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The concept of Maps

Functions are basic tools of mathematics to describe relations, dependencies and developments. The handling of functions is one of the main topics of mathematical analysis.

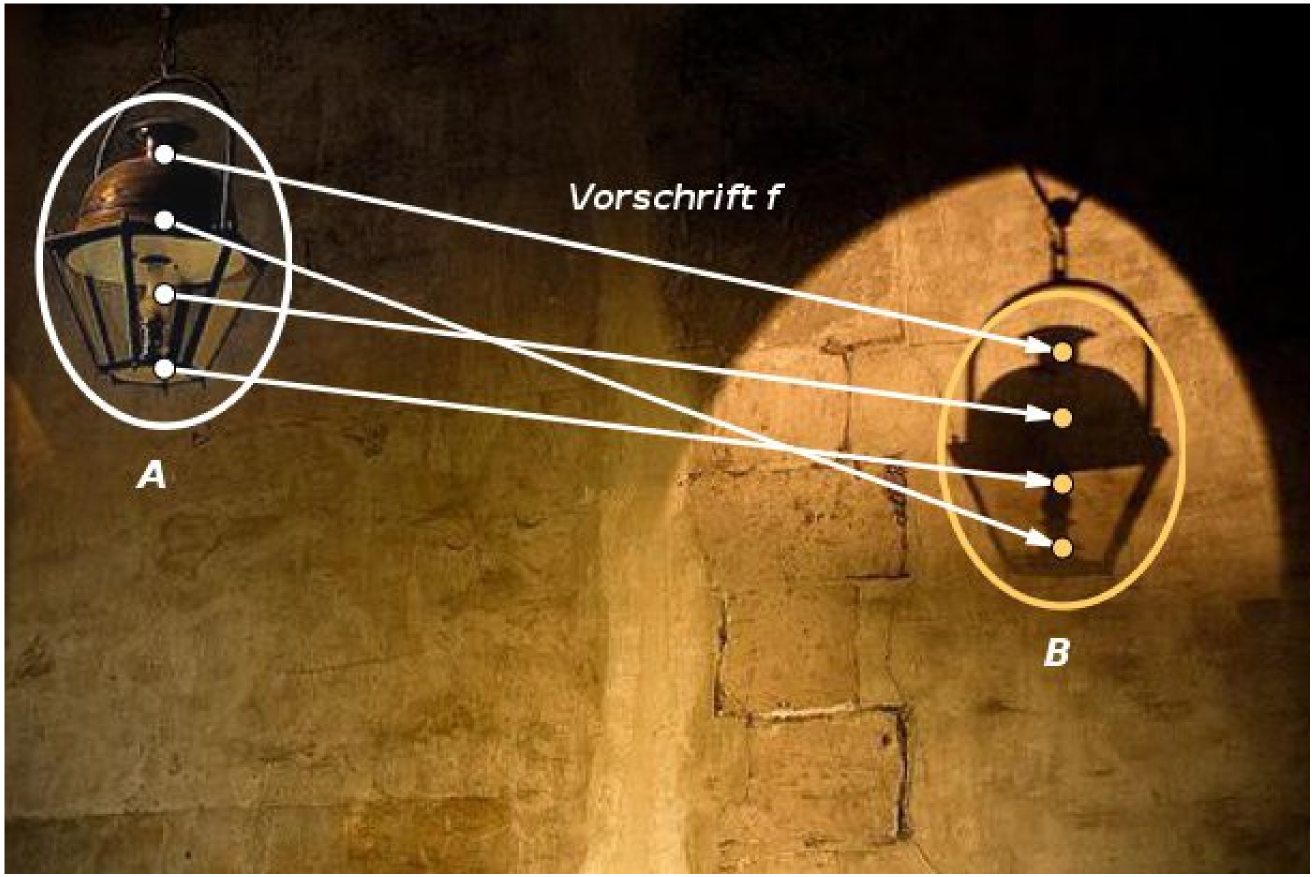
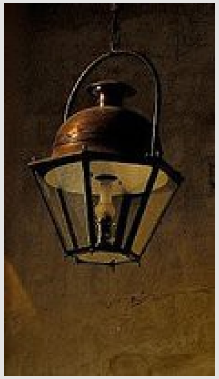


Fig. 1-1: Illustration of mapping

Very often, one wants to assign elements of one set to elements of another set in a well defined way. Such assignments are called maps or mapping. Mapping is a fundamental concept of mathematics.



Notation:

A map f from set A to set B involves a general instruction, which assigns each element a of A to one and only one element $f(a)$ of B.

The term "map" is very general and tells nothing on the type of objects which are assigned to each other. The term "function" is used more restrictively when the mapping refers to real or complex numbers. In this case we talk about real or complex functions.

