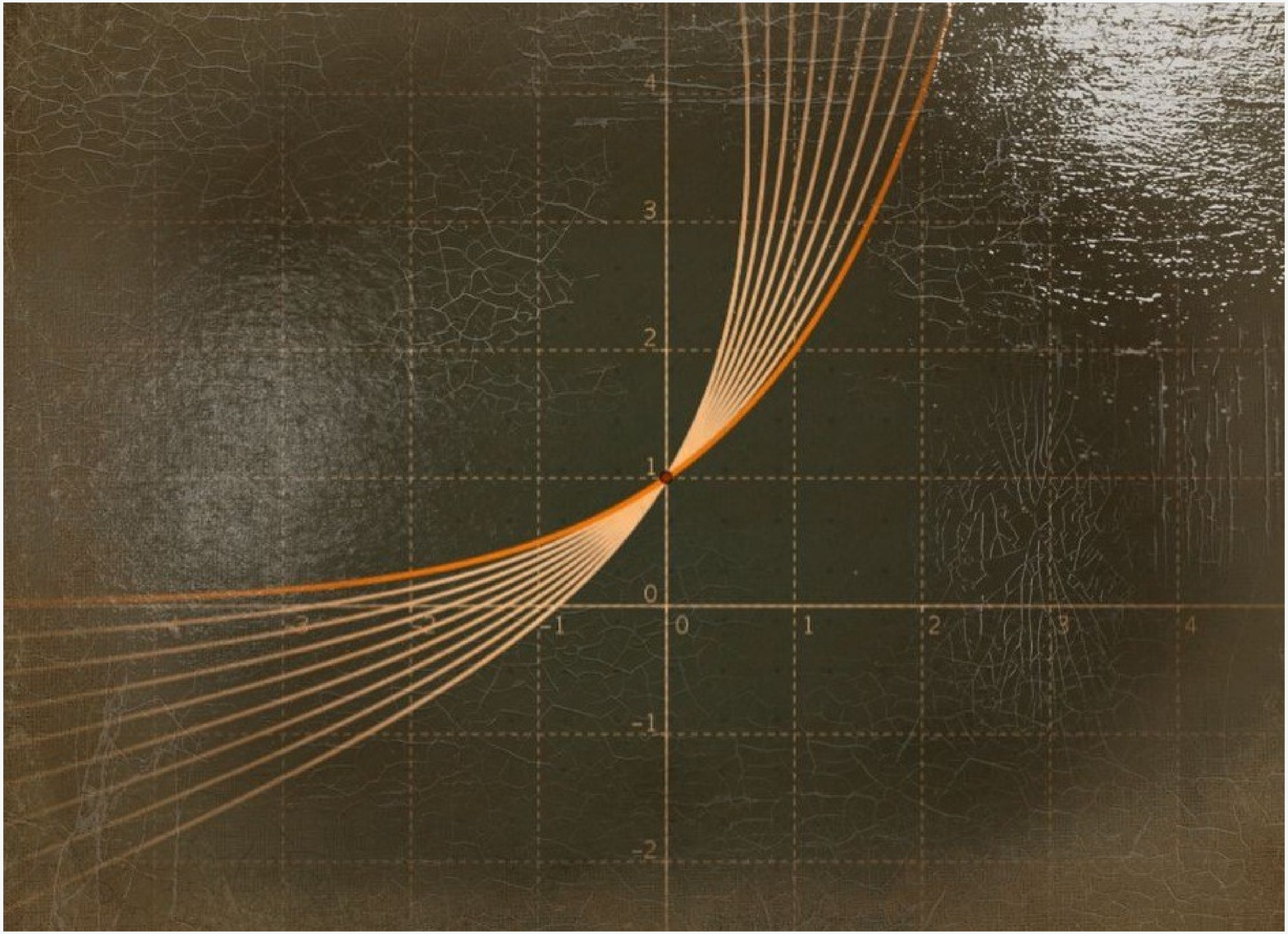

Monotonicity

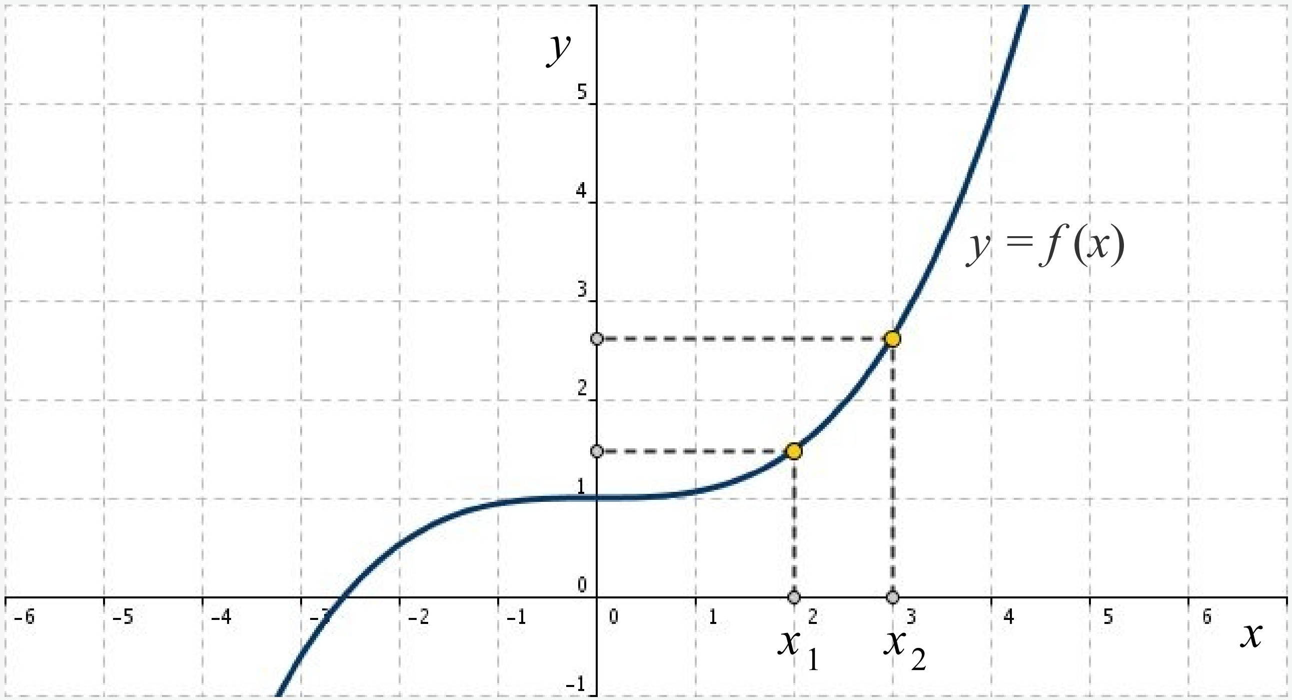

Fig. 1: Example of a monotonically increasing function $f(x) = 0.06\, x^3 + 1$

Definition:

A function $y = f(x)$ is monotonically increasing in an interval I of the domain D, if

$$f(x_1) \leqslant f(x_2)$$

for all $x_1,\ x_2 \in I \subset D, \quad x_1 < x_2$

It is strictly monotonically increasing, if $f(x_1) < f(x_2)$.

