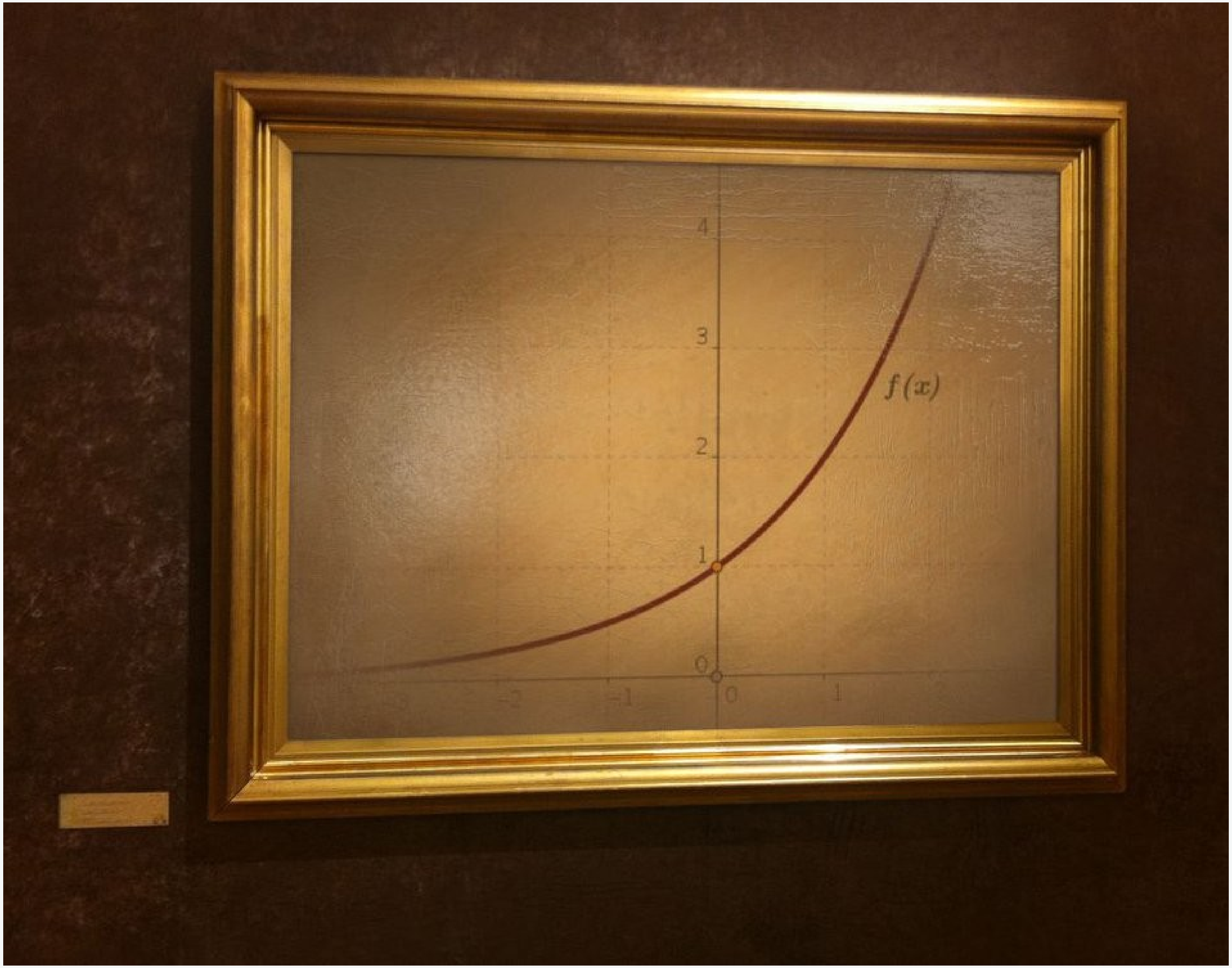
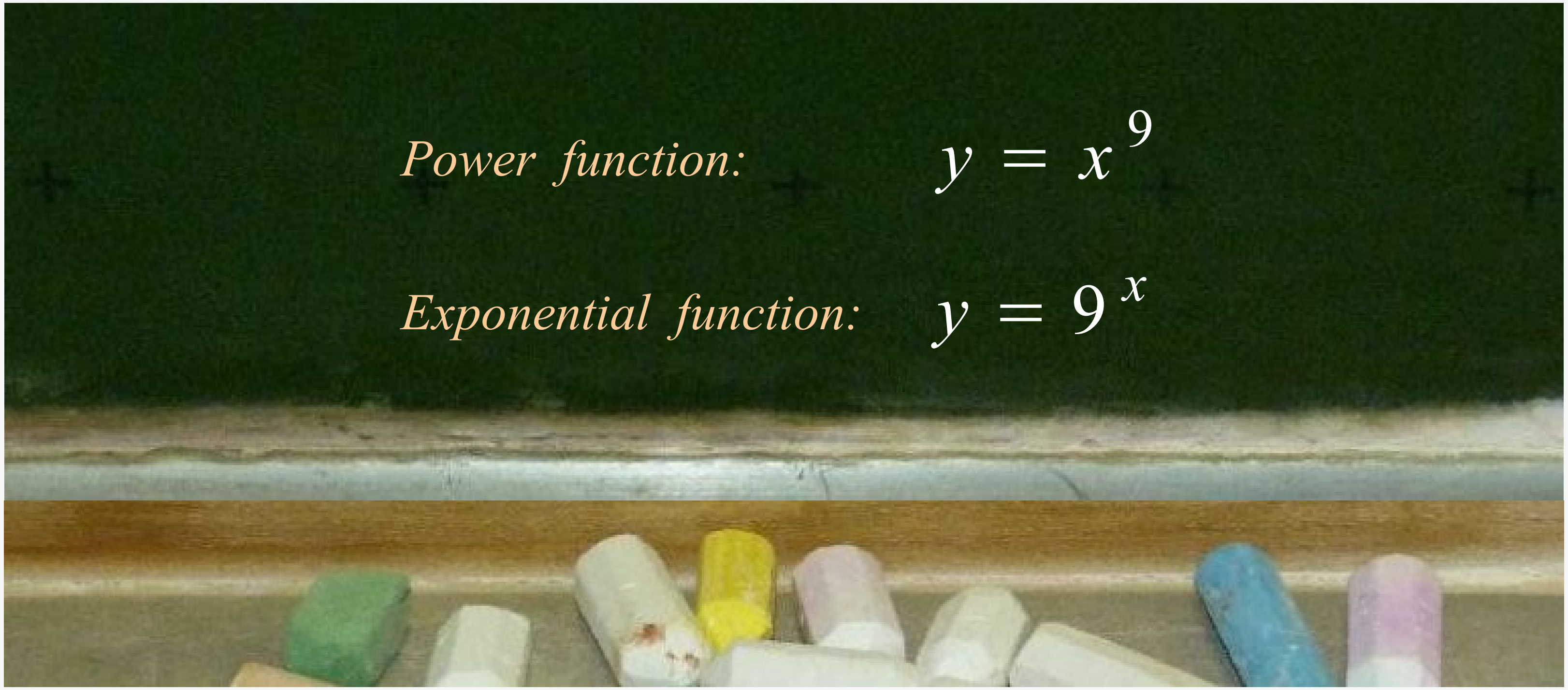


Exponential Functions: *Properties, Graphs*

Exponential functions



The formal structure of power functions and exponential functions is very similar. In both cases a base is exponentiated. Whilst with power functions, the variable is the base which is then exponentiated by a constant, it is the other way round with exponential functions: the base is a constant and the exponent contains the variable.

Fig. 1: Some applications of exponential functions with base $a = e$

Definition: Functions like $y = a^x$ with positive base a $(a \neq 1)$ are called exponential functions.