	Argument x	**Funktionswert**
1.	$x = -2$	$f(-2) = 0$
2.	$x = -1$	$f(-1) = -1.5$
3.	$x = 0$	$f(0) = -2$
4.	$x = 1$	$f(1) = -1.5$
5.	$x = 2$	$f(2) = 0$
6.	$x = 3$	$f(3) = 2.5$
7.	$x = 4$	$f(4) = 6$

Table 1: x-values and corresponding function values $f(x) = 0.5\,x^2 - 2$

The assignments of a function can be presented as a table.

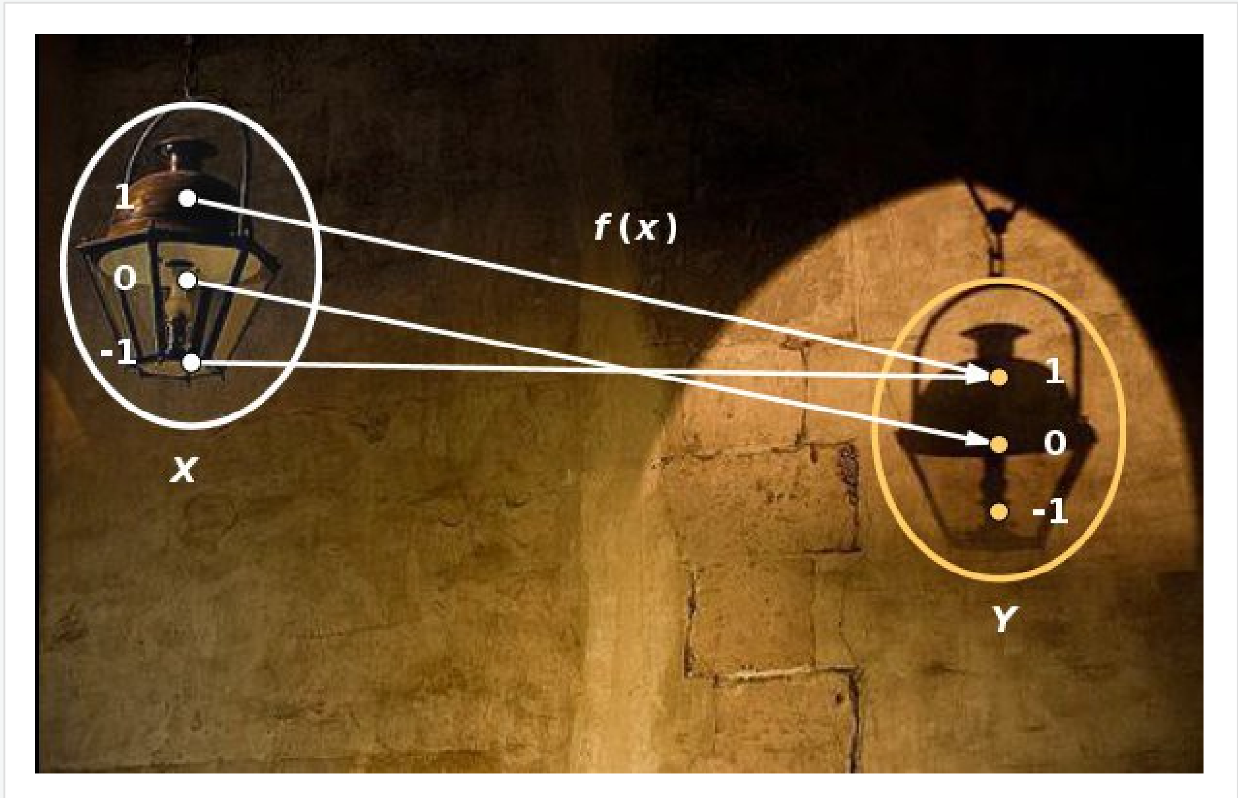


Fig. 1-2: Illustration of a map.

Mapping f from X to Y can symbolically be written as follows:

$$X \overset{f}{\longrightarrow} Y, \qquad x \rightarrow f(x)$$

The first expression shows, that f is some mapping from X to Y, the second specifies how x is assigned to the element $f(x)$.