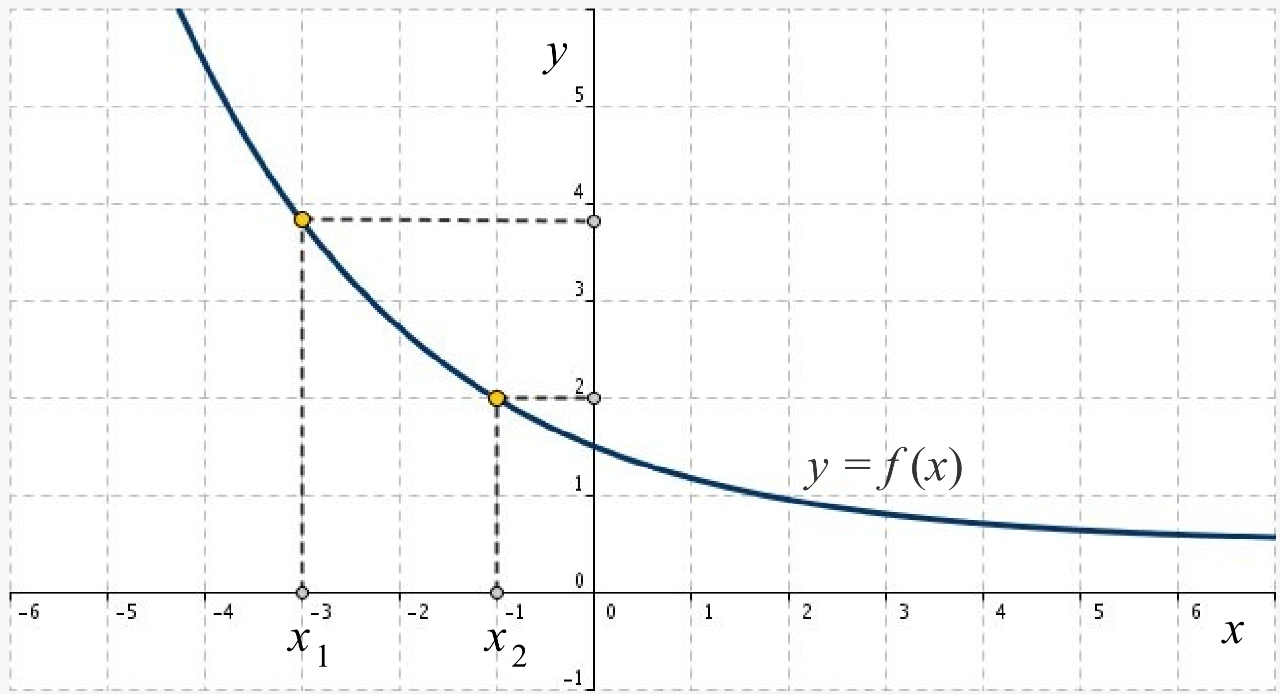

Fig. 2: Example of a monotonically decreasing function $f(x) = \exp(-0.4\,x) + 0.5$

Definition:

A function $y = f(x)$ is monotonically decreasing in an interval I of the domain D, if

$$f(x_1) \geqslant f(x_2)$$

for all $x_1,\ x_2 \in I \subset D,\quad x_1 < x_2$

It is strictly monotonically decreasing, if $f(x_1) > f(x_2)$.

Are the following functions monotonic in the given intervals I?