Draw the exponential functions $y = f(x)$ and examine their Properties:

Exercise 1: $f(x) = 2^{x}$

Exercise 2: $f(x) = 2^{-x}$

Exercise 3: $f(x) = 2^{x}, \quad g(x) = e^{x}, \quad h(x) = 4^{x}$

Exercise 4: $f(x) = 1.5^{-x}, \quad g(x) = 2^{-x}, \quad h(x) = 3^{-x}$

Exercise 5: $f(x) = 2^{-x}, \quad g(x) = 2^{x}$

Exercise 6: $f(x) = e^{x} - 2, \quad g(x) = e^{x} + 1$

Exercise 7: $f(x) = 2e^{x}, \quad g(x) = 0.5e^{x}$

Exponential functions: Exercise 8-12

Exercise 8: $f(x) = e^{2x}, \quad g(x) = e^{\frac{x}{2}}$

Exercise 9: $f(x) = 2^{-x}, \quad g(x) = 2^{-x+1}$

Exercise 10: $f(x) = e^{\cos x}$

Exercise 11: $f(x) = e^{\cos^2 x}$

Exercise 12: $f(x) = \cos(e^x)$

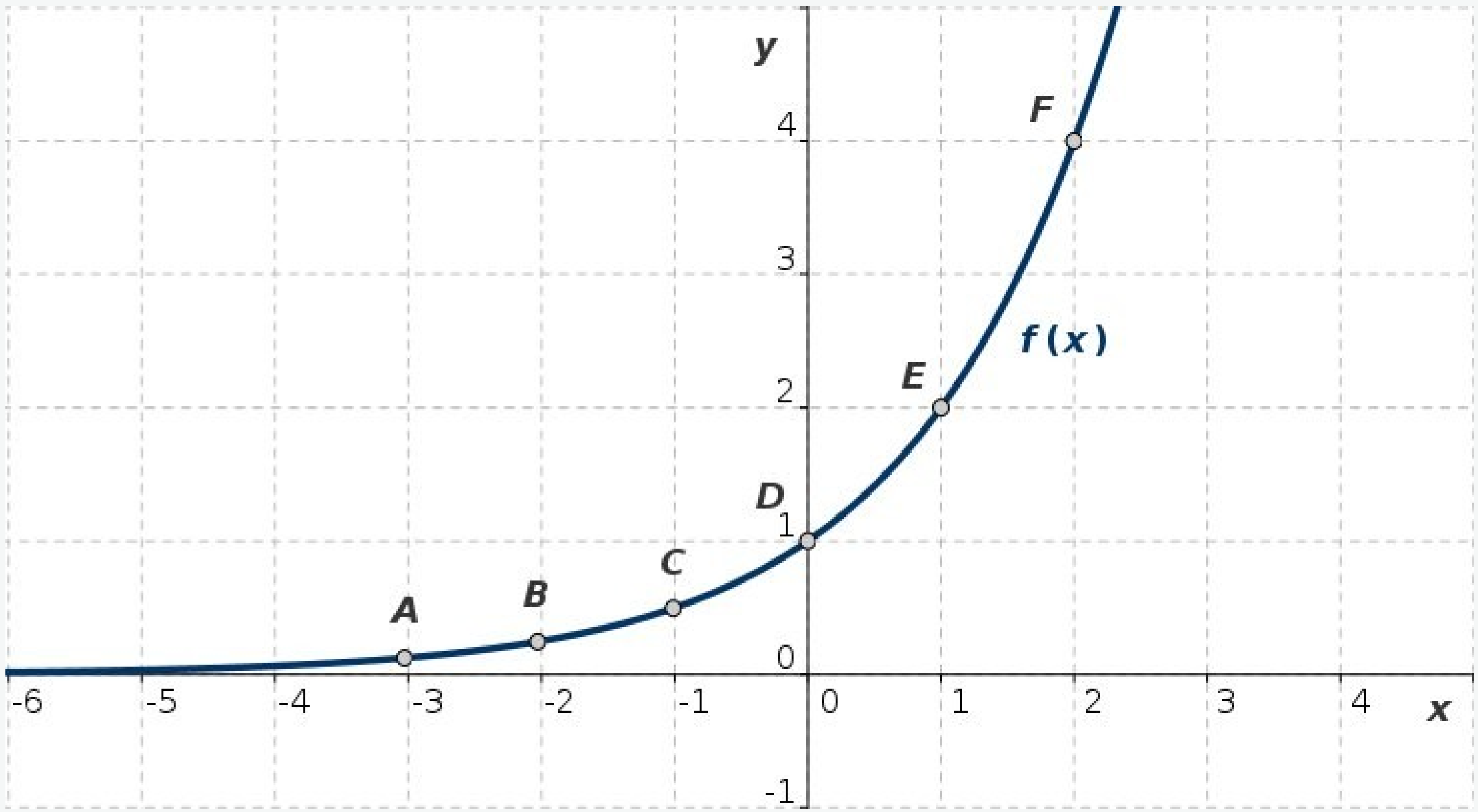


Fig. A1-1: Exponential function $y = f(x)$

$$f(x) = 2^{x}$$

$$A = \left(-3,\ \frac{1}{2^3}\right) = \left(-3,\ \frac{1}{8}\right), \qquad B = \left(-2,\ \frac{1}{2^2}\right) = \left(-2,\ \frac{1}{4}\right)$$

$$C = \left(-1,\ \frac{1}{2}\right), \qquad D = (0,\ 1), \qquad E = (1,\ 2), \qquad F = (2,\ 4)$$

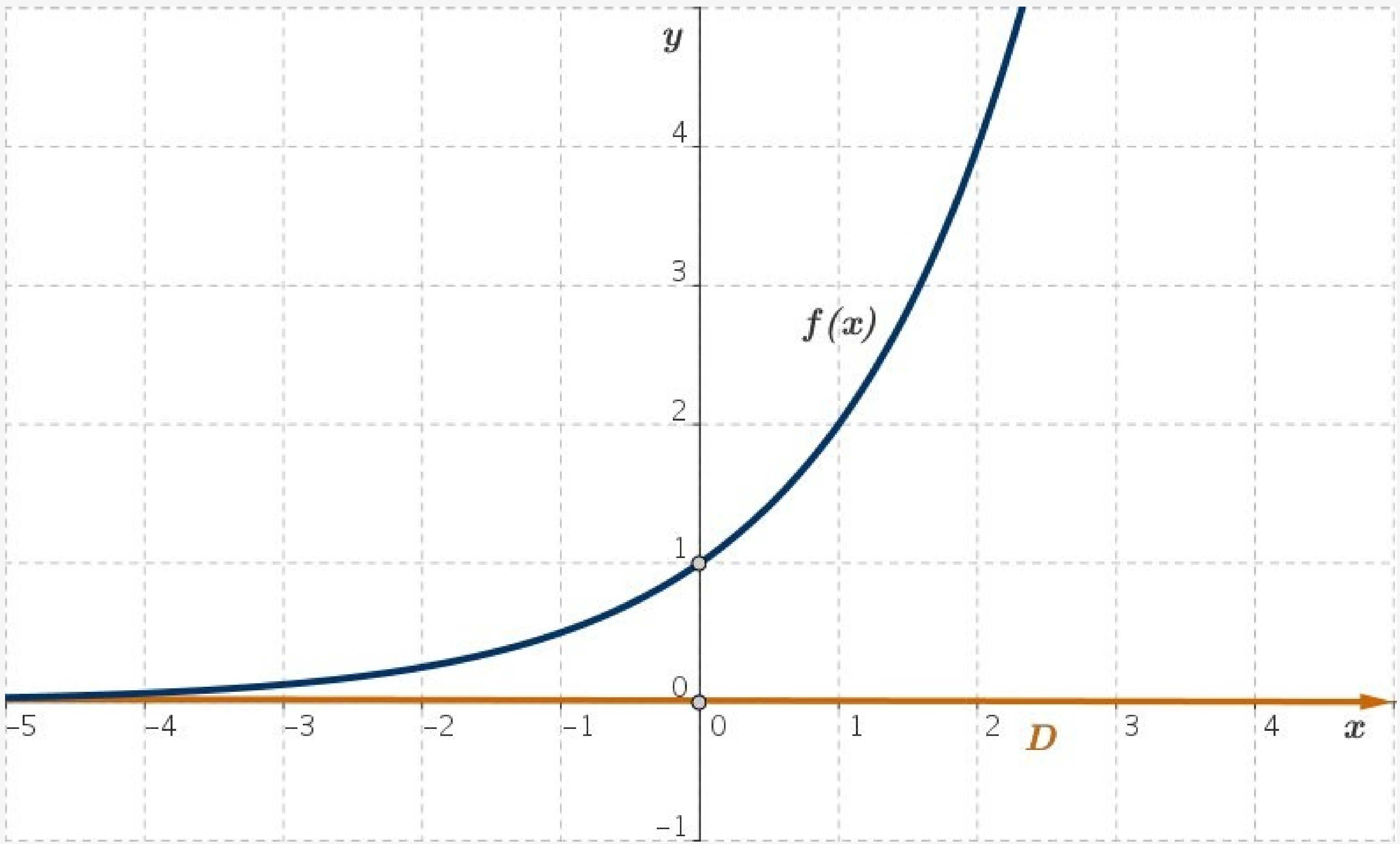


Fig. A1-2: Illustration of the domain of the exponential function $y = f(x)$

The exponential function $y = f(x)$ is defined for all real x :