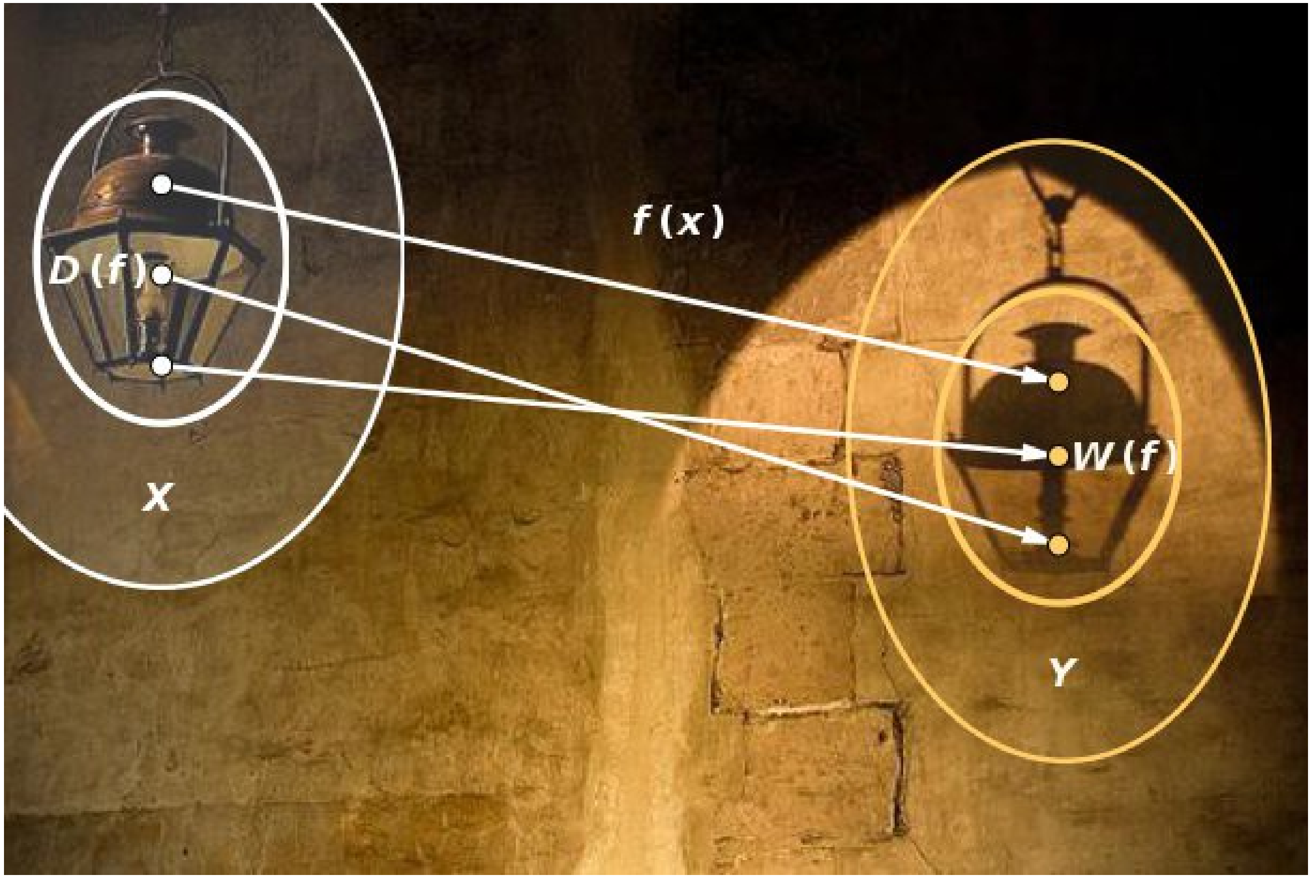
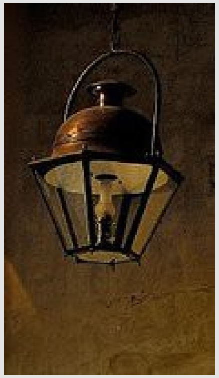


Fig.. 1-3: Illustration of a map f(x)

$D(f)$ – domain, $R(f)$ – range



Each x of $D(f)$ is assigned to only one element $y = f(x)$.

x – is an input value from the domain of f

y – is the output value $f(x)$ ("f of x").

f – is the definition of the mapping.

Several input values may be assigned to the same output y of $R(f)$.

Such maps are called functions.

If $X = R$ and $Y = R$, then f is real function.

Usually on writes shortly

$f(x) = x^2$, e.g., instead of $f : \mathbb{R} \rightarrow \mathbb{R}, \quad x \rightarrow x^2$

That is fine, but the domain D and the range R need to be defined.

