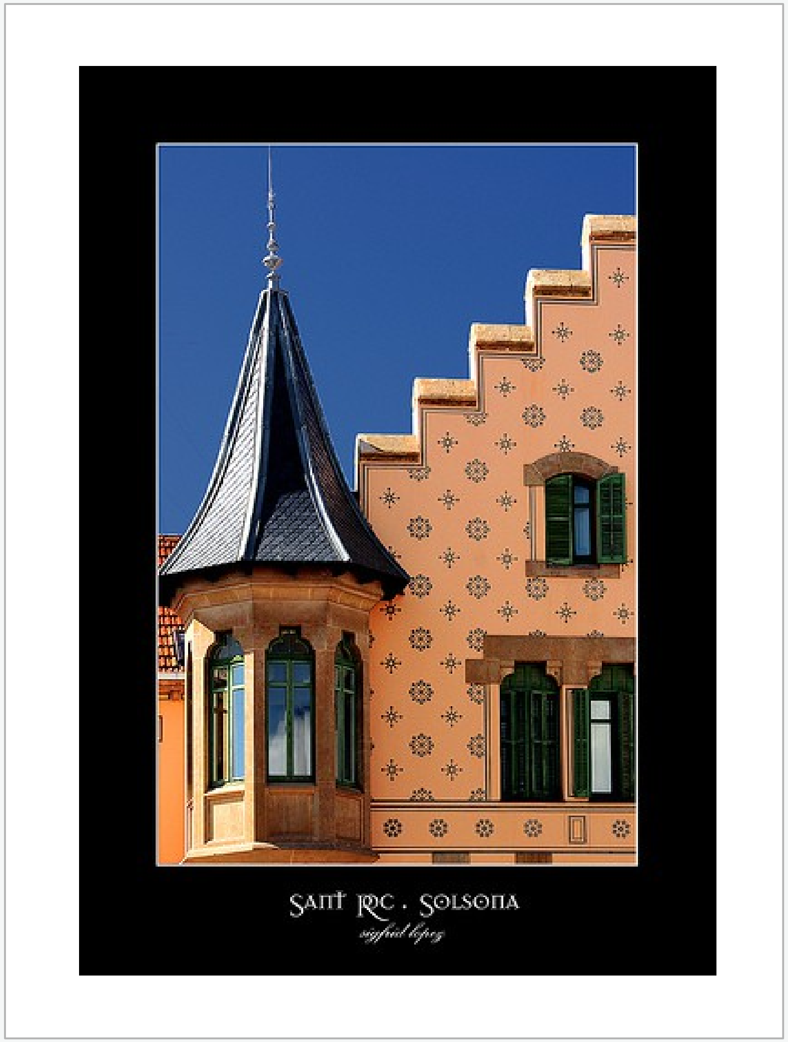

http://www.flickr.com/photos/sigfrid/3428162159/

Derivative of Power Functions

Ma 1 – Lubov Vassilevskaya

Derivative of the function $y = 1/x^2$

$$f(x) = \frac{1}{x^2}, \qquad f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left(\frac{1}{(x + \Delta x)^2} - \frac{1}{x^2} \right) =$$

$$= \lim_{\Delta x \to 0} \frac{2x + \Delta x}{x^2 (x + \Delta x)^2} = \frac{\lim\limits_{\Delta x \to 0} (2x + \Delta x)}{\lim\limits_{\Delta x \to 0} x^2 (x + \Delta x)^2} = -\frac{2}{x^3}$$

The derivative is positive for $x < 0$ and negative for $x > 0$.

