http://www.youtube.com/watch?v=9FFDJK8jmms

Tangent lines, Gradients and Derivatives

Let us begin with velocities: How can we measure the velocity of a moving object at a given instant in time? Or more fundamentally, what do we mean by the term velocity or speed?

We shall define a speed that has general relevance – not just for the movement of objects, but for measuring the rate of change of any quantity. This will lead to the important concept of the derivative .

The derivative can be interpreted geometrically as the slope of a curve, and more generally as a rate of change. Derivatives can be used to present many different things from fluctuations in interest rates to the variations of populations or movements of molecules. Therefore they have found applications throughout science and engineering.

Average velocity

If $s(t)$ is the position of an object at time t, then the average velocity of the object over the interval $[t, t + \Delta t]$ is

$$Average\ velocity = \frac{Change\ in\ position}{Change\ in\ time} = \frac{s(t + \Delta t) - s(t)}{\Delta t}$$

The instantaneous velocity at the moment t is given by the limit of the average velocity over an interval, as the interval shrinks around t

$$\lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$

This expression forms the foundation of the calculus. To estimate this number, i.e. the limit, we look at intervals of smaller and smaller, but never zero length.

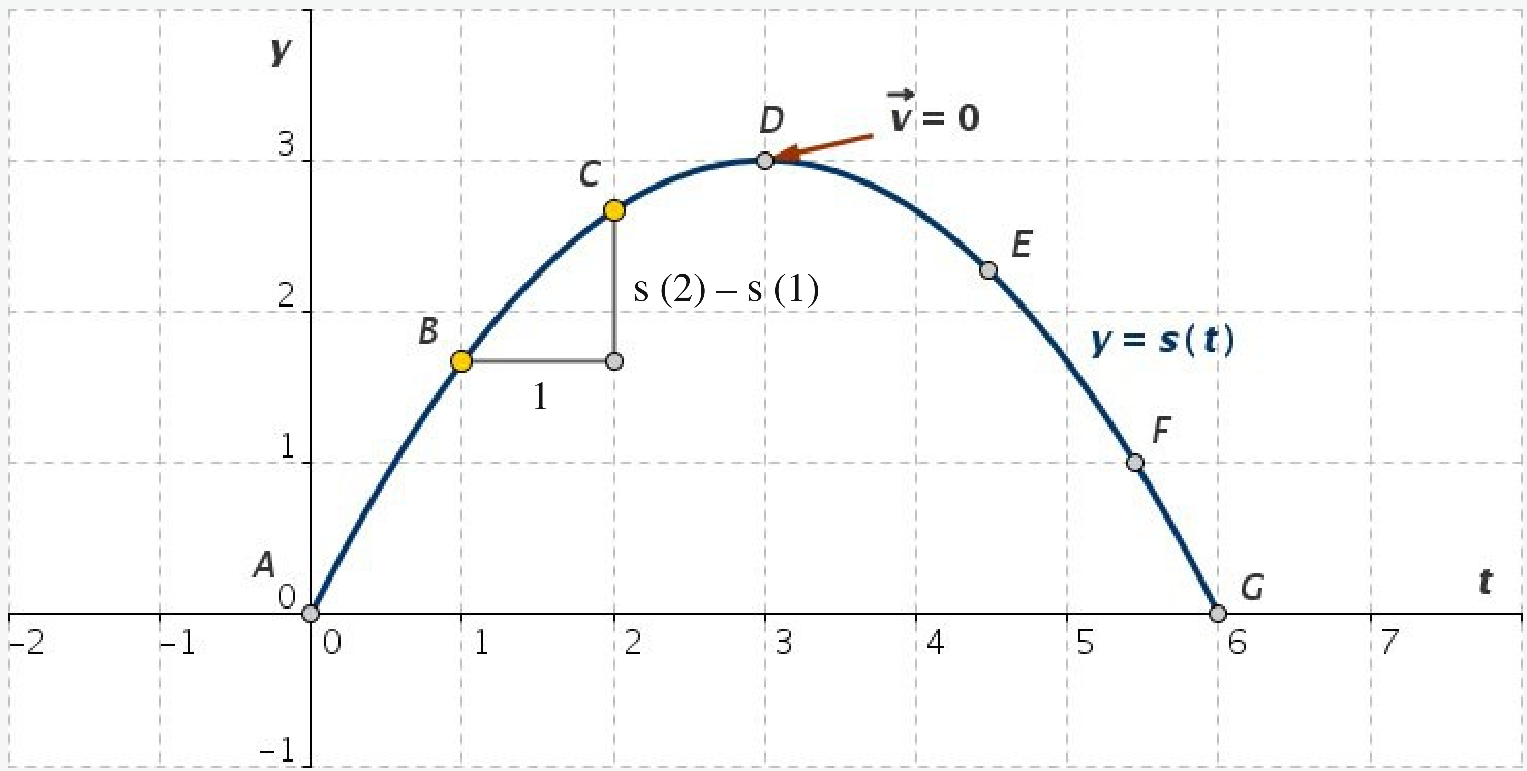

Fig. 2-1: Function s (t) = - t (t - 6) / 3

How can we visualize the average velocity on this graph $y = s(t)$?
Let us consider the interval $1 \le t \le 2$:

$$\textit{Average velocity} = \frac{s(2) - s(1)}{2 - 1} = \frac{2.67 - 1.67}{1} = 1 \; \frac{m}{s}$$

s (2) - s (1) is the change in position over the interval, and it is marked vertically in Fig. 2-1. The 1 in the denominator is the time during which this change took place and is marked horizontally. Therefore

$$\textit{Average velocity} = \frac{\textit{Change in position}}{\textit{Change in time}} = \textit{Slope of line joining } B, C$$

More general: the average velocity over any time interval $a \le t \le b$ is the slope of the line joining the points on the graph of $s(t)$ corresponding to $t = a$ and $t = b$.

Velocity: *Slope of the Curve*

http://www.flickr.com/photos/pixelpixel/2582277400/in/set-72157605637154802/

The next question is:

How to visualize the instantaneous velocity ?

To answer this question let us think about, how we found the instantaneous velocity. We took average velocities over smaller and smaller intervals. Three such velocities are represented by the slopes of the lines in Figure 2-2. As the lengths of the intervals shrink, the slope of the lines get closer to the slope of the curve at $t = 1$.

