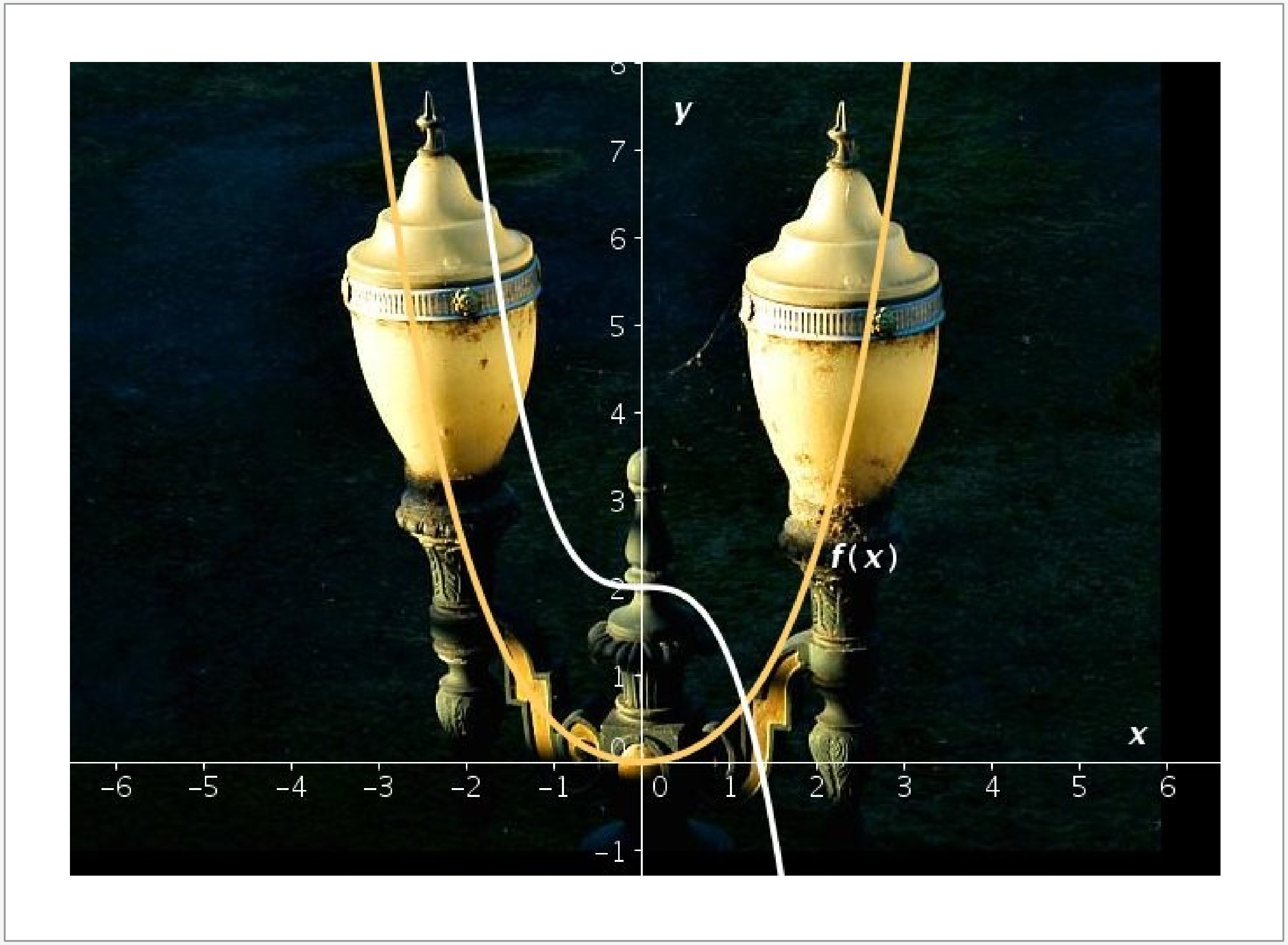

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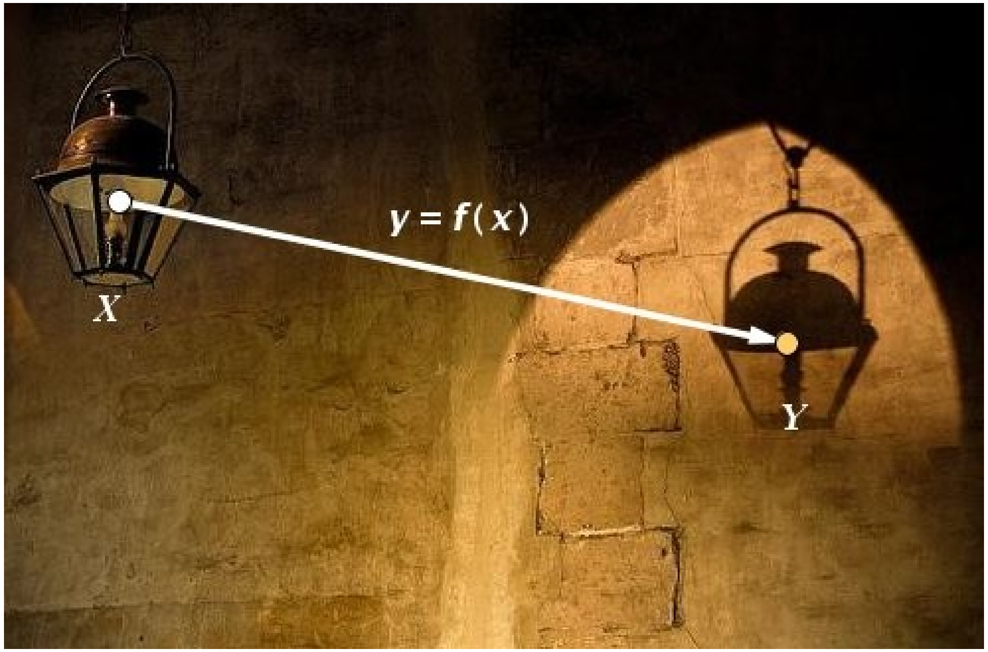
Inverse function



