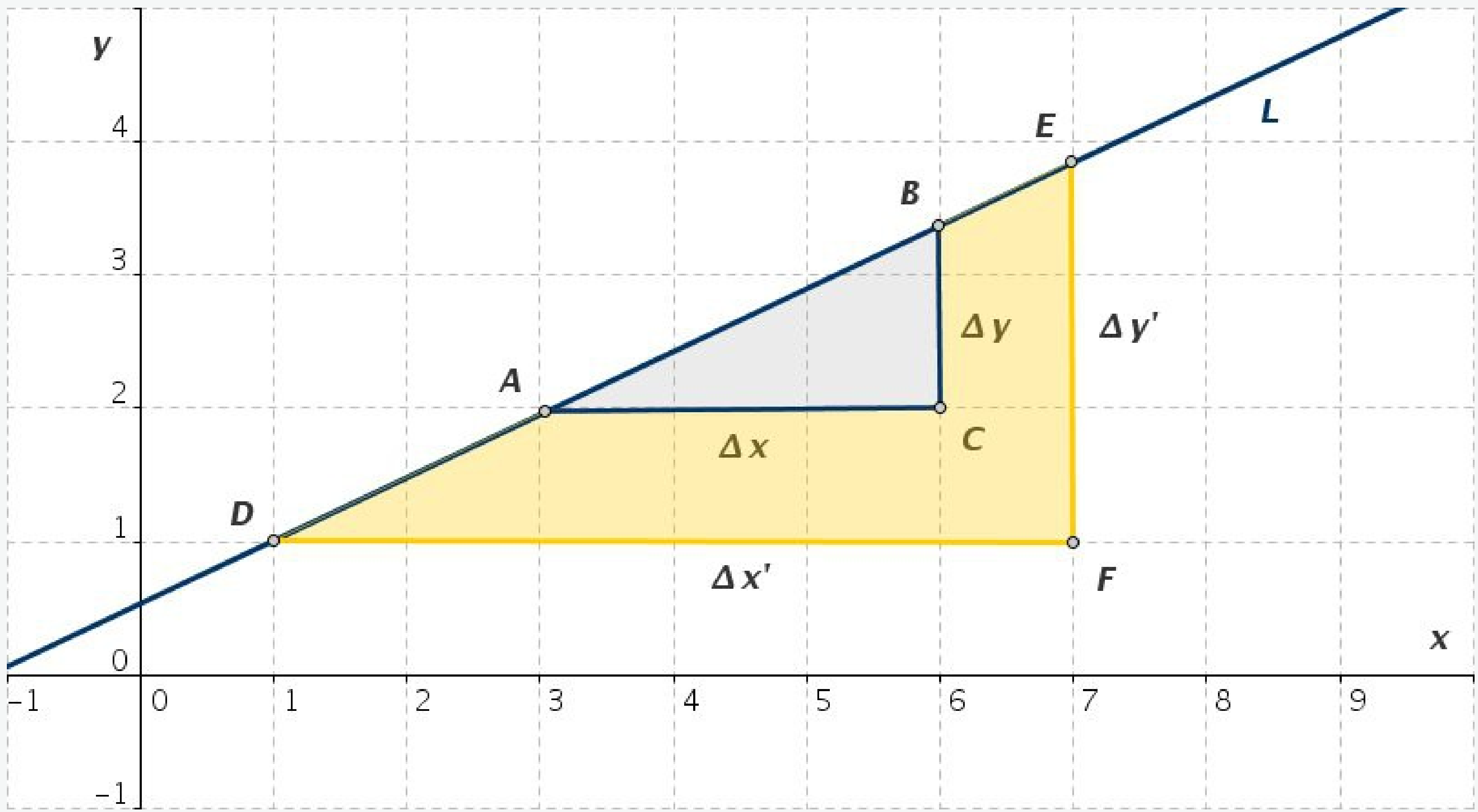

Fig. 1-3: Slope of a nonvertical straight line

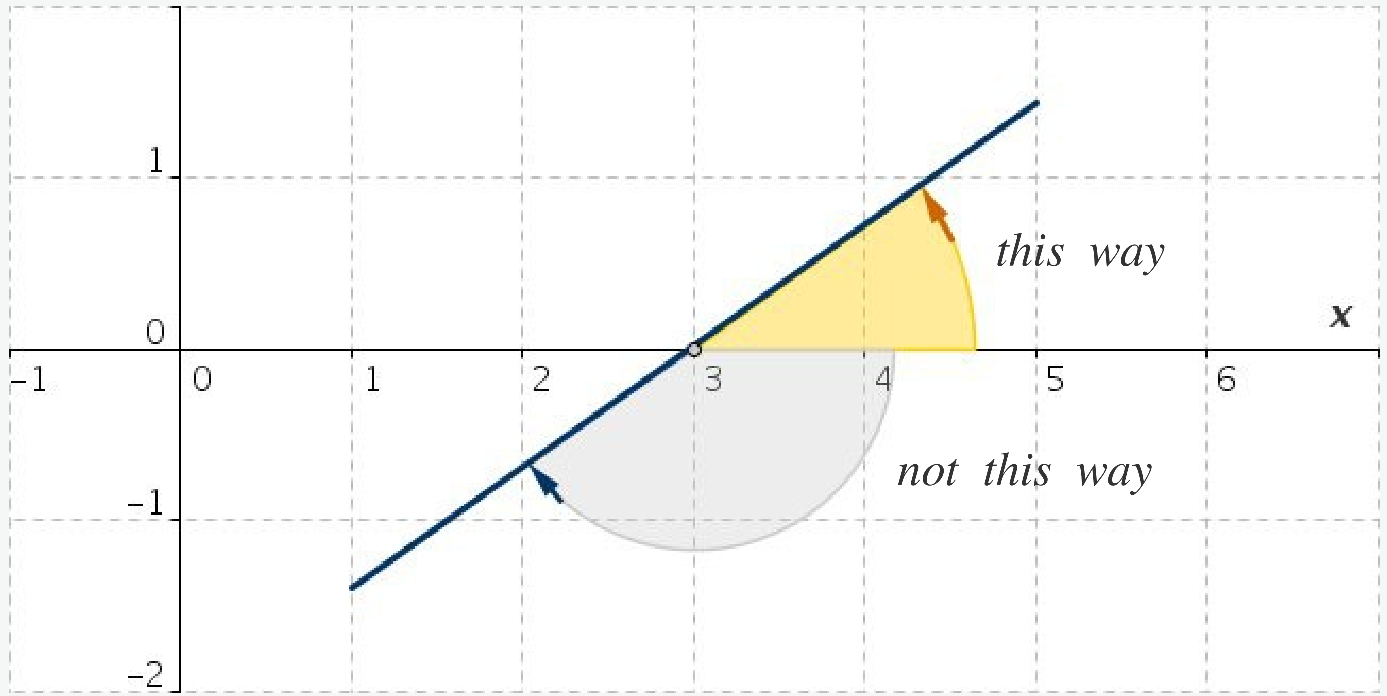

Fig. 1-4: The angle of inclination of a straight line

The angel of inclination of a straight line is given by the smallest angle measured counter clock wise with respect to the x-axis (figure *1-4*,*1-5*). The angle of inclination of a horizontal line is *0*°. Therefore the inclination angle φ of a straight line may take any value in the range

