Solve the following exponential equations applying the power rules

Exercise 6: $e^{x+1} - e^{x} = 1$

Exercise 7: $2^{x+2} - 6 \cdot 2^{x+1} = -64$

Exercise 8: $4^{x+2} = (4e)^{x}$

Solve the following exponential equations using substitutions

Exercise 9: $e^{2x} - 2e^{x} - 3 = 0$

Exercise 10: $e^{4x} - 5e^{2x} + 6 = 0$

Exercise 11: $e^{x} + e^{-x} = 2$

Exercise 12: $e^{2x} - 2e^{x} - 15 = 0$

Solution 6:

$$e^{x+1} - e^{x} = 1 \quad \Leftrightarrow \quad e\, e^{x} - e^{x} = 1 \quad \Leftrightarrow \quad e^{x}(e-1) = 1$$

$$e^{x}(e-1) = 1 \quad \Rightarrow \quad e^{x} = \frac{1}{e-1} \quad \Rightarrow \quad x = \ln\left(\frac{1}{e-1}\right) = -\ln(e-1)$$

Solution 7:

$$2^{x+2} - 6 \cdot 2^{x+1} = -64 \quad \Leftrightarrow \quad 2^{2} \cdot 2^{x} - 6 \cdot 2 \cdot 2^{x} = -64 \quad \Rightarrow$$

$$2^{x}(1-3) = -16 \quad \Leftrightarrow \quad 2^{x} = 8 = 2^{3} \quad \Rightarrow \quad x = 3$$

Solution 8:

$$4^{x+2} = (4e)^{x} \quad \Leftrightarrow \quad 4^{2} \cdot 4^{x} = 4^{x} \cdot e^{x} \quad \Rightarrow \quad e^{x} = 4^{2} = 2^{4}$$

$$x = 4 \ln 2$$

Solutions 9:

$$e^{2x} - 2e^{x} - 3 = 0, \qquad u = e^{x}$$

$$u^2 - 2u - 3 = 0, \qquad u_1 = -1, \qquad u_2 = 3$$

The function *exp (x)* is positive everywhere. The condition $e^{x} = -1$ can not be satisfied. There is one solution only :

$$e^{x} = 3, \qquad x = \ln 3$$

Solutions 10:

$$e^{4x} - 5\,e^{2x} + 6 = 0, \qquad u = e^{2x}$$

$$u^2 - 5\,u + 6 = 0, \qquad u_1 = 2, \qquad u_2 = 3$$

$$\Rightarrow \quad e^{2x} = 2 \quad \Rightarrow \quad x = \frac{\ln 2}{2}$$

$$e^{2x} = 3 \quad \Rightarrow \quad x = \frac{\ln 3}{2}$$

Solution 11:

$$e^{x} + e^{-x} = 2 \quad | \quad \times \; e^{x}$$

$$e^{2x} + 1 = 2e^{x} \quad \Rightarrow \quad e^{2x} - 2e^{x} + 1 = 0, \qquad u = e^{x}$$

$$u^2 - 2u + 1 = 0 \quad \Rightarrow \quad u_1 = u_2 = 1$$

There is only one solution :

$$e^{x} = 1 \quad \Rightarrow \quad x = 0$$

Solution 12:

$$e^{2x} - 2e^{x} - 15 = 0, \qquad u = e^{x}$$

$$u^2 - 2u - 15 = 0, \qquad u_1 = -3, \qquad u_2 = 5$$

The function *exp (x)* is positive everywhere. The condition $e^{x} = -3$ can not be satisfied. There is one solution only :

$$e^{x} = 5, \qquad x = \ln 5$$

Solve the following exponential equations applying the power rules

Exercise 13: $e^{x+2} - e^{x} = 5$

Exercise 14: $2^{x+3} + 2^{x+1} = 4$

Exercise 15: $3^{x+3} - 6 \cdot 3^{x+1} = 81$

Exercise 16: $-2e^{2x} - 4e^{x} + 6 = 0$

Exercise 17: $2e^{2x} + 4e^{x} + 2 = 0$

Exercise 18: $e^{2x} - 6e^{x} + 5 = 0$

Solution 13: $e^{x+2} - e^{x} = 5, \qquad x = \ln\left(\dfrac{5}{e^2 - 1}\right)$

Solution 14: $2^{x+3} + 2^{x+1} = 4, \qquad x = \log_2\left(\dfrac{2}{5}\right) = -\log_2 5$