a) $f(x) = -x^2 + 4x - 2, \quad I = \mathbb{R}$

b) $f(x) = \sin x, \quad I = [0, 2\pi]$

c) $f(x) = \sqrt{x+2}, \quad I = \mathbb{R}$

$g(x) = -\frac{3}{5}\sqrt{x+1}, \quad I = \mathbb{R}$

d) $f(x) = e^x, \quad g(x) = e^x + 1, \quad I = \mathbb{R}$

$h(x) = e^{-x}, \quad j(x) = e^{-x} - 2, \quad I = \mathbb{R}$

e) $f(x) = \frac{1}{x}, \quad g(x) = -\frac{2}{x}, \quad I = \mathbb{R}$

f) $f(x) = -0.2\,x^3, \quad g(x) = \frac{1}{x^2}, \quad I = \mathbb{R}$

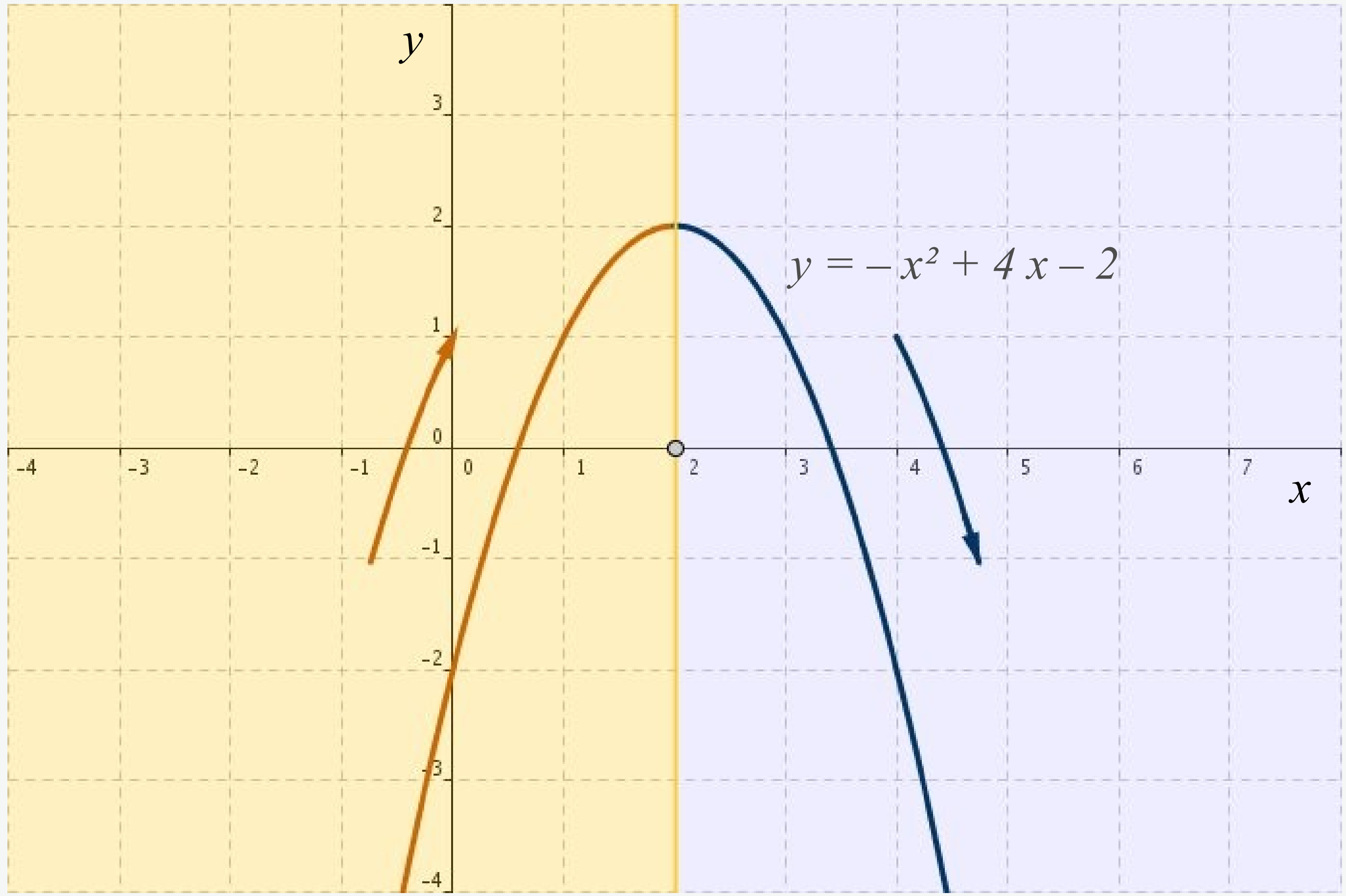

Fig. 3-1: Quadratic function $f(x) = -x^2 + 4x - 2$

The function $f(x)$ is monotonically increasing in the interval $I_1 = (-\infty, 2]$ and monotonically decreasing in the interval $I_2 = [2, \infty)$