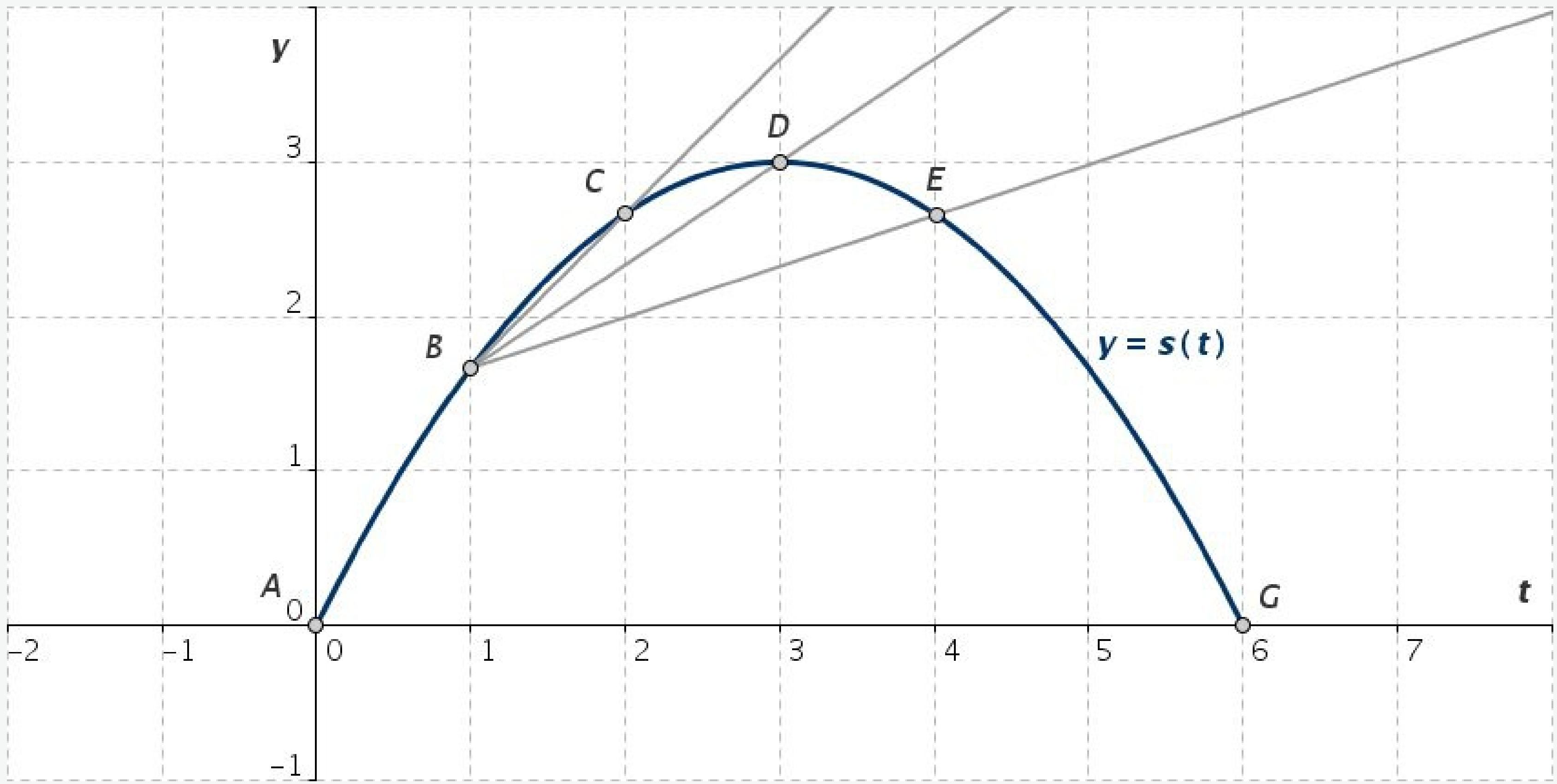

Fig. 2-2: Average velocities over small intervals

B - C : Slope = Average velocity over $1 \leq t \leq 2$

B - D : Slope = Average velocity over $1 \leq t \leq 3$

B - E : Slope = Average velocity over $1 \leq t \leq 4$