Solution 15: $3^{x+3} - 6 \cdot 3^{x+1} = 81, \qquad x = 2$

Solution 16: $-2e^{2x} - 4e^{x} + 6 = 0, \quad e^{x} = 1, \quad x = 0$

Solution 17: $2e^{2x} + 4e^{x} + 2 = 0$

$e^{x} = -1$ – no real solution

Solution 18: $e^{2x} - 6e^{x} + 5 = 0, \qquad x = 0, \quad x = \ln 5$

Solve the following equations:

Exercise 19: $3^{x} = 3^{5x-4}$

Exercise 20: $2^{x} = 4^{x-1}$

Exercise 21: $7^{x^2} = 7^{6-x}$

Exercise 22: $4^{5-9x} = \dfrac{1}{8^{3x+2}}$

Exercise 23: $5^{x} = 9$

Exercise 24: $2^{4x+1} - 3^{x} = 0$

Exercise 25: $3^{2x+3} = 4^{2x}$

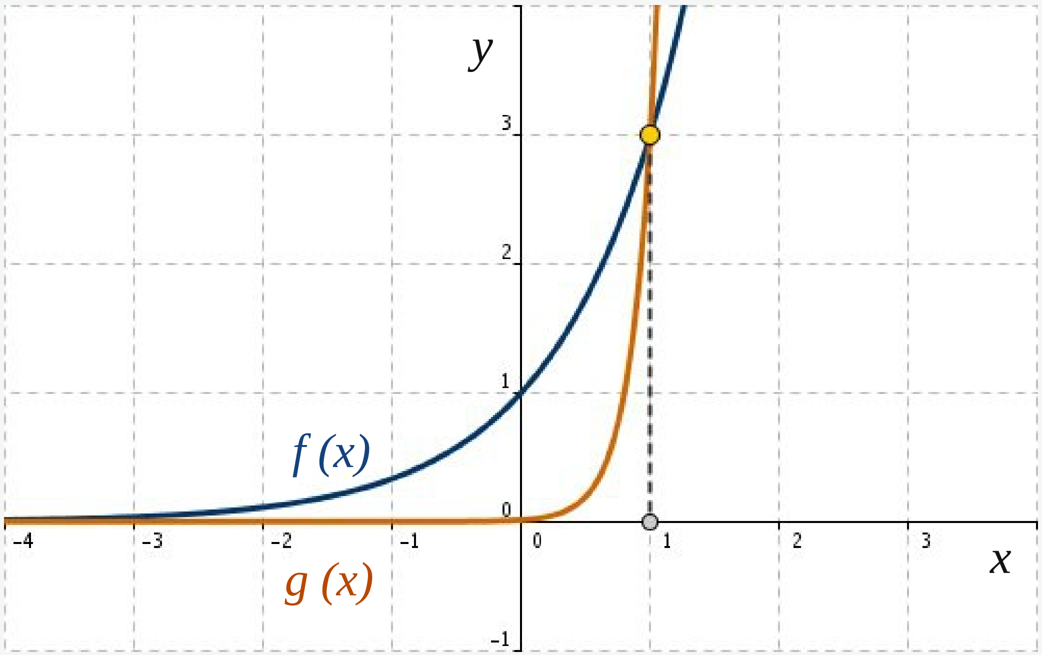

Fig. L19: Functions f (x) and g (x)

$$f(x) = 3^{x}, \qquad g(x) = 3^{5x-4}$$

$$3^{x} = 3^{5x-4} \quad \Rightarrow \quad x = 5x - 4 \quad \Rightarrow \quad x = 1$$

Two powers with the same base are equal, if the exponents are equal.