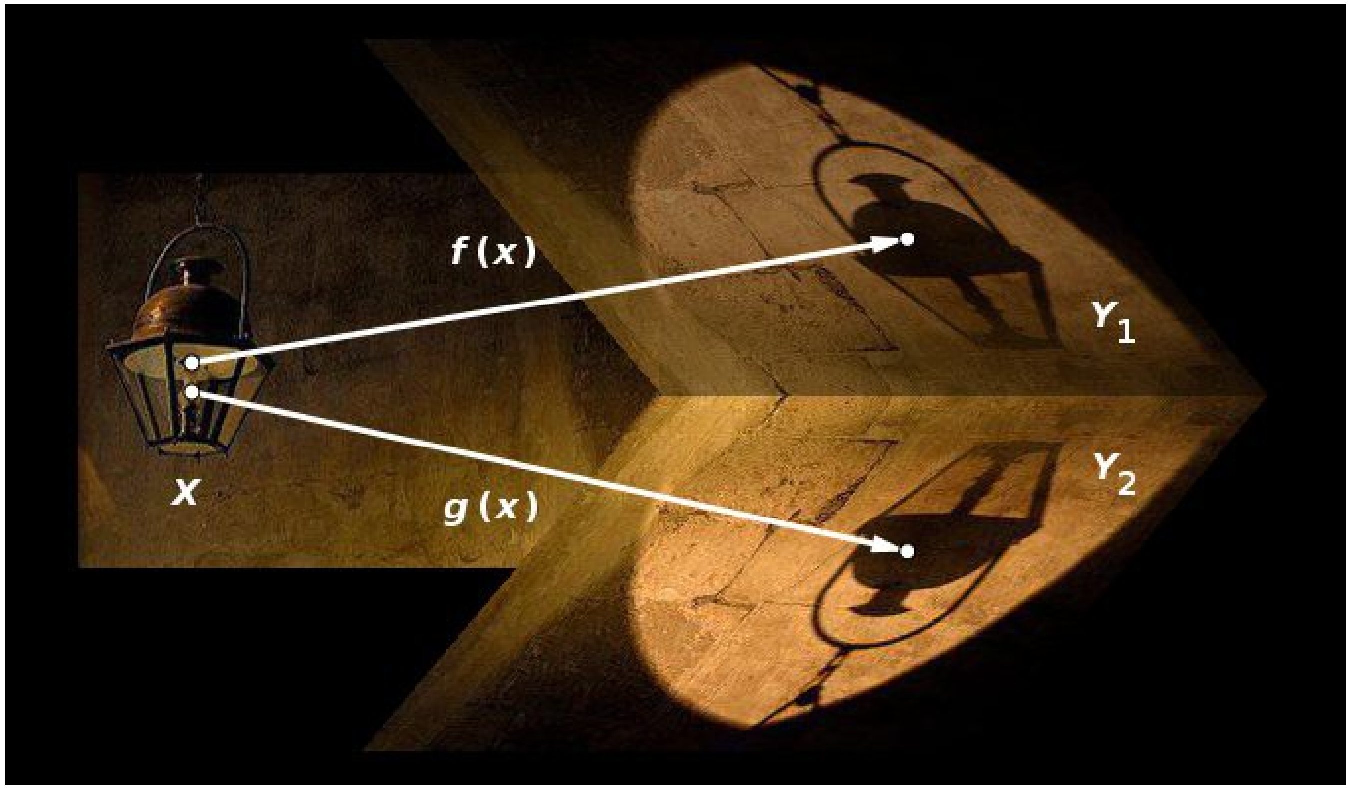
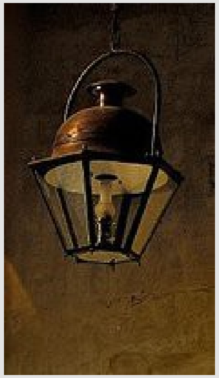


Fig. 1-4: Illustration of two mappings, f (x) and g(x)

Two different function definitions $f(x)$ and $g(x)$ map the set X into the two different sets Y_1 and Y_2.



Give examples for maps (as illustrated in Fig. 1-4)

a) $Y_1 \cap Y_2 = \{ \emptyset \}$

the two output sets (ranges) have no common element

b) $Y_1 \cap Y_2 = Y_3$

the intersection of the two output sets is not empty.

c) $Y_1 = Y_2$

a) $X = \{-1, 0, 1\}$, $f(x) = x^2 + 1$, $g(x) = -x^2$

$Y_1 = \{1, 2\}$, $Y_2 = \{-1, 0\}$, $Y_1 \cap Y_2 = \{\varnothing\}$

b) $X = \{-1, 0, 1\}$, $f(x) = x^2$, $g(x) = -x^2 + 2$

$Y_1 = \{0, 1\}$, $Y_2 = \{1, 2\}$, $Y_1 \cap Y_2 = \{1\}$

c) $X = \{-1, 0, 1\}$, $f(x) = x^2$, $g(x) = -x^2 + 1$

$Y_1 = \{0, 1\}$, $Y_2 = \{0, 1\}$, $Y_1 = Y_2$