Derivative of the function $y = 1/x^2$

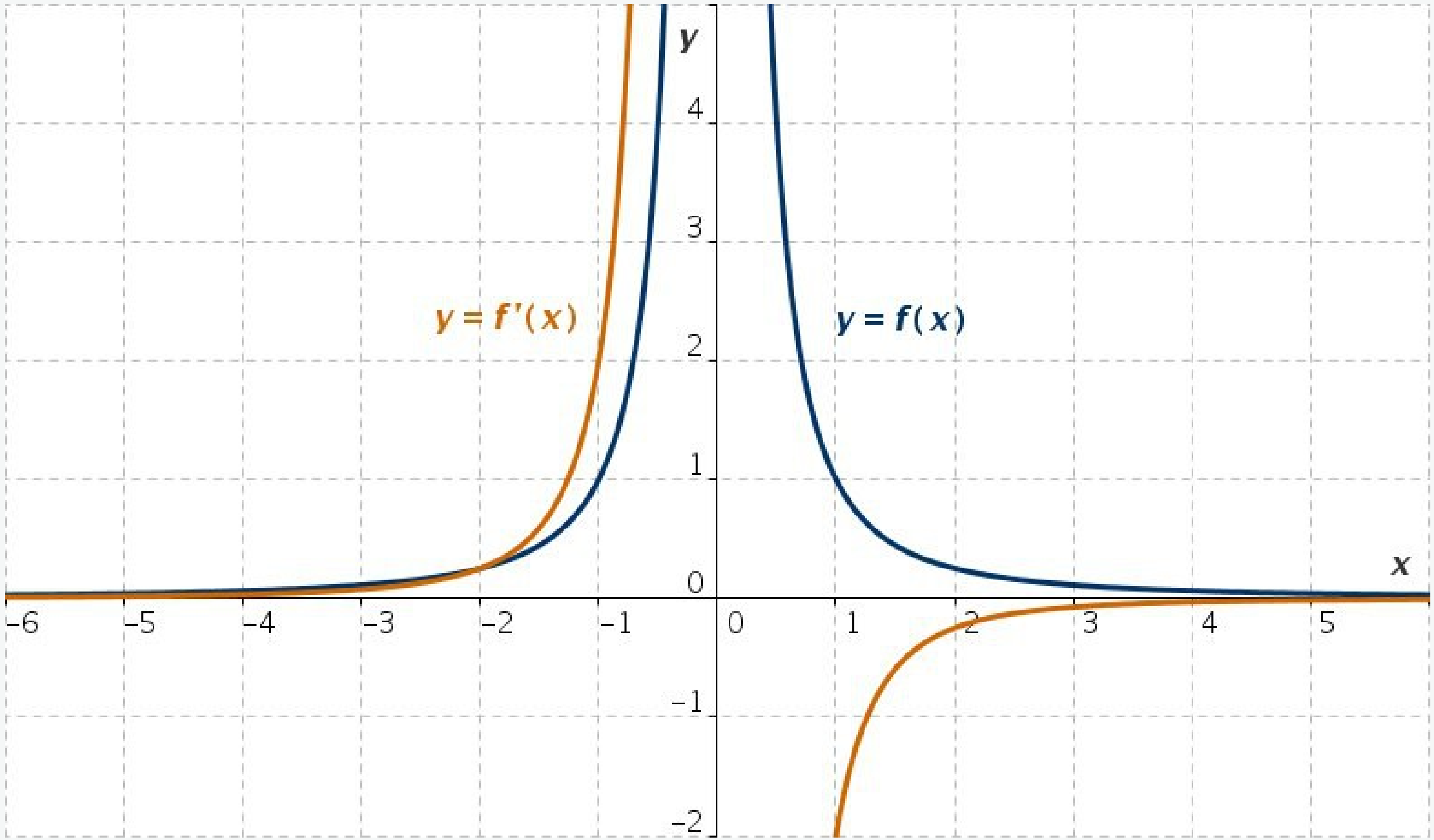

Fig. 1-6: Function $f(x) = 1/x^2$ (blue) and its derivative $f'(x) = -2/x^3$ (red)

Derivative of a function $y = x^n$

$$f(x) = x^n, \qquad f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^n - x^n}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{x^n + n\,x^{n-1}\Delta x + C\,x^{n-2}\Delta x^2 + \ldots + \Delta x^n - x^n}{\Delta x} =$$

