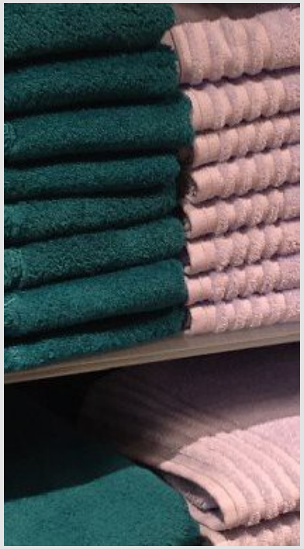
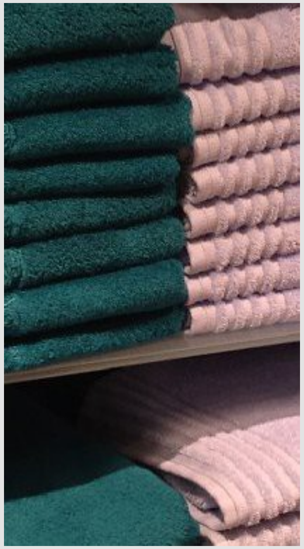
Classification of Functions



Definition:

A function $y = f(x)$ is called rational, if and only if a finite number of basic arithmetic operations (addition, subtraction, multiplication and division) are applied to the independent variable x. Another common definition is, that the function can be written as the ratio of two polynomials.

Other functions are not rational or irrational. “Irrational” is not as generally used for functions as it is for numbers.



Rational functions are:

- the linear functions

  $y = a\,x + b \quad (x\,,\ a\,,\ b \in \mathbb{R})$

- the quadratic functions

  $y = a\,x^2 + b\,x + c \quad (x\,,\ a\,,\ b\,,\ c \in \mathbb{R}\,,\quad a \neq 0)$