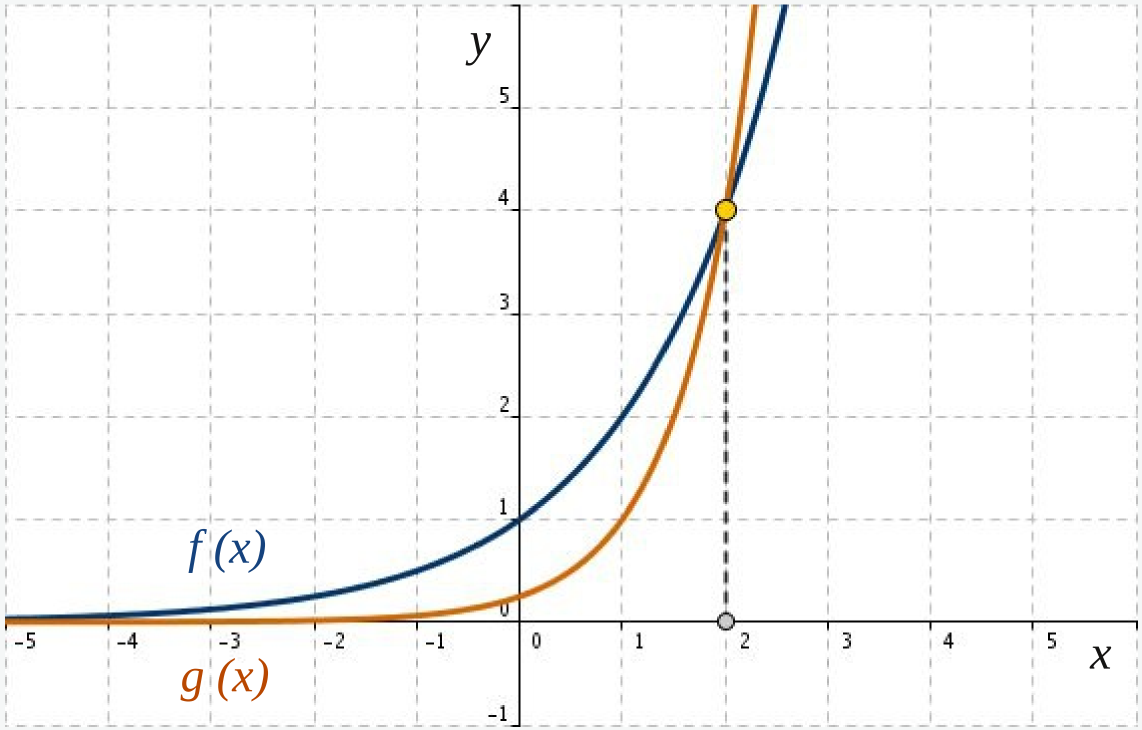

Fig. L20: Functions f (x) and g (x)

$$f(x) = 2^{x}, \qquad g(x) = 4^{x-1}$$

$$2^{x} = 4^{x-1} \quad \Leftrightarrow \quad 2^{x} = 2^{2(x-1)} \quad \Leftrightarrow \quad x = 2(x-1) \quad \Rightarrow$$

$$x = 2$$

Exponential equations: Solutions 21,23

Solution 21: $$7^{x^2} = 7^{6-x} \quad \Rightarrow \quad x^2 = 6 - x \quad \Leftrightarrow \quad x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0 \quad \Rightarrow \quad x_1 = -3, \quad x_2 = 2$$

Solution 22: $$4^{5-9x} = \frac{1}{8^{3x+2}} \quad \Leftrightarrow \quad 2^{2(5-9x)} = \frac{1}{2^{3(3x+2)}} \quad \Leftrightarrow$$

$$2^{2(5-9x)} = 2^{-3(3x+2)} \quad \Rightarrow \quad 2(5-9x) = -3(3x+2)$$

$$\Rightarrow \quad 9x = 16 \quad \Rightarrow \quad x = \frac{16}{9}$$

Solution 23: $$5^x = 9 \quad \Rightarrow \quad \ln 5^x = \ln 9 \quad \Leftrightarrow \quad x \ln 5 = \ln 9 \quad \Rightarrow$$

$$x = \frac{\ln 9}{\ln 5} \simeq \frac{2.197}{1.609} \simeq 1.365$$

$$1.365 \simeq \frac{\ln 9}{\ln 5} \neq \ln\left(\frac{9}{5}\right) \simeq 0.5888$$

Solution 24: $2^{4x+1} - 3^{x} = 0$

$$\ln 2^{4x+1} = \ln 3^{x} \quad \Leftrightarrow \quad (4x+1)\ln 2 = x \ln 3 \quad \Rightarrow$$

$$4x + 1 = c\,x \qquad c \equiv \frac{\ln 3}{\ln 2} \quad \Rightarrow \quad x = \frac{1}{c-4} = \frac{\ln 2}{\ln 3 - 4\ln 2}$$

$$x = \frac{1}{c-4} = \frac{\ln 2}{\ln 3 - 4\ln 2} \simeq \frac{0.693}{1.099 - 4 \cdot 0.693} \simeq -0.414$$

Solution 25: $3^{2x+3} = 4^{2x}$

$$\ln 3^{2x+3} = \ln 4^{2x} \quad \Leftrightarrow \quad (2x+3)\ln 3 = 2x \ln 4 \quad \Rightarrow$$

$$2x + 3 = 2c\,x\,, \qquad c \equiv \frac{\ln 4}{\ln 3} \quad \Rightarrow \quad x = \frac{3}{2(c-1)}$$

$$x = \frac{3}{2\left(\frac{\ln 4}{\ln 3} - 1\right)} \simeq \frac{3}{2(1.261 - 1)} \simeq 5.693$$