$$= n\,x^{n-1}$$

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

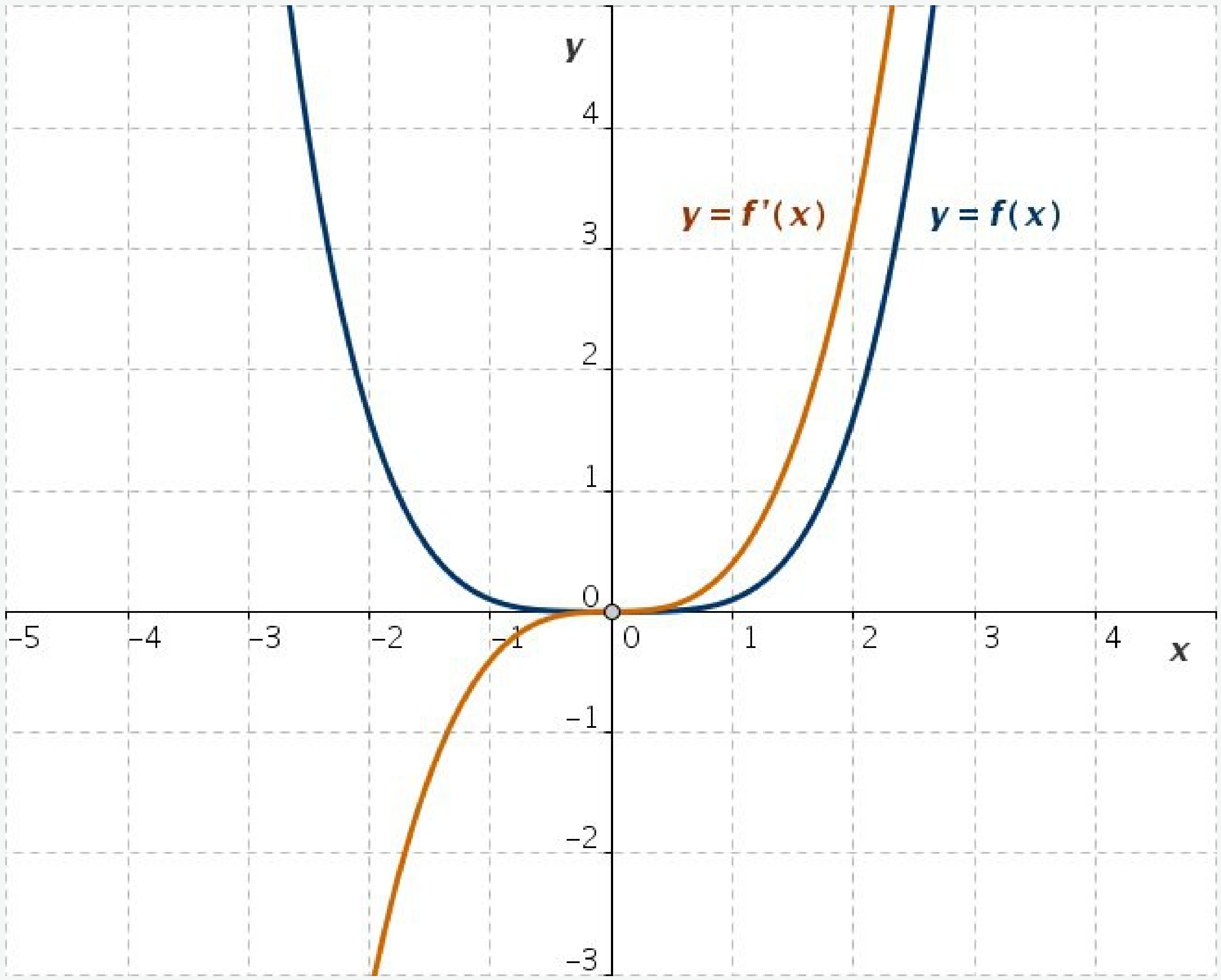

Fig. 1-8a: Function y = f (x) (blue) and its derivative y = f '(x) (red)