- $y = x^n\,, \quad y = \dfrac{1}{x^n} \quad (n \in \mathbb{N})$

- $y = \dfrac{x}{x^2 + 1}\,, \quad y = -\dfrac{5\,x}{x^3 - 7}$

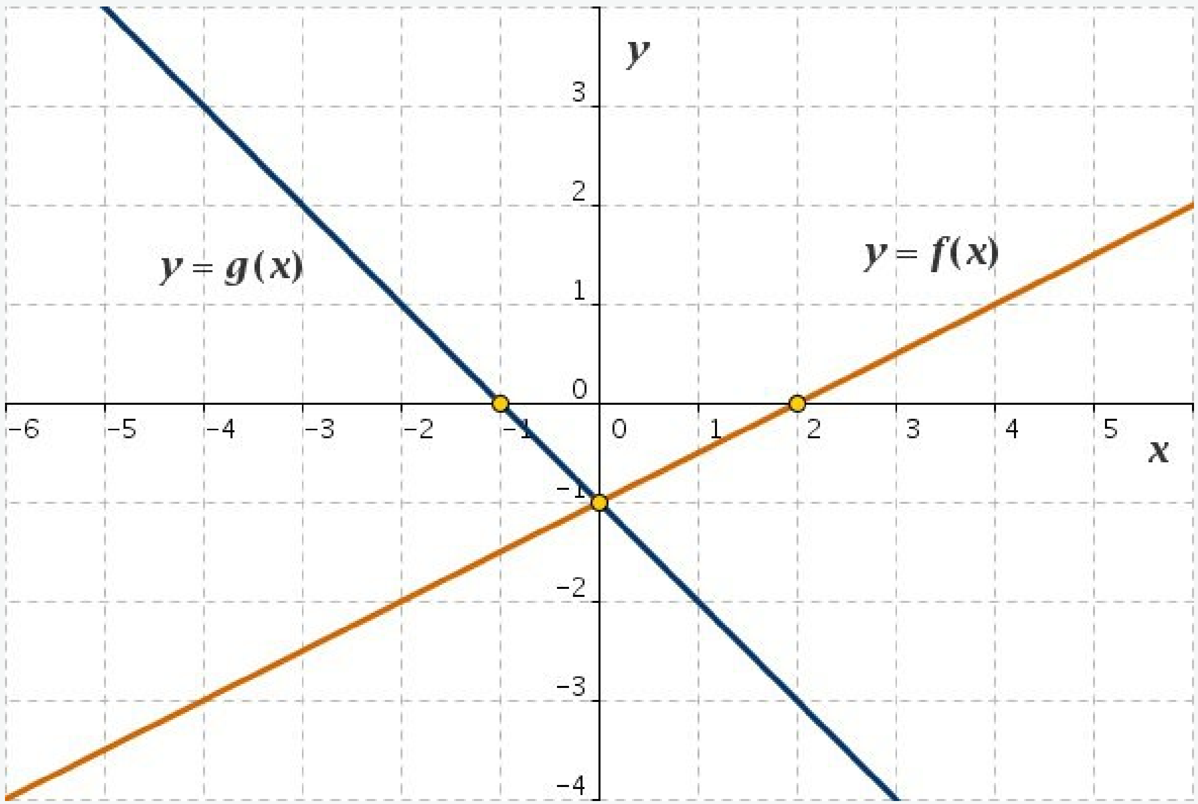


Fig. 1: Linear functions $y = f(x)$ (red) and $y = g(x)$ (blue)

$y = a\,x + b \quad (a\,,\ b \in \mathbb{R})$ – general form

$y = f(x): \quad f(x) = \dfrac{x}{2} - 1, \quad a = \dfrac{1}{2}, \quad b = -1$

$y = g(x): \quad g(x) = -x - 1, \quad a = -1, \quad b = -1$

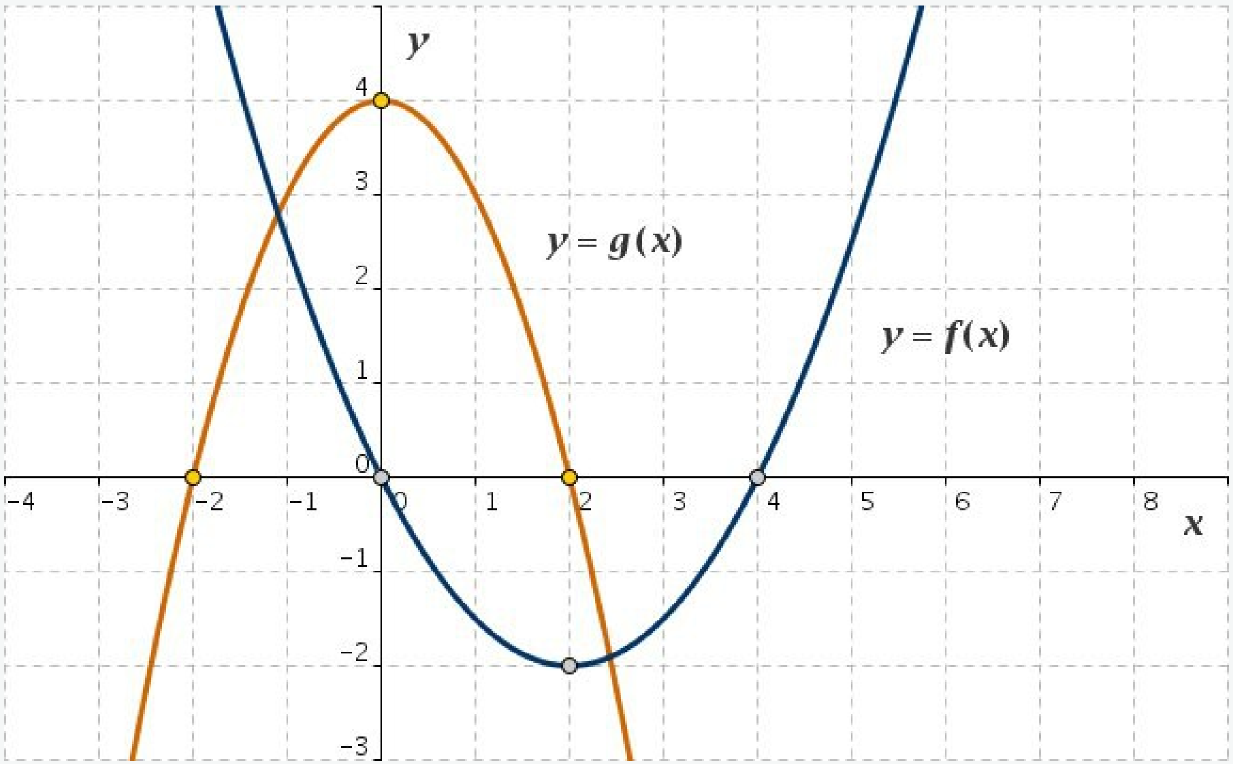


Fig. 2: Quadratic functions y = f (x) (blue) and y = g (x) (red)