$$0^\circ \leqslant \varphi < 180^\circ$$

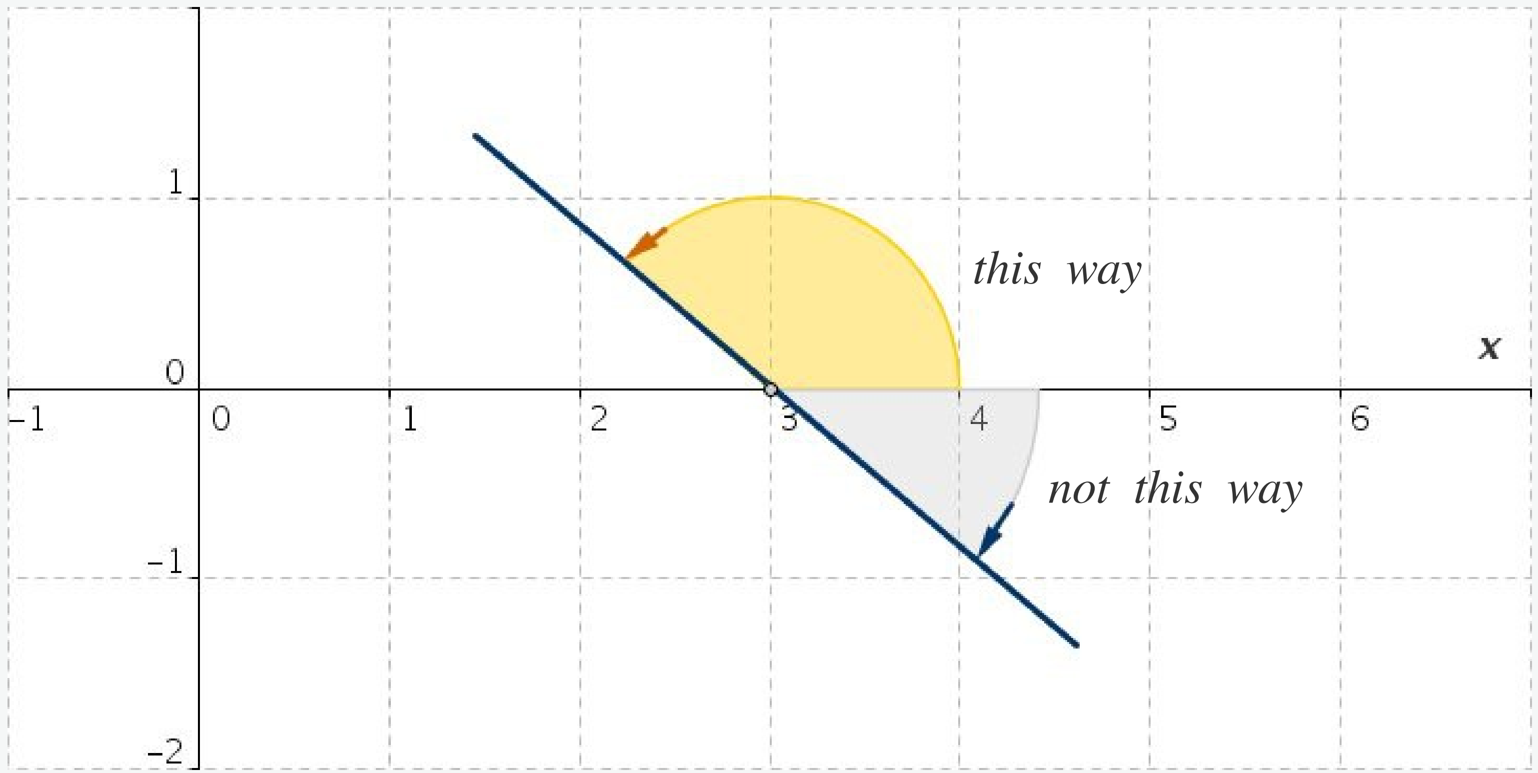

Fig. 1-5: The angle of inclination of a straight line

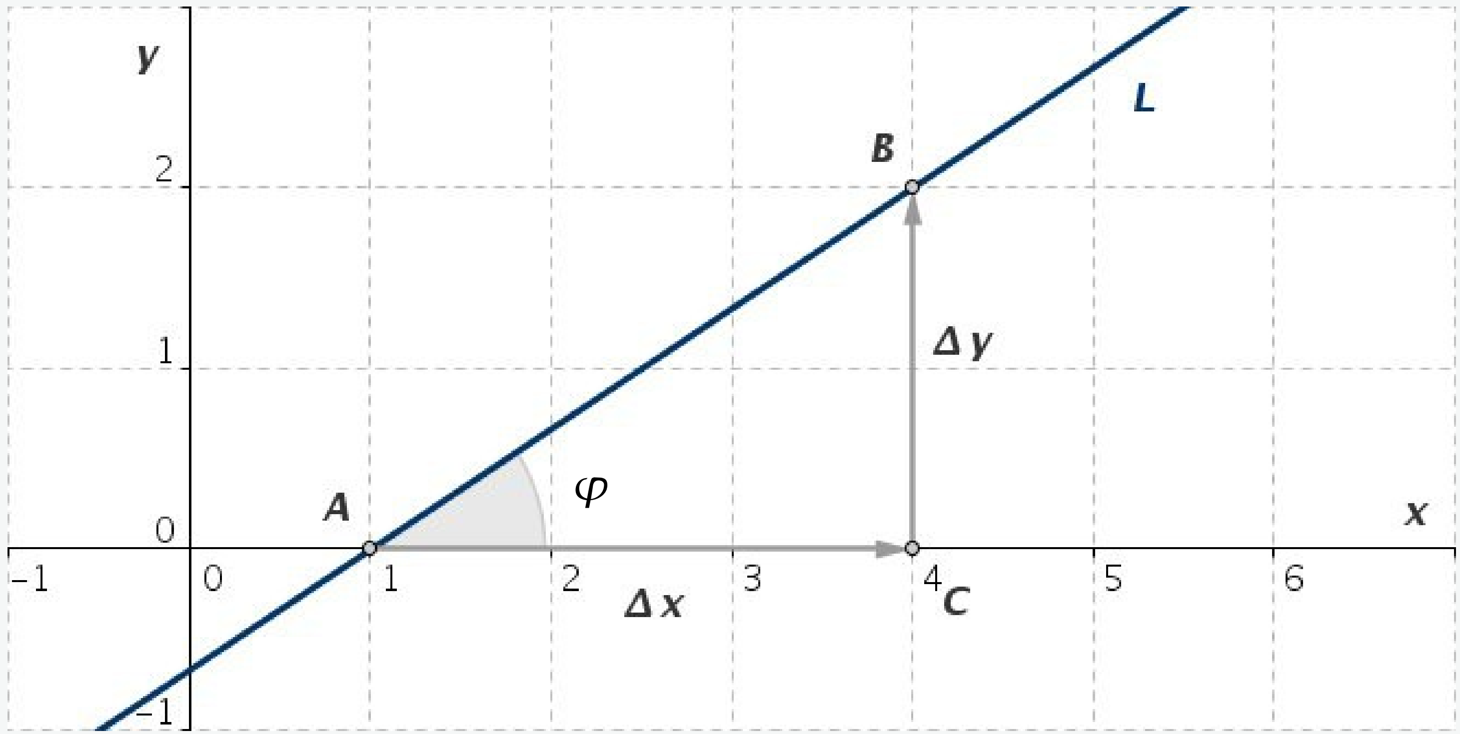

Fig. 1-6: gradient and angle of inclination

The gradient (or slope) of a non-vertical straight line is given by the tangent of the angle of inclination

$$m = \frac{\Delta y}{\Delta x} = \tan \varphi$$