$$f(x) = \frac{x^4}{10}, \qquad f'(x) = \frac{2}{5}\, x^3$$

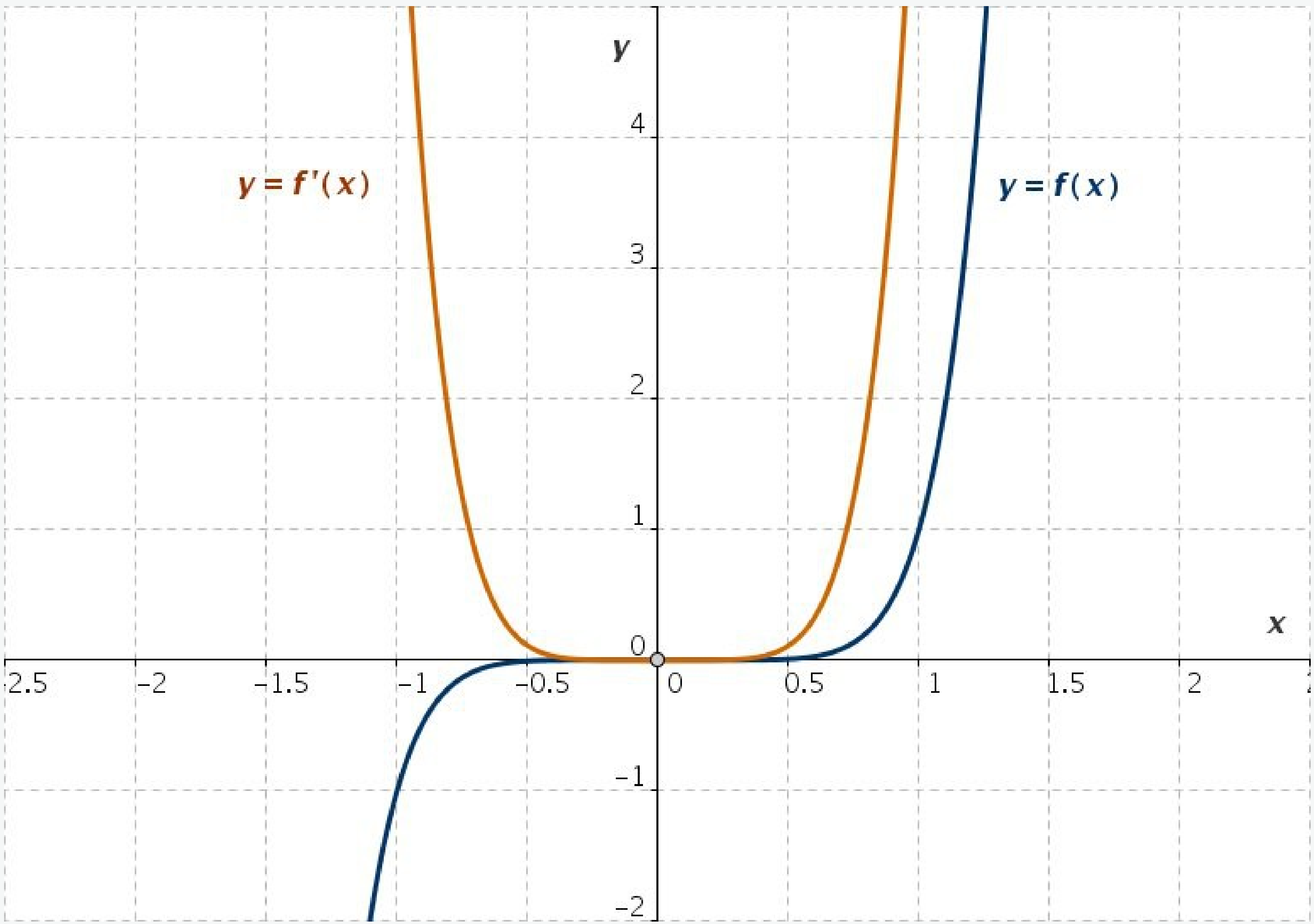

Fig. 1-8b: Function y = f (x) (blue) and its derivative y = f '(x) (red)