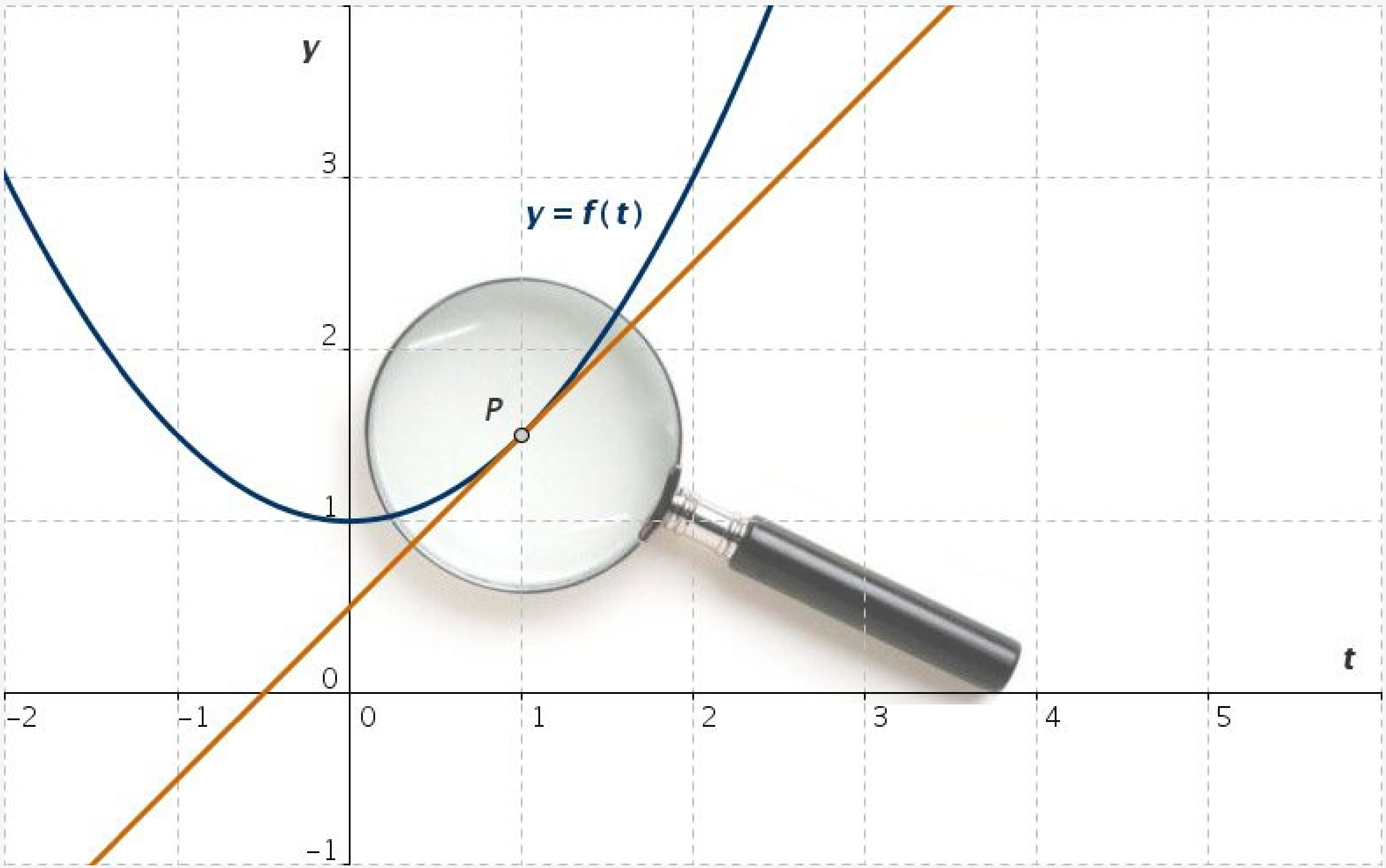

Fig. 3-1: Estimating the slope of the curve by "zooming in"

The cornerstone of the idea is the fact that, on a very small scale, most functions look almost like straight lines when "zooming in" to get a close-up view as indicated in Fig. 3-1