$$f(x) = 2^x, \qquad D_f = \mathbb{R}$$

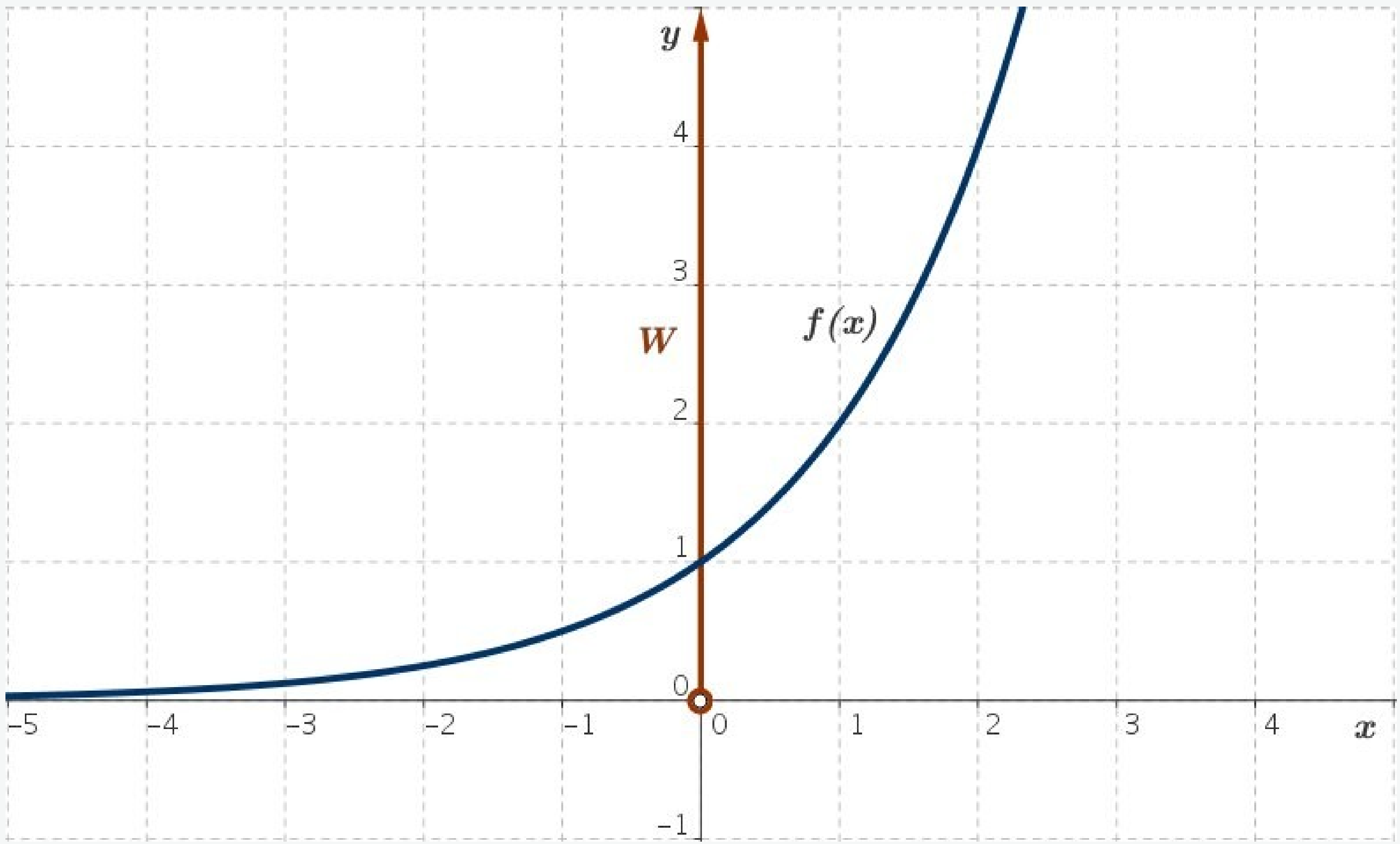


Fig. A1-3: Illustration of the range of the exponential function $y = f(x)$

Range of the exponential function $y = f(x)$: all positive real numbers:

$$f(x) = 2^x, \qquad R_f = \mathbb{R}^+$$

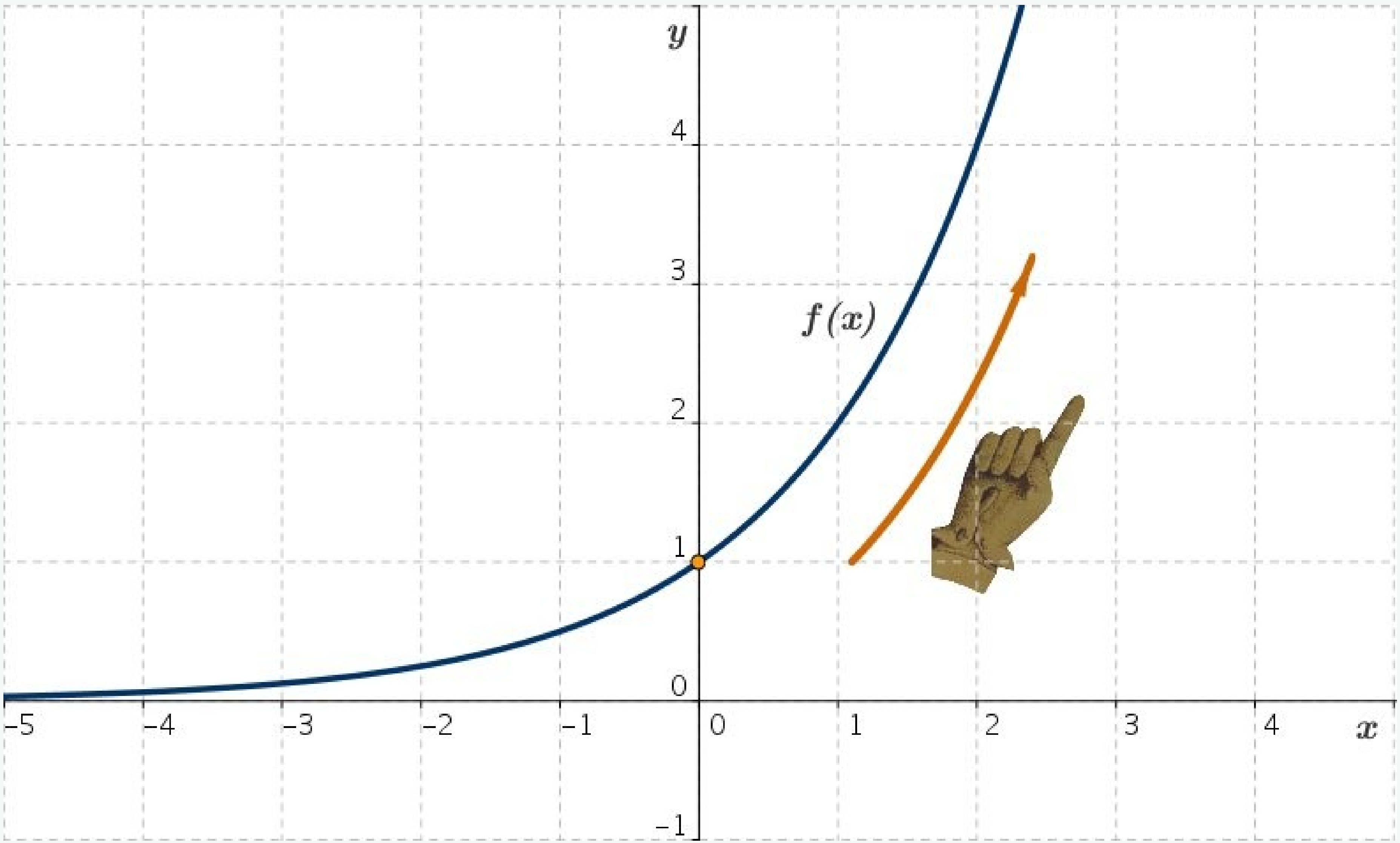


Fig. A1-4: Exponential function $y = f(x)$

The function $y = f(x)$ is monotonically increasing. There is no symmetry:

$$f(x) = 2^x$$

Exponential function of Exercise 2

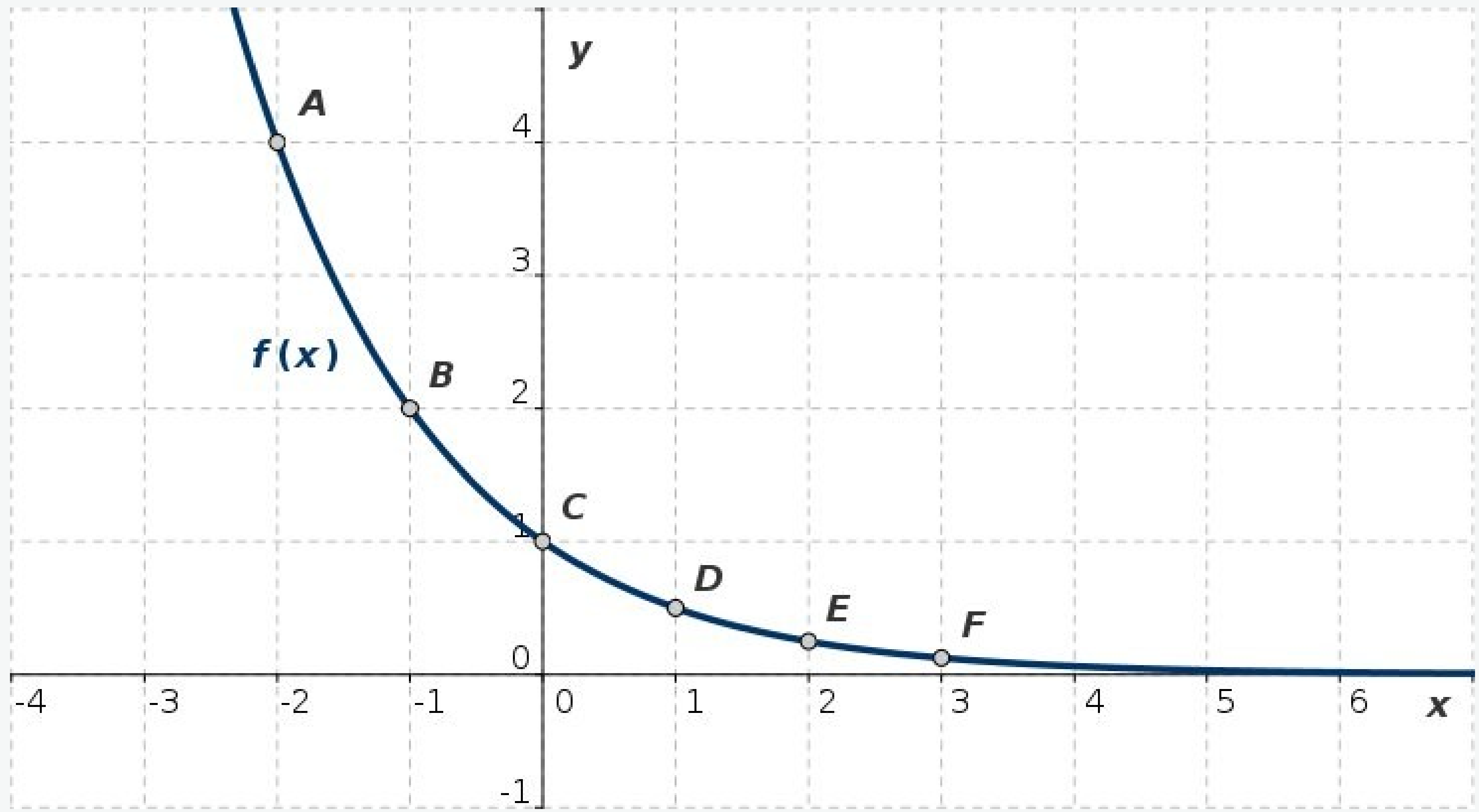


Fig. A2: Exponential function y = f (x)

$$f(x) = 2^{-x}$$

$$A = (-2,\ 4),\qquad B = (-1,\ 2),\qquad C = (0,\ 1)$$

$$D = \left(1,\ \frac{1}{1}\right),\qquad E = \left(2,\ \frac{1}{4}\right),\qquad F = \left(3,\ \frac{1}{8}\right)$$

Exponential functions of Exercise 3

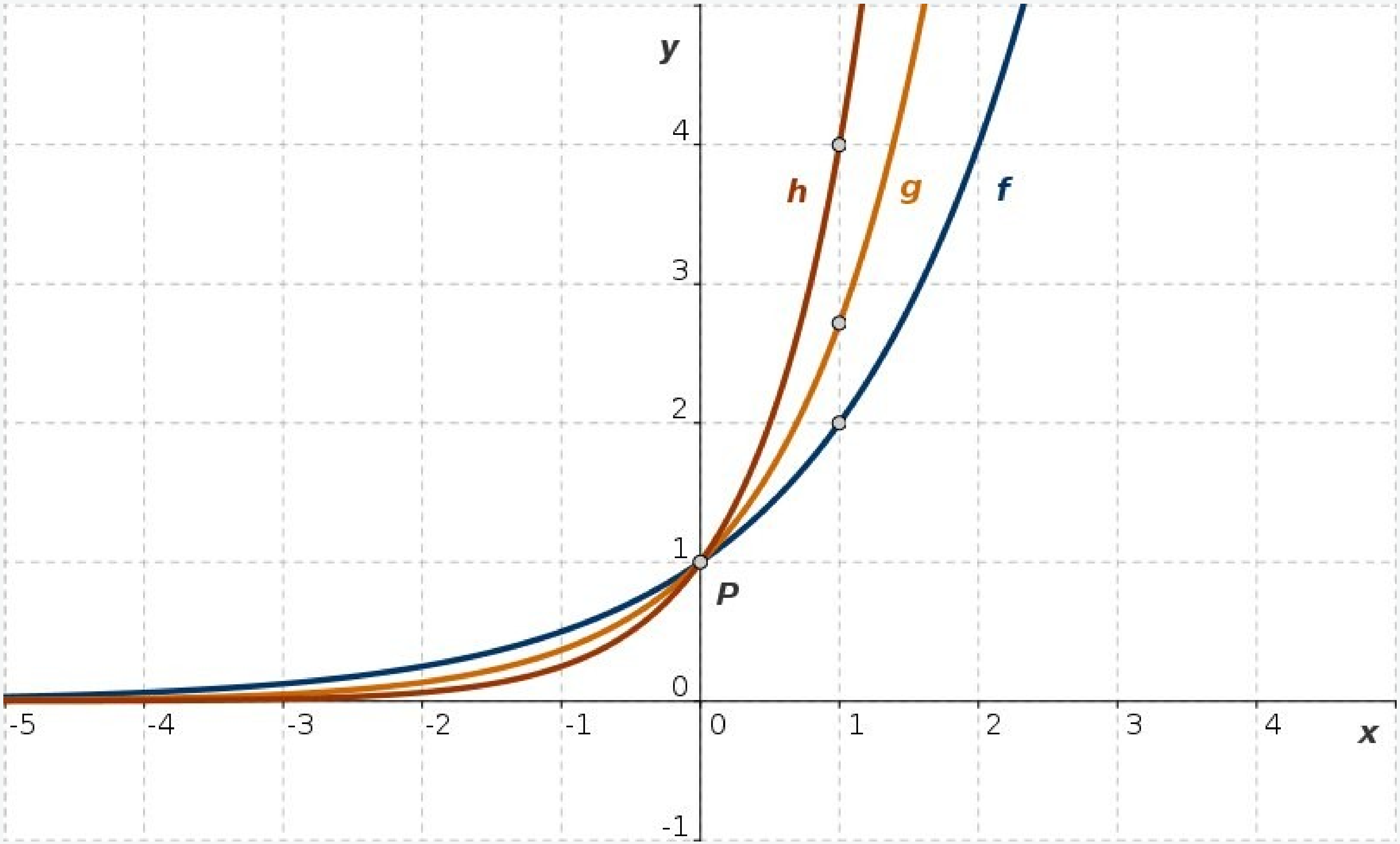


Fig. A3: Exponential functions y = f (x), y = g (x) and y = h (x)