$$f(x) = x^7, \qquad f'(x) = 7x^6$$

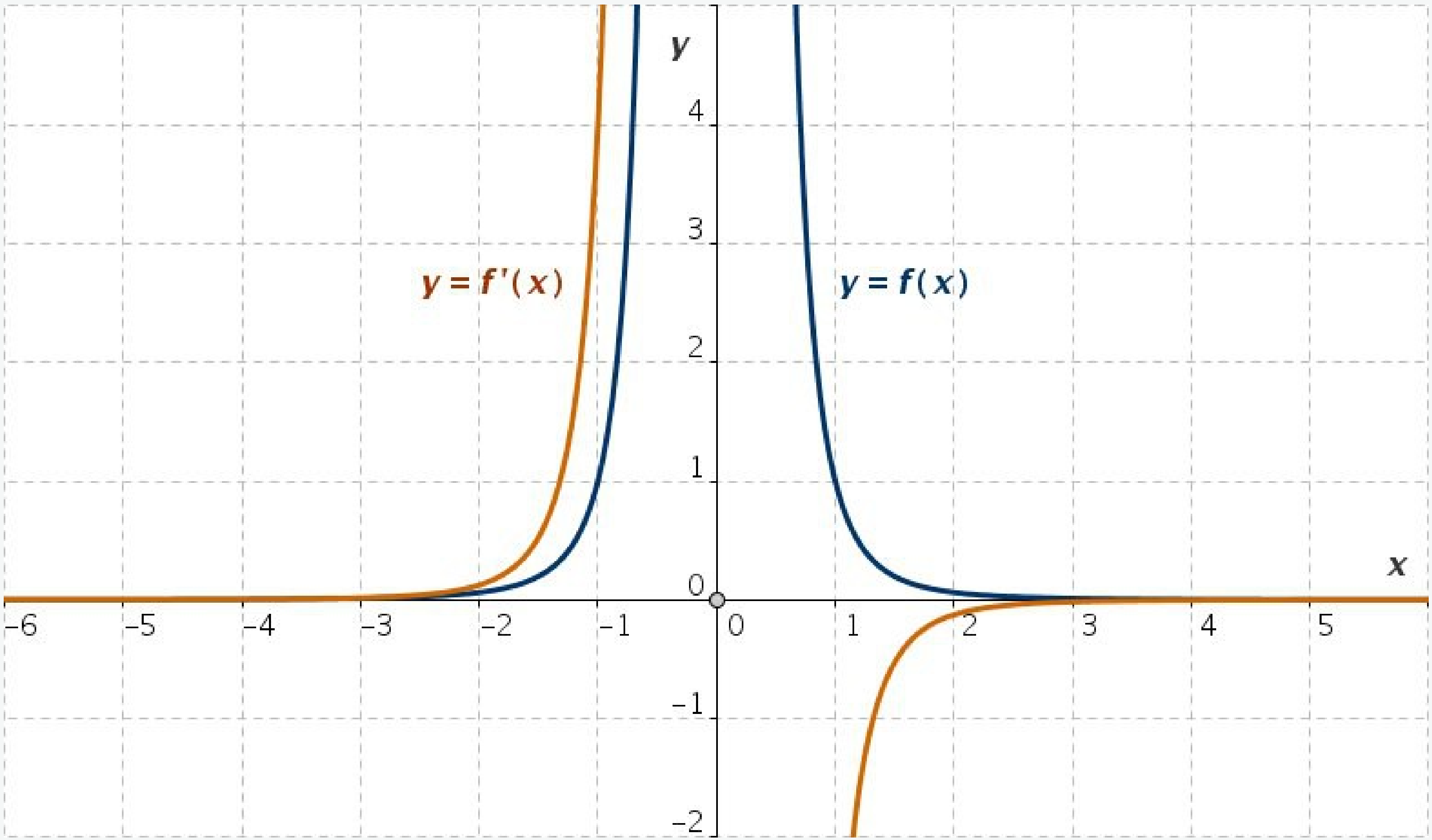

Fig. 1-8c: Function y = f (x) (blue) and its derivative y = f '(x) (red)

$$f(x) = \frac{1}{x^4}, \qquad f'(x) = -\frac{4}{x^5}$$

Derivative of elementary funktions

$$C' = 0 \quad (C = const), \qquad (x^n) = n \cdot x^{n-1} \quad (n \in \mathbb{R})$$

$$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0), \qquad (\ln x)' = \frac{1}{x} \quad (x > 0)$$

$$(e^x)' = e^x, \qquad (a^x)' = (\ln a) \cdot a^x$$

$$(\sin x)' = \cos x, \qquad (\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}, \qquad \cot(x)' = -\frac{1}{\sin^2 x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}, \qquad (\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

$$(\sinh x)' = \cosh x, \qquad (\cosh x)' = \sinh x$$

$$(\tanh x)' = \frac{1}{\cosh^2 x}, \qquad (\coth x)' = -\frac{1}{\sinh^2 x}$$

$$(\operatorname{arsinh} x)' = \frac{1}{\sqrt{x^2+1}}, \qquad (\operatorname{arcosh} x)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\operatorname{artanh} x)' = \frac{1}{1-x^2}, \qquad (\operatorname{arcoth} x)' = \frac{1}{1-x^2}$$