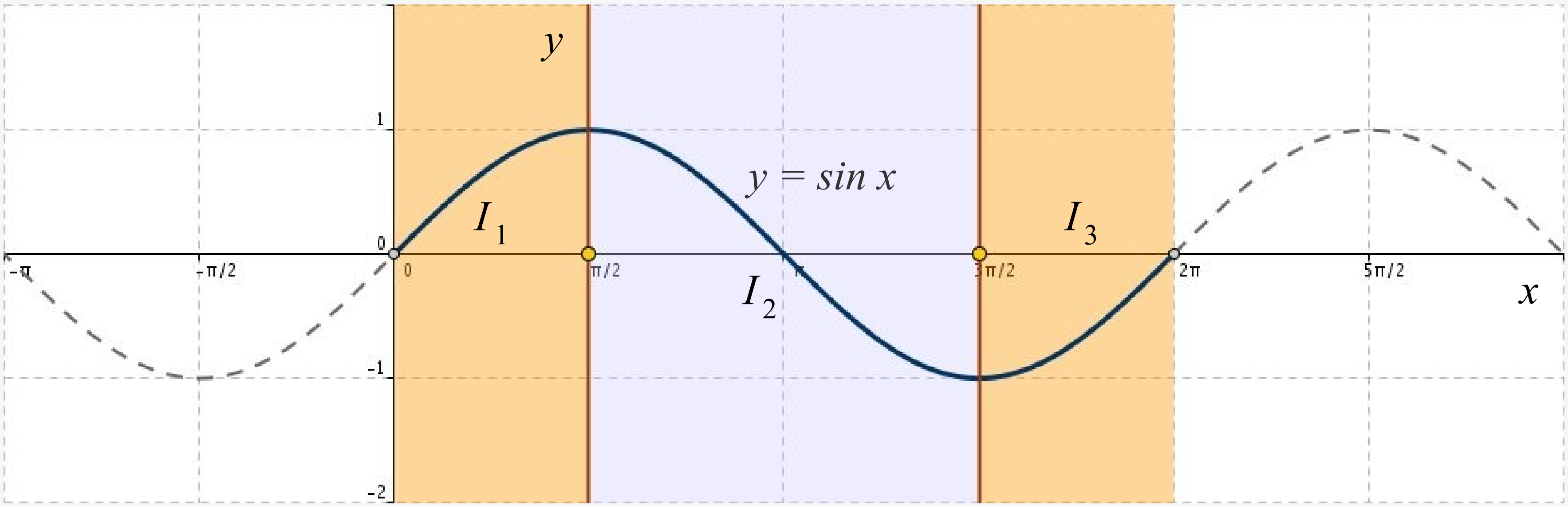

Fig. 3-2: Function $f(x) = \sin x$ in interval $[0, 2\pi]$

The function $f(x)$ is monotonically increasing in the intervals

$$I_1 = \left[0, \frac{\pi}{2}\right], \qquad I_3 = \left[\frac{3\pi}{2}, 2\pi\right]$$

and monotonically decreasing in the interval $I_2 = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