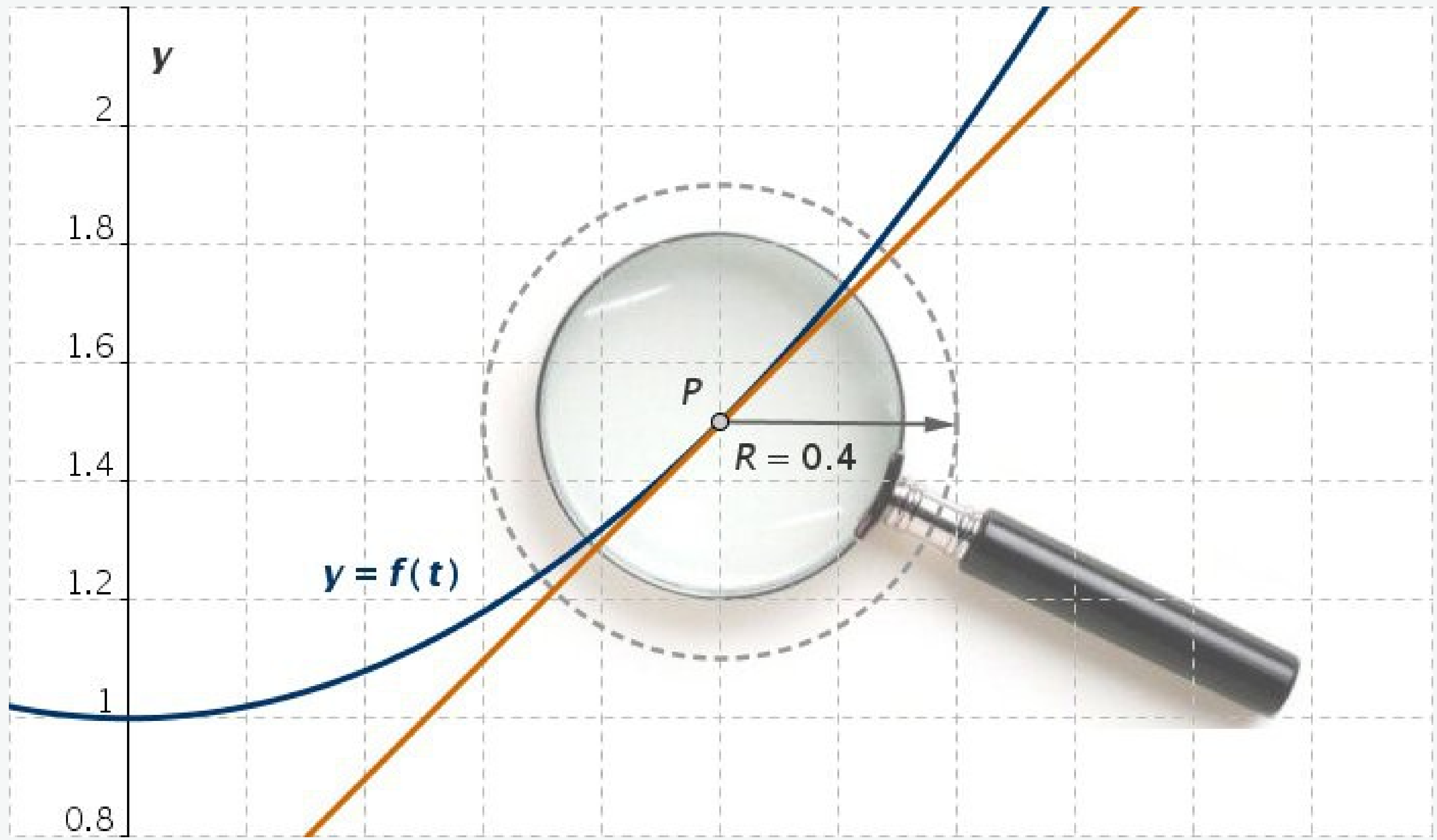

Fig. 3-2: The "zooming in"

By this and the following steps we will show, that the more we zoom in, the more the curve will appear to be a straight line.

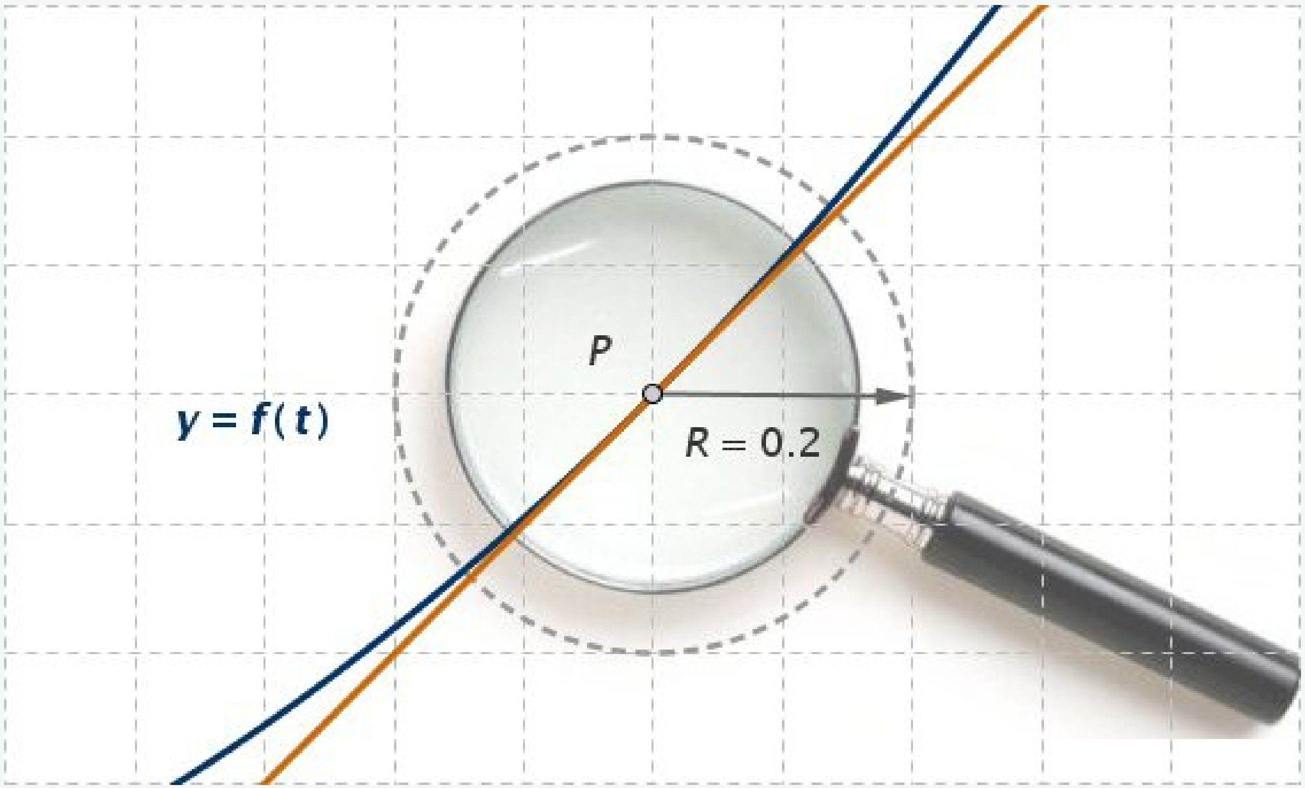

Fig. 3-3: The "zooming in"

If we repeatedly zoom in on a section of the curve centered at a point of interest, the section of the curve will eventually look like a straight line. We call the slope of this line the slope of the curve at the point.

Therefore:
the instantaneous velocity is the slope of the curve at a point.