$$y = a\,x^2 + b\;x + c \quad (x\,,\,a\,,\,b\,,\,c \in \mathbb{R}\,, \quad a \neq 0)$$

$$y = f\,(x):\quad f\,(x) = \frac{x^2}{2} - 2\,x\,,\qquad a = \frac{1}{2}\,,\quad b = -2,\quad c = 0$$

$$y = g\,(x):\quad f\,(x) = -\,x^2 + 4,\qquad a = -1,\quad b = 0,\quad c = 4$$

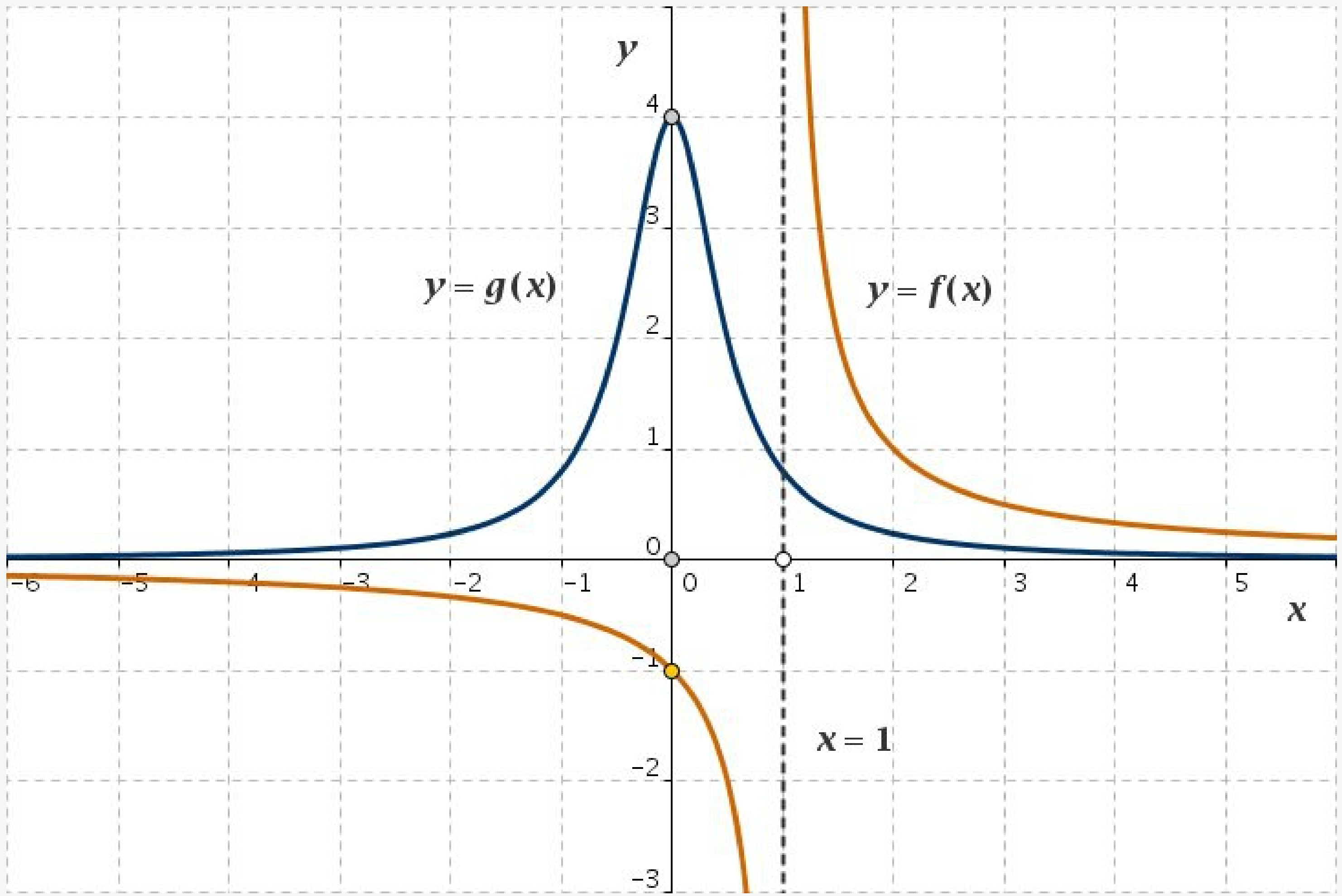
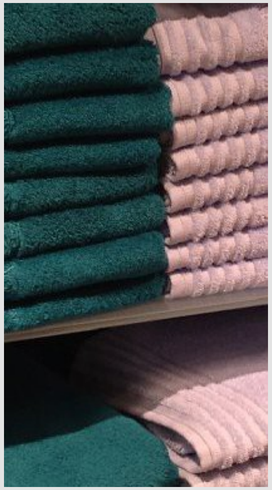