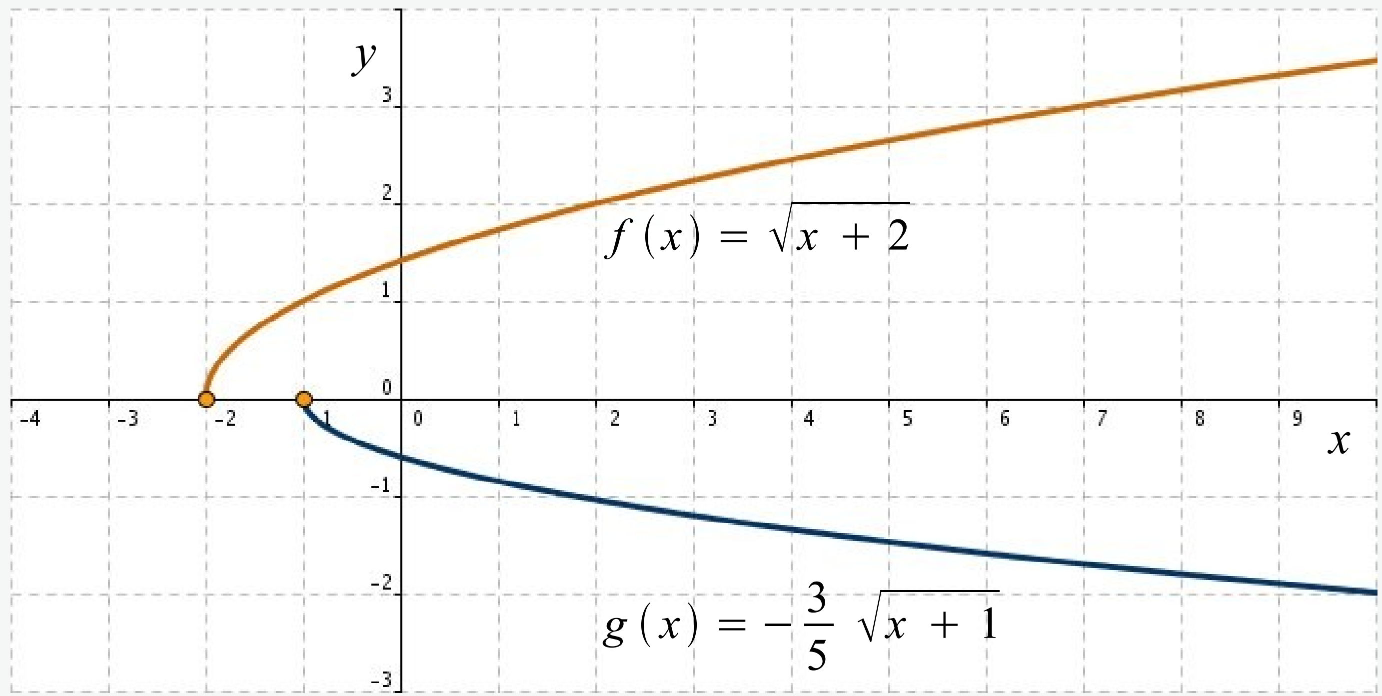

Fig. 3-3: The root functions of exercise 1c

The function $f(x)$ is monotonically increasing in the domain

$$D(f) = [-2, \infty)$$

The function $g(x)$ is monotonically decreasing in the domain

$$D(g) = [-1, \infty)$$

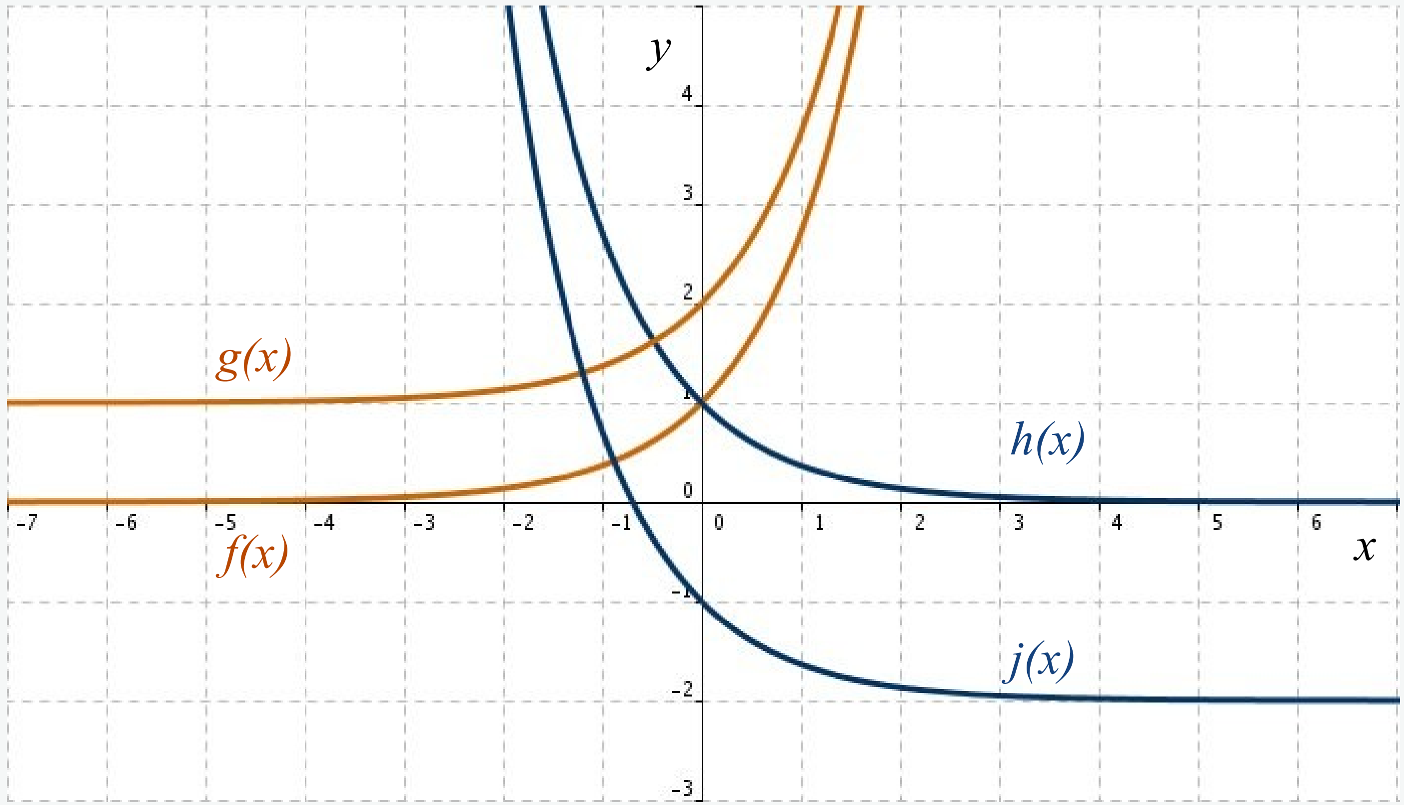

Fig. 3-4: Exponential functions of exercise 1d

$$f(x) = e^x, \quad g(x) = e^x + 1, \quad h(x) = e^{-x}, \quad j(x) = e^{-x} - 2$$

$$D(f) = D(g) = D(h) = D(j) = \mathbb{R}$$

Functions $f(x)$ and $g(x)$ increase monotonically in the full domain, functions $h(x)$ and $j(x)$ decrease monotonically in the full domain.