$$f(x) = 2^x, \qquad g(x) = e^x, \qquad h(x) = 4^x$$

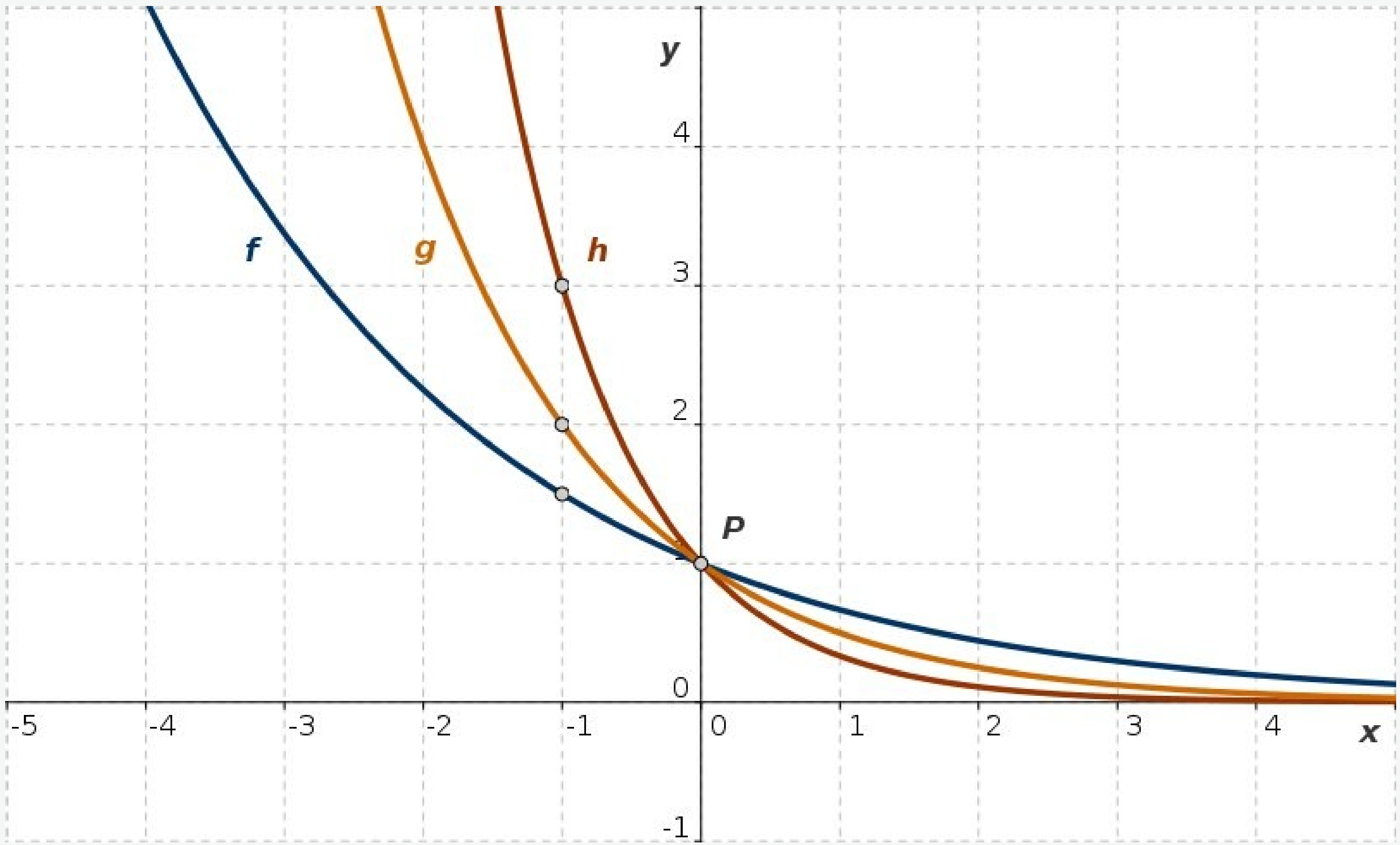


Fig. A4: Exponential functions y = f (x), y = g (x) and y = h (x)

$$f\,(x) = 1.5^{-x}, \qquad g\,(x) = 2^{-x}, \qquad h\,(x) = 3^{-x}$$

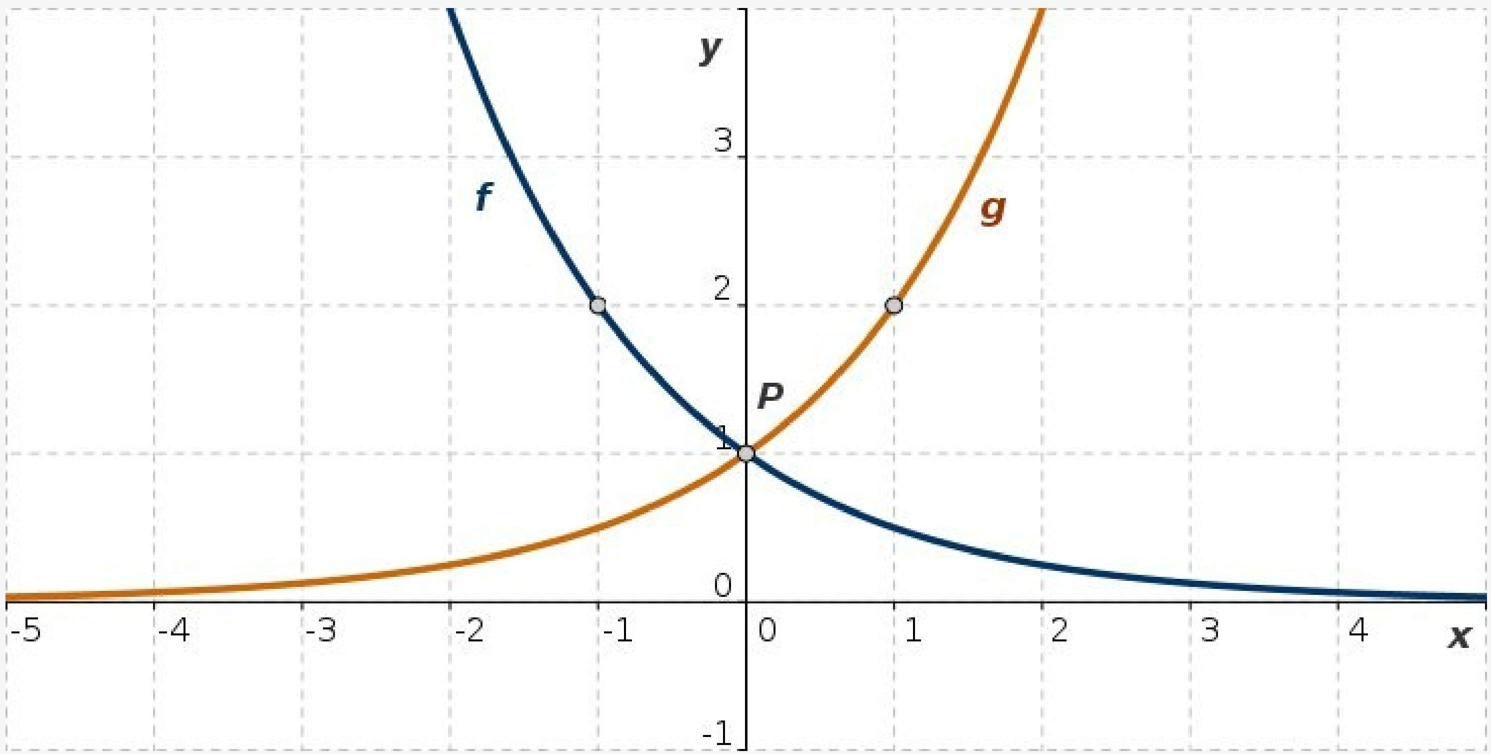


Fig. A5: Exponential functions y = f (x) and y = g (x)

$$f(x) = 2^{-x}, \qquad g(x) = 2^{x}$$

$y = a^x$: Domain: $\mathbb{R}$, Range: $(0, \infty)$

Symmetry: none

Monotony: monotonically increasing ($a > 1$)

monotonically decreasing ($0 < a < 1$)