Fig. 3: Rational functions $y = f(x)$ (red) and $y = g(x)$ (blue)

$$f(x) = \frac{1}{x - 1}, \qquad g(x) = \frac{4}{4x^2 + 1}$$



Definition:

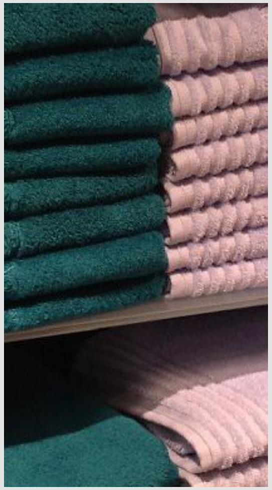
Functions of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \qquad (x \in \mathbb{R})$$

are polynomial functions or polynomials. The coefficients $a_0, a_1, \ldots, a_n$ are real numbers. The degree of the polynomial is given by the highest exponent n of the function equation.

Only the operations addition, subtraction, multiplication and division are applied to the independent variable x

$y = a_0$	constant function
$y = a_1 x + a_0$	lineare function
$y = a_2 x^2 + a_1 x + a_0$	quadratic function
$y = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	cubic function



Irrational functions are:

- $y = x^n \quad (n \notin \mathbb{Z})$

- the trigonometric functions and their inverse functions, e.g.

  $y = \sin x, \quad y = \tan x, \quad y = \arcsin x$

- the exponential functions and their inverse functions, e.g.

  $y = e^x, \quad y = \ln x$

- all composite functions which contain an irregular piece, e.g.