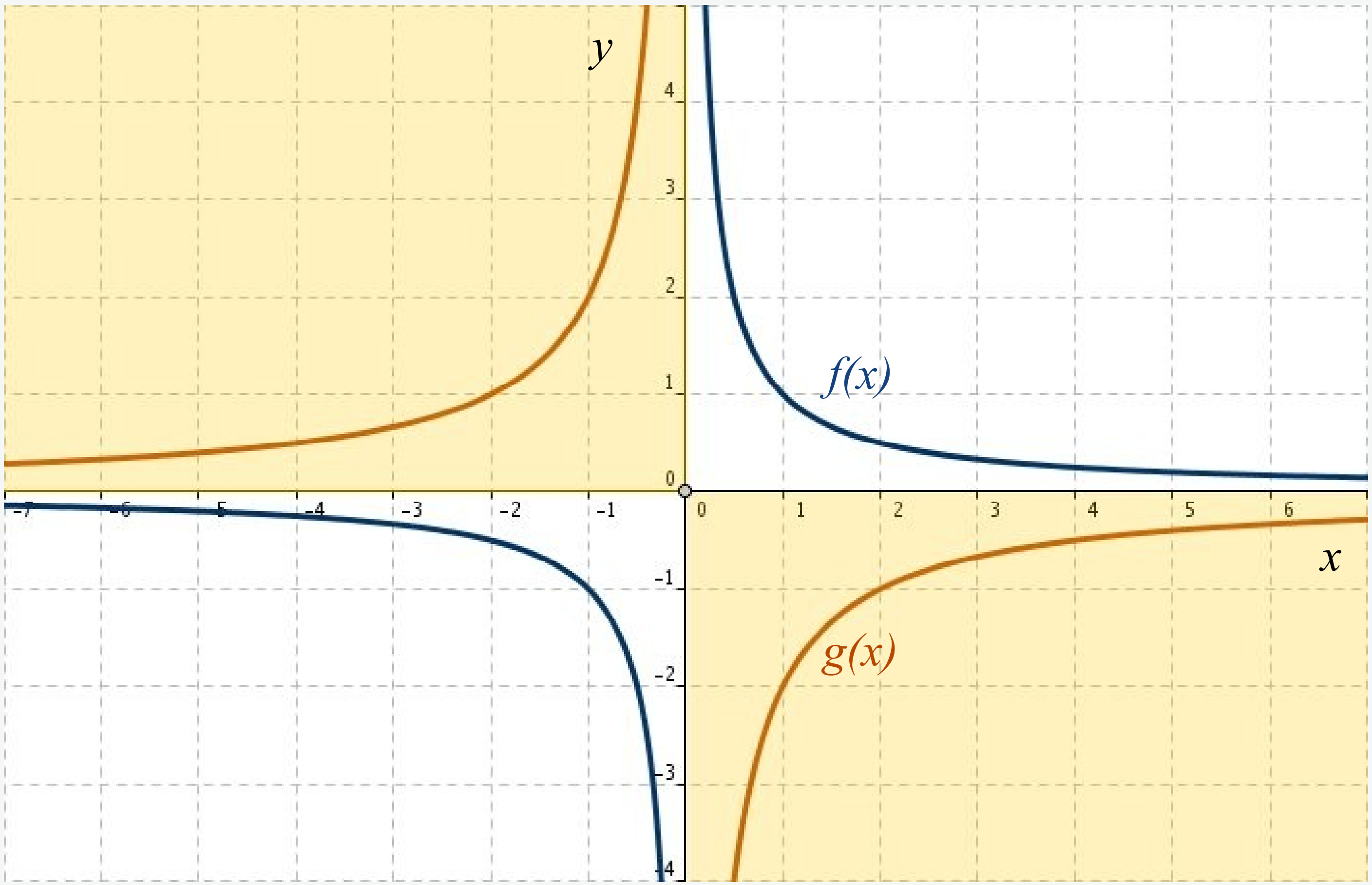

Fig. 3-5: Rational functions of exercise 1e

$$f(x) = \frac{1}{x}, \quad D(f) = \mathbb{R} \setminus \{0\}, \qquad g(x) = -\frac{2}{x}, \quad D(g) = \mathbb{R} \setminus \{0\}$$