Common point: $P(0, 1)$

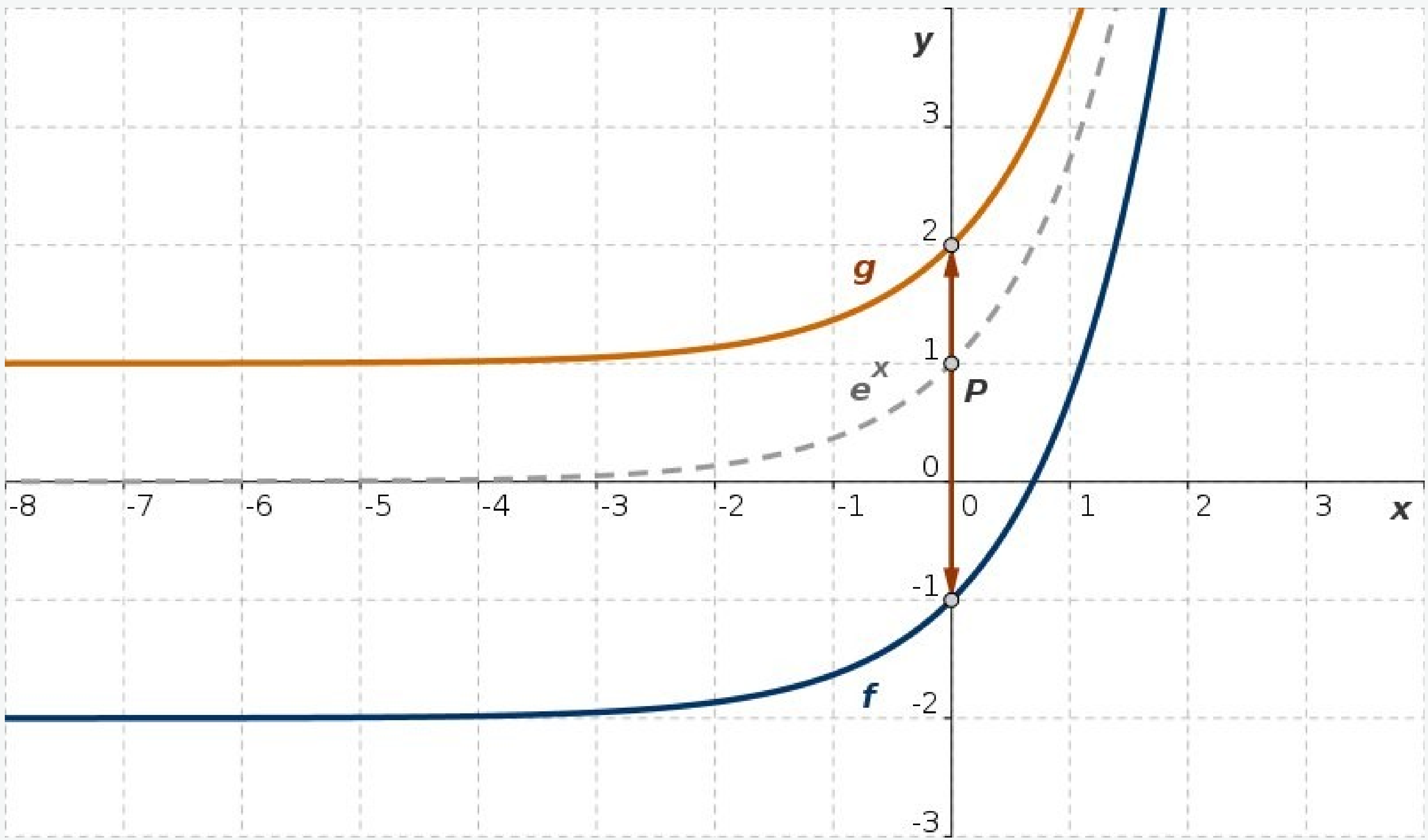


Fig. A6: Exponential functions y = f (x) and y = g (x)

$$f(x) = e^x - 2, \qquad g(x) = e^x + 1$$

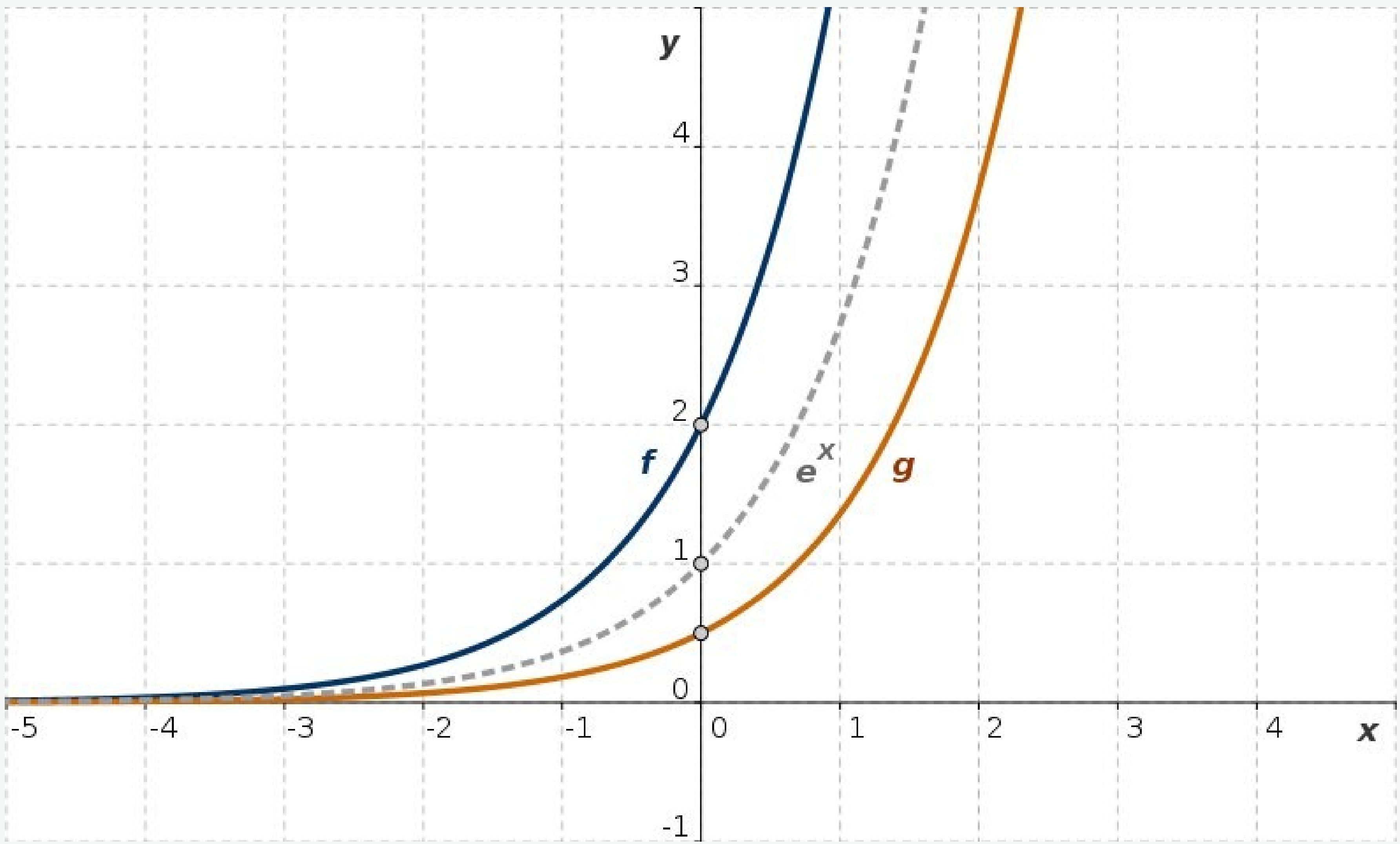


Fig. A7: Exponential functions y = f (x) and y = g (x)