  $y = x \cdot e^x + 2, \quad y = \ln x - 2x^3 + 3$

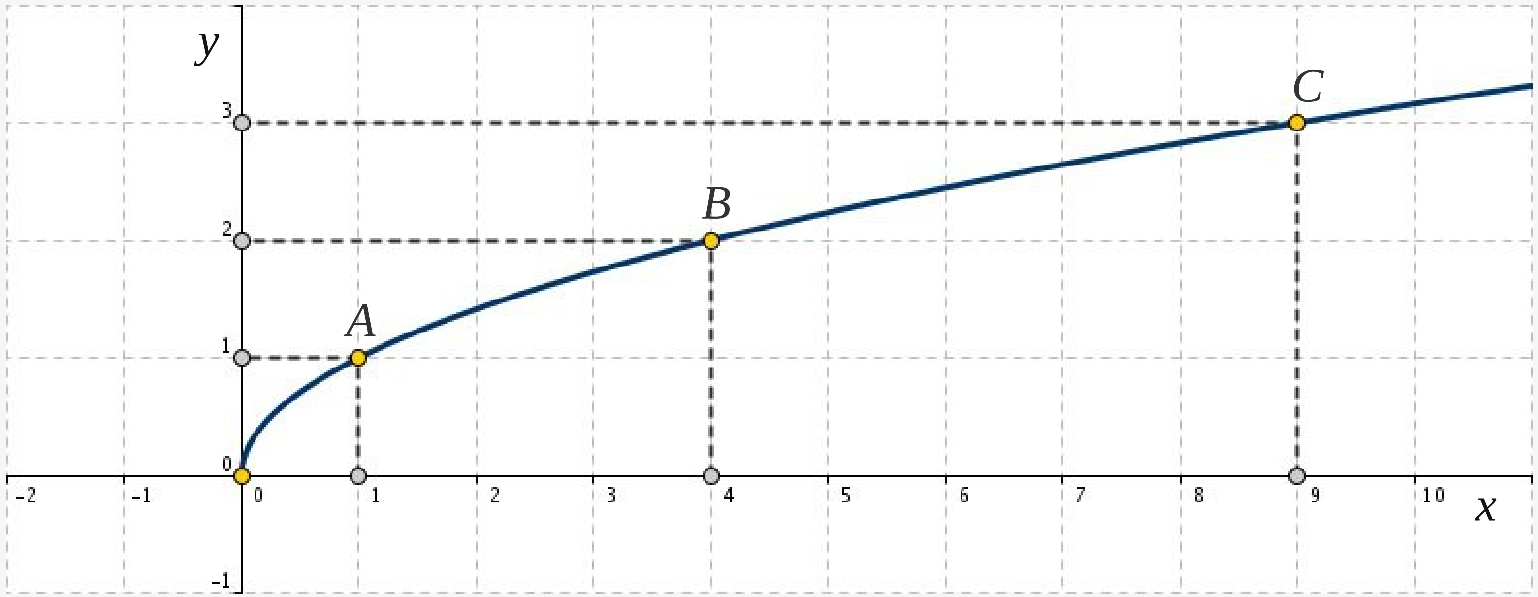


Fig 4: Root function $f(x) = \sqrt{x}$

$$y = \sqrt{x}$$

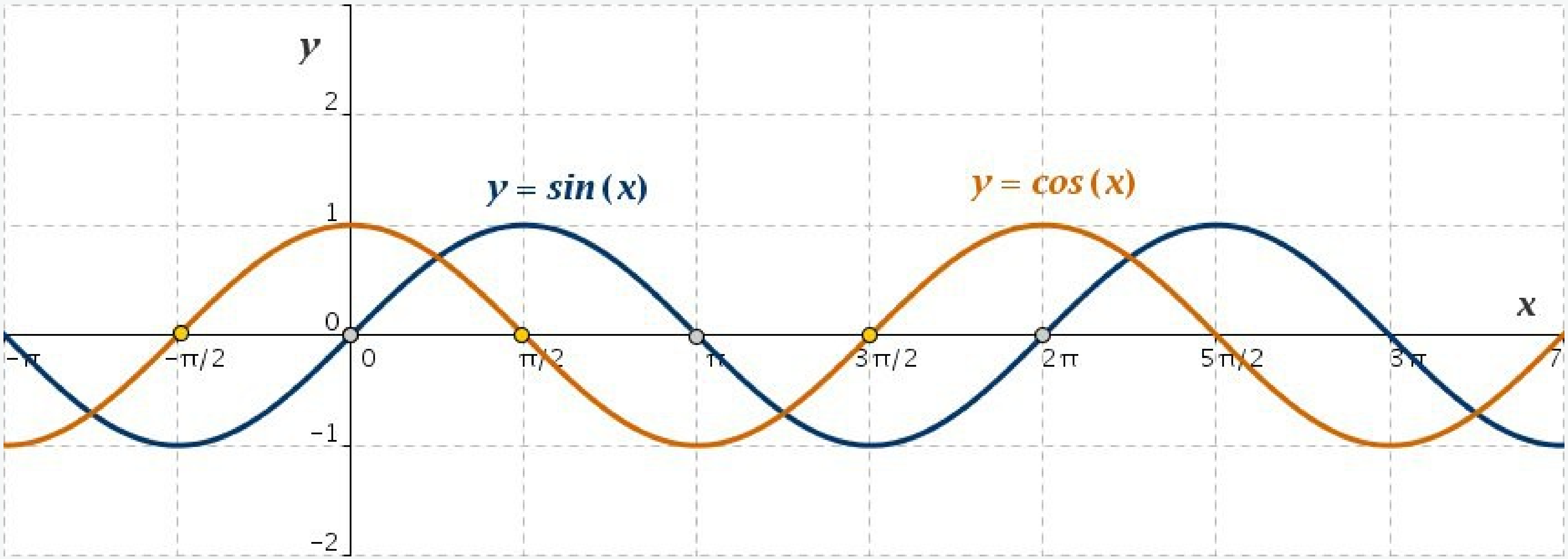