Function $f(x)$ is monotonically decreasing in the full domain

Function $g(x)$ is monotonically increasing in the full domain

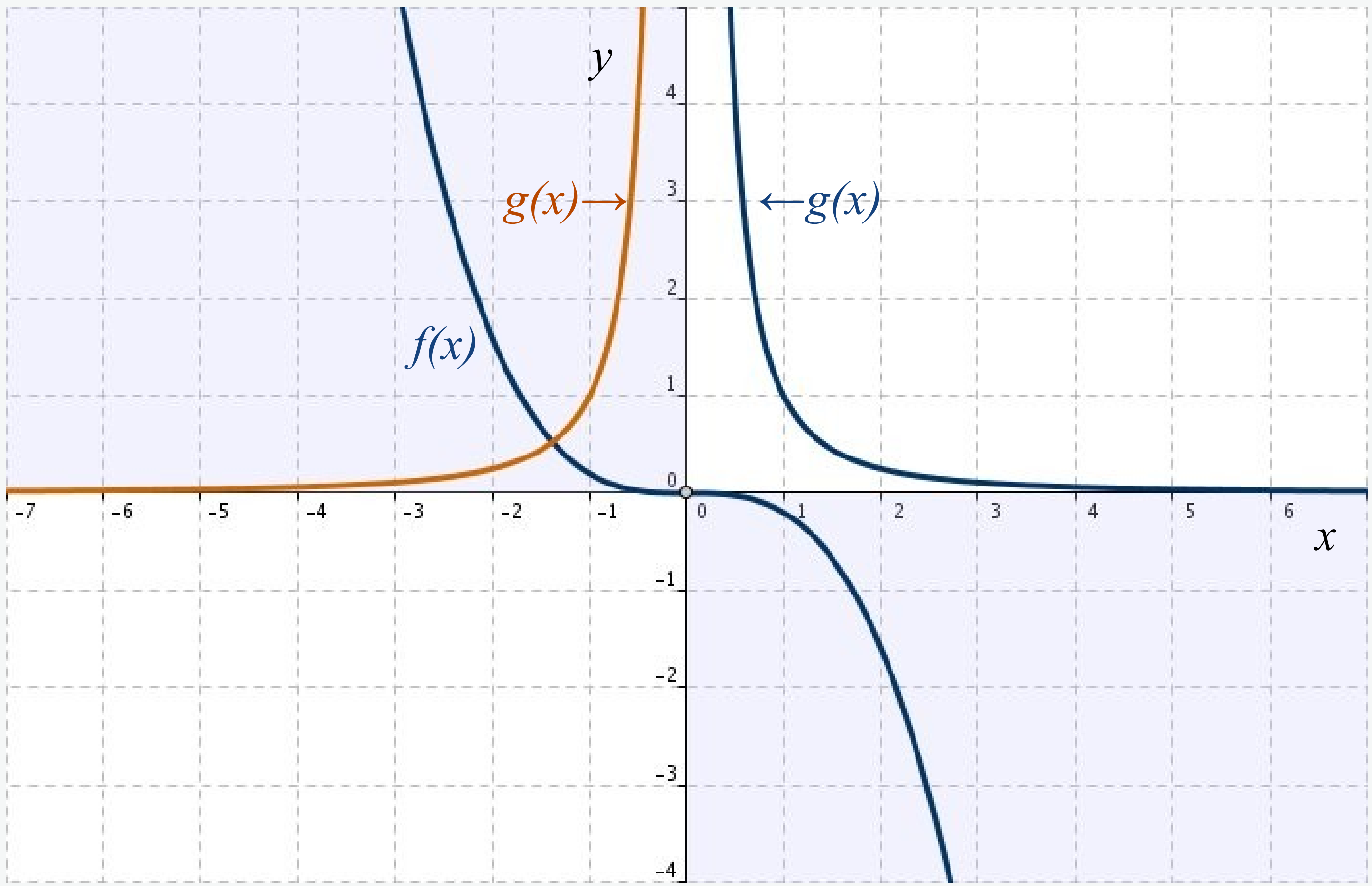

Fig. 3-5: Rational functions of exercise 1e

$$f(x) = -0.2\,x^3, \quad D(f) = \mathbb{R}, \qquad g(x) = \frac{1}{x^2}, \quad D(g) = \mathbb{R} \setminus \{0\}$$

The function *f (x)* is monotonically decreasing in the full domain, the function *g (x)* is monotonically increasing in the range of negative real numbers (red) and monotonically decreasing for positive real numbers (blue).

Some other Monotony

http://fsinfo.cs.uni-sb.de/~lynx/images/pfox.jpg

"My life is monotonous", the fox said. "I am chasing chickens, men are chasing me. All chickens are alike, and all men are alike. So I am bored a bit".

"The little prince" by Antoine de Saint-Exupéry