$$f(x) = 2\,e^x, \qquad g(x) = 0.5\,e^x$$

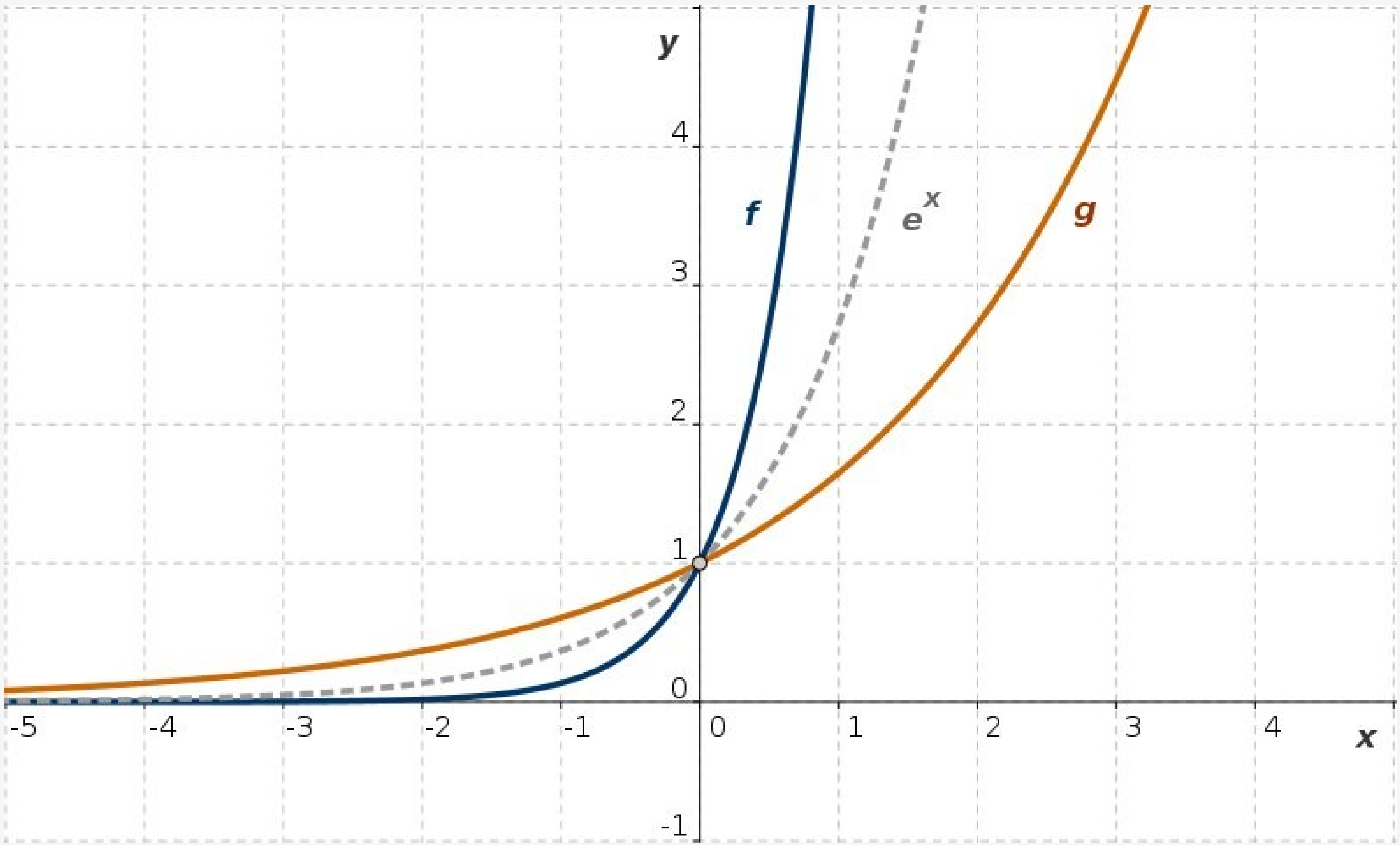


Fig. A8: Exponential funktions y = f (x) and y = g (x)

$$f(x) = e^{2x}, \qquad g(x) = e^{\frac{x}{2}}$$

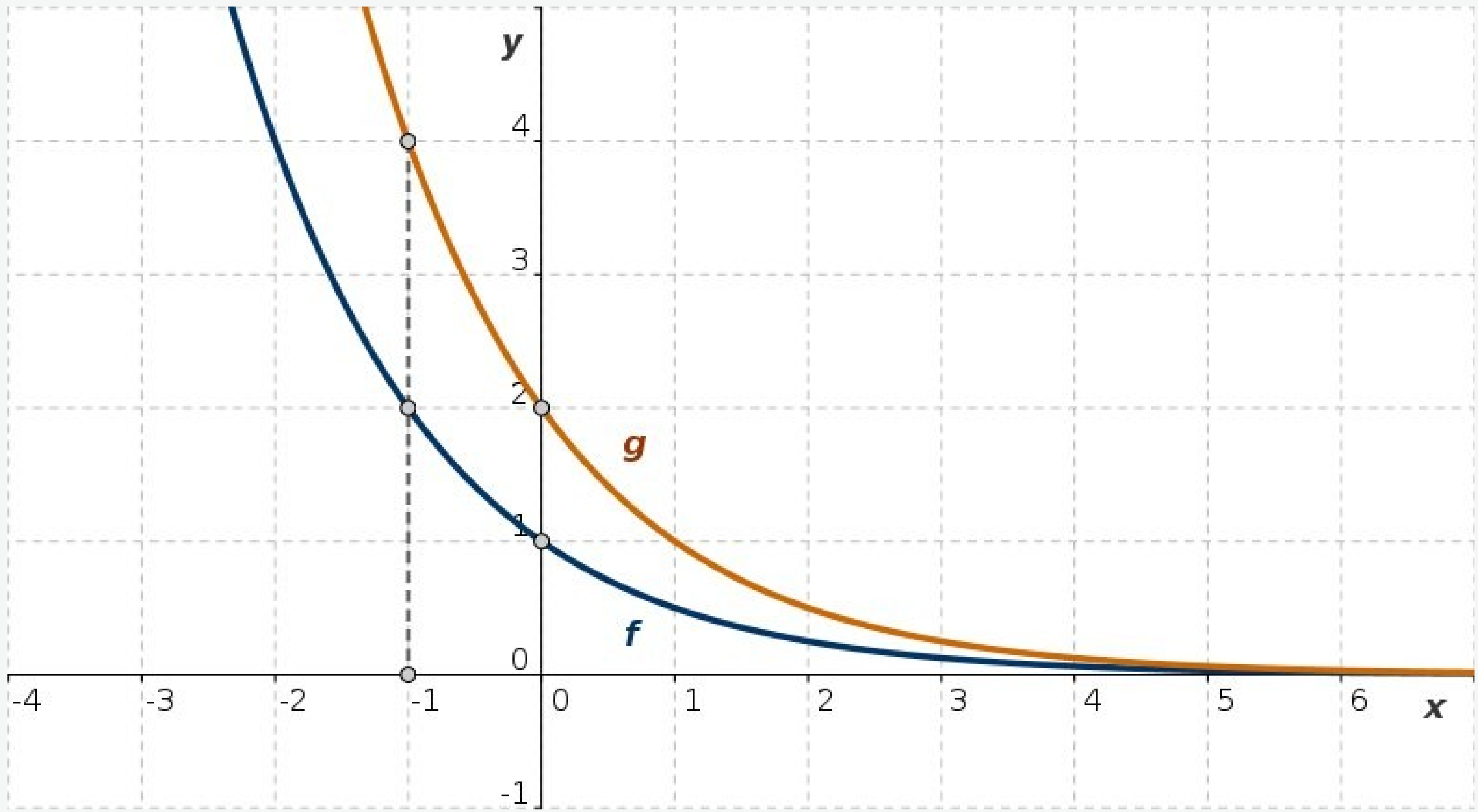


Fig. A9: Exponential functions y = f (x) and y = g (x)

$$f\,(x) = 2^{-x}, \qquad g\,(x) = 2^{-x+1} = 2 \cdot 2^{-x}$$

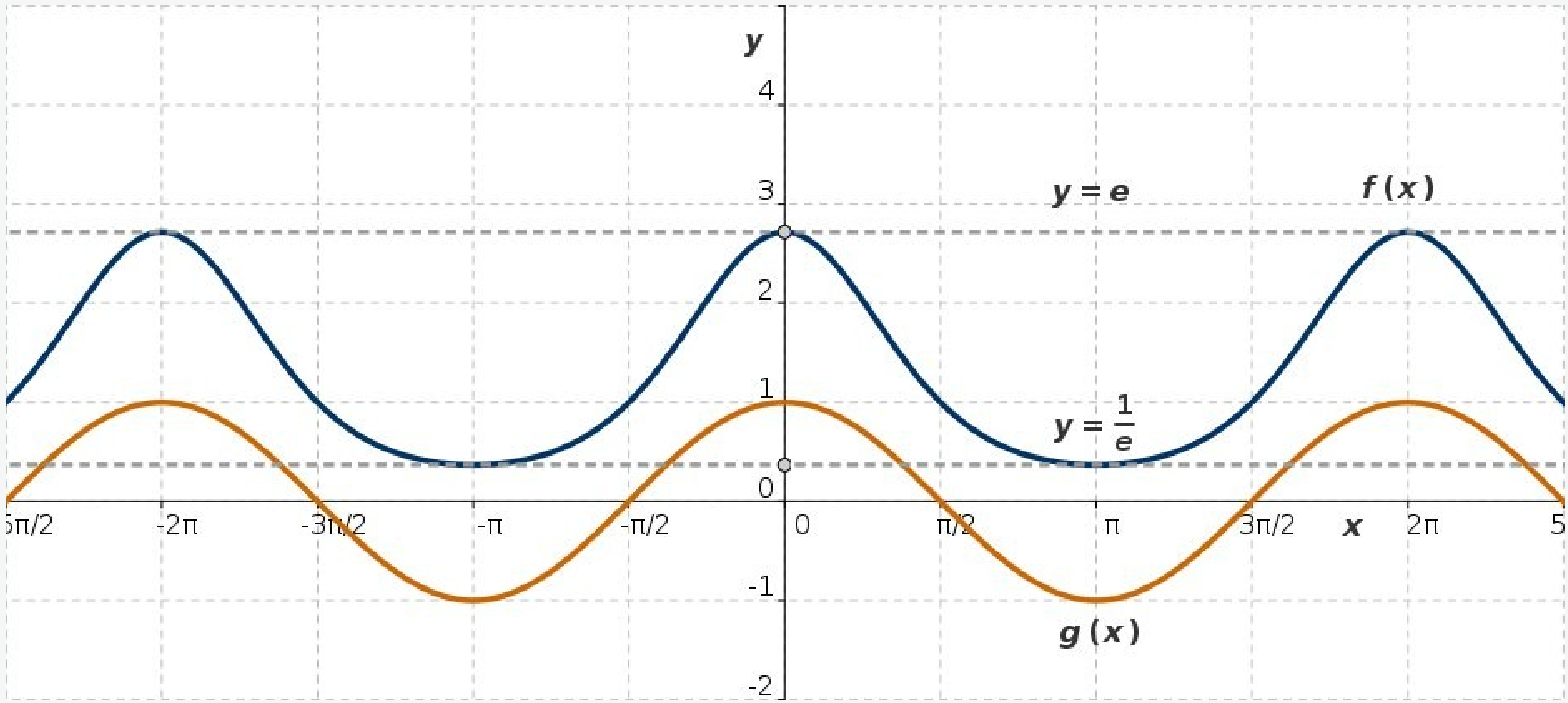


Fig. A10: Functions y = f (x) and y = g (x)

$$f(x) = e^{\cos x}, \qquad g(x) = \cos x$$

$$f_{max}(x) = f\left(x = (\cos x)_{max}\right) = e, \qquad f_{min}(x) = f\left(x = (\cos x)_{min}\right) = \frac{1}{e}$$