Fig. 5: Trigonometric functions $y = \sin x$ and $y = \cos x$

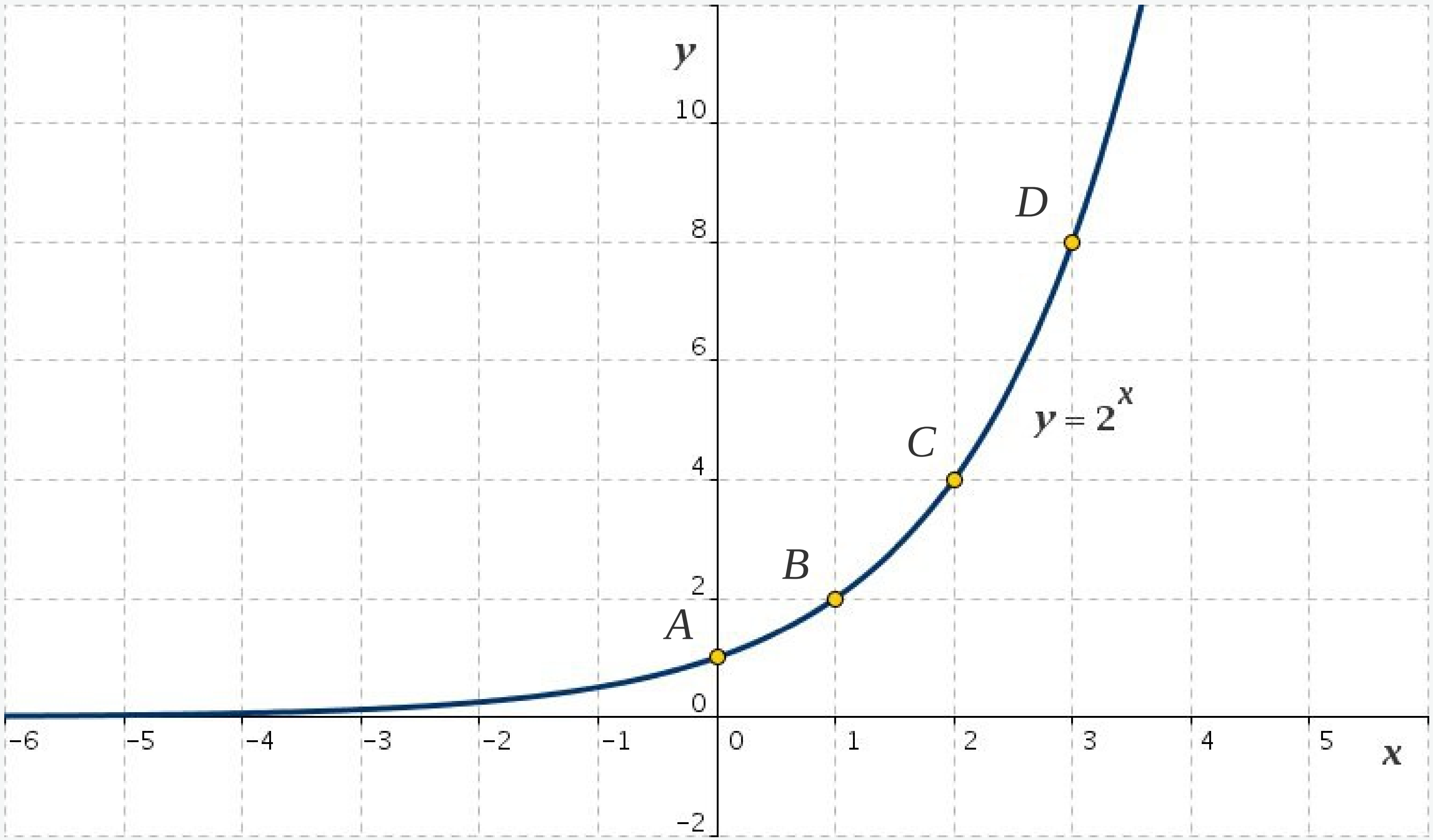


Fig. 6: exponential function $y = f(x)$

$$A = (0,\ 2^0) = (0,\ 1), \qquad B = (1,\ 2^1) = (1,\ 2)$$

$$C = (2,\ 2^2) = (2,\ 4), \qquad D = (3,\ 2^3) = (3,\ 8)$$

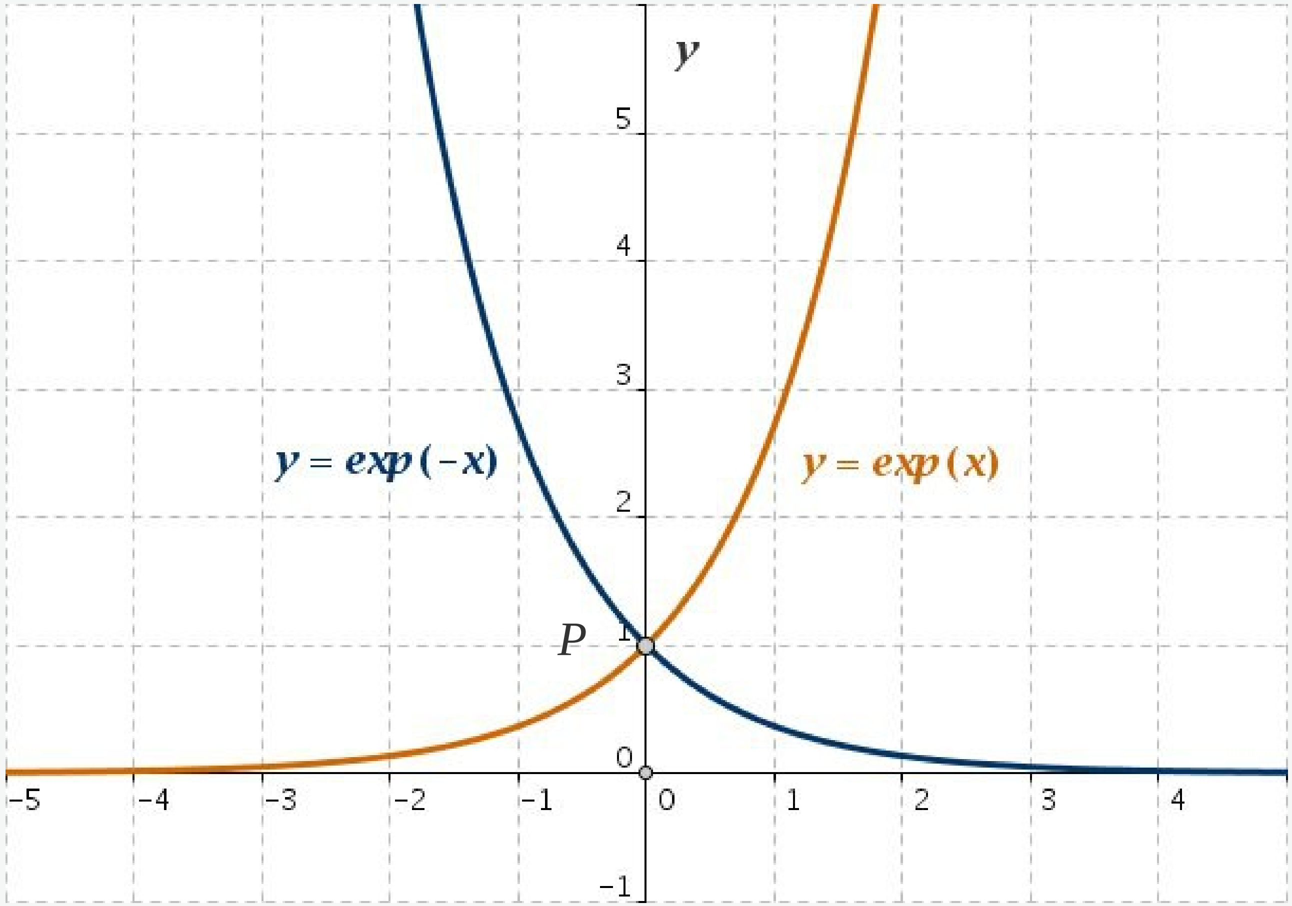


Fig. 7: Exponential functions y = exp (x) (ret) and y = exp (-x) (blue)

The point P is common to the functions $y = exp\,(x)$ and $y = exp\,(-x)$.