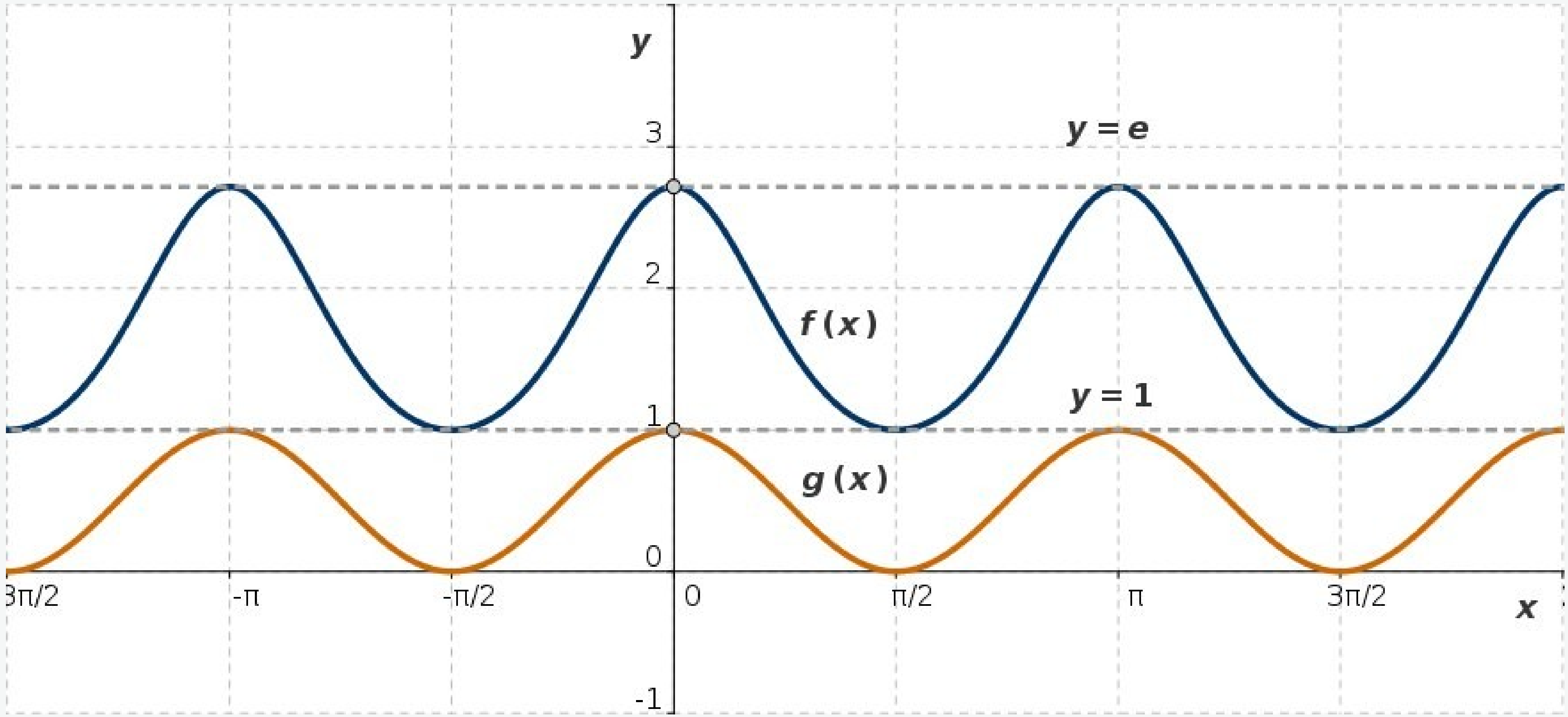


Fig. A11: Functions y = f (x) and y = g (x)

$$f\,(x) \;=\; e^{\cos^2 x}\,, \qquad g\,(x) \;=\; \cos^2 x$$

$$f_{max}\,(x) \;=\; f\left(x = (\cos^2 x)_{max}\right) \;=\; e\,, \qquad f_{min}\,(x) \;=\; f\left(x = (\cos^2 x)_{min}\right) \;=\; 1$$

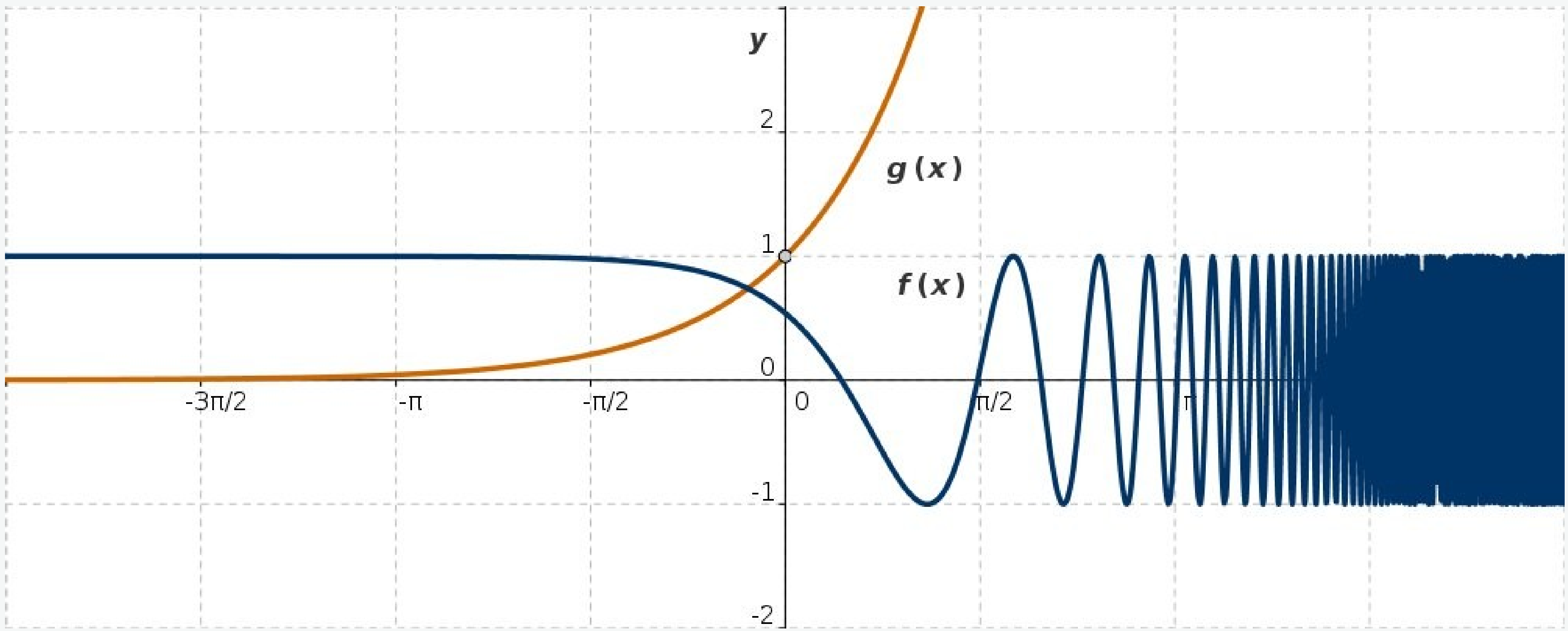


Fig. A12: Funktions $y = f(x)$ and $y = g(x)$

$$f(x) = \cos(e^x), \qquad g(x) = e^x$$