http://www.flickr.com/photos/sabriirmak/2193323529/

Fig. 1-1: Representation of a mapping

A function $f(x)$ describes a mapping of X to Y

$$X \overset{f}{\overbrace{\rightarrow}} Y, \qquad x \rightarrow f(x)$$

The first expression states that f maps X to Y, the second specifies which element of Y is assigned to x by $f(x)$ $(X = D(f),\ Y = R(f))$.

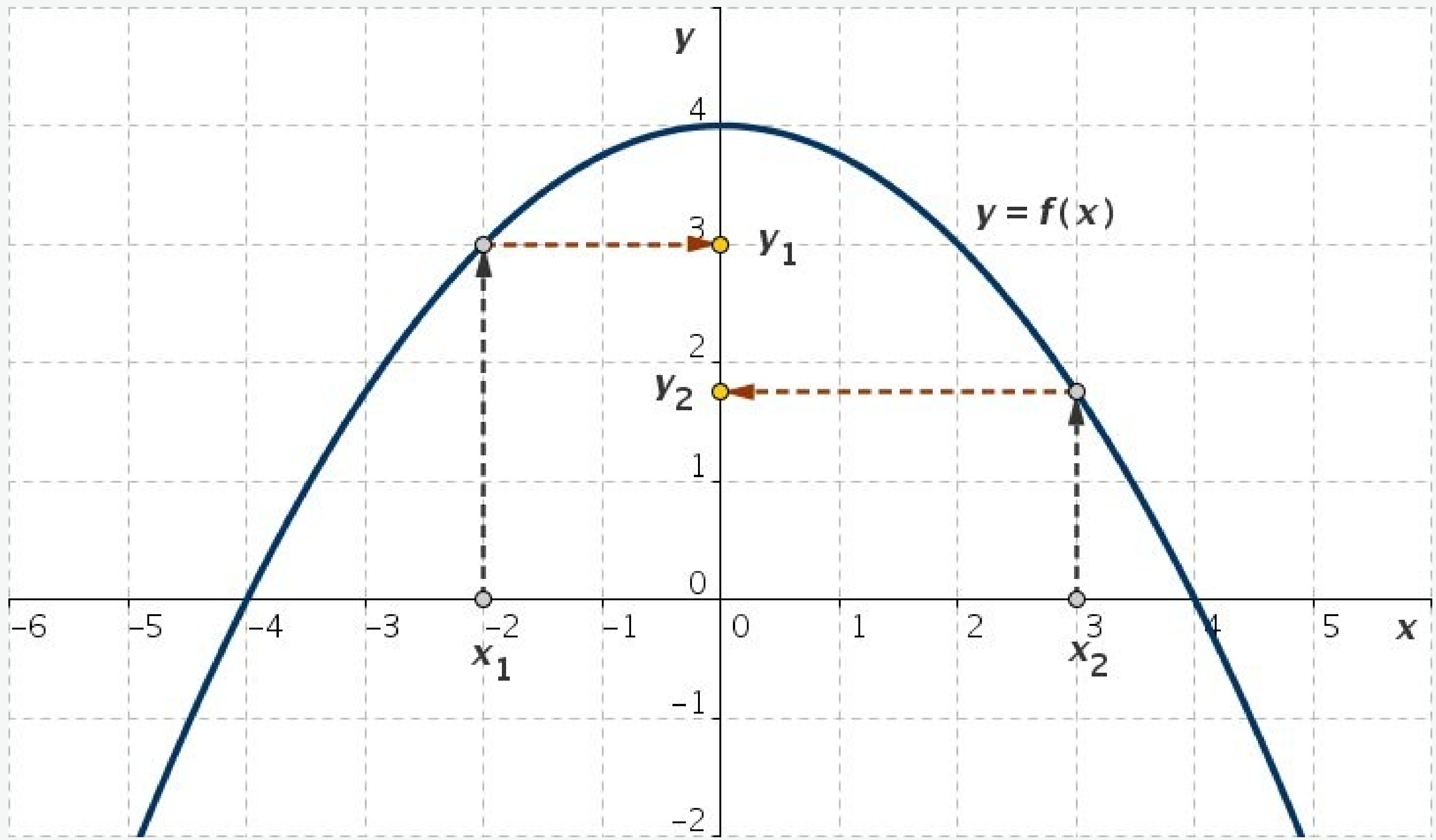


Fig. 1-2: The quadratic function $y = -0.25\,x^2 + 4$. The function assigns to each x a unique y.

The prescription given by the function assigns to each element x of X one and only one element $y = f(x)$ of Y. The function input x results in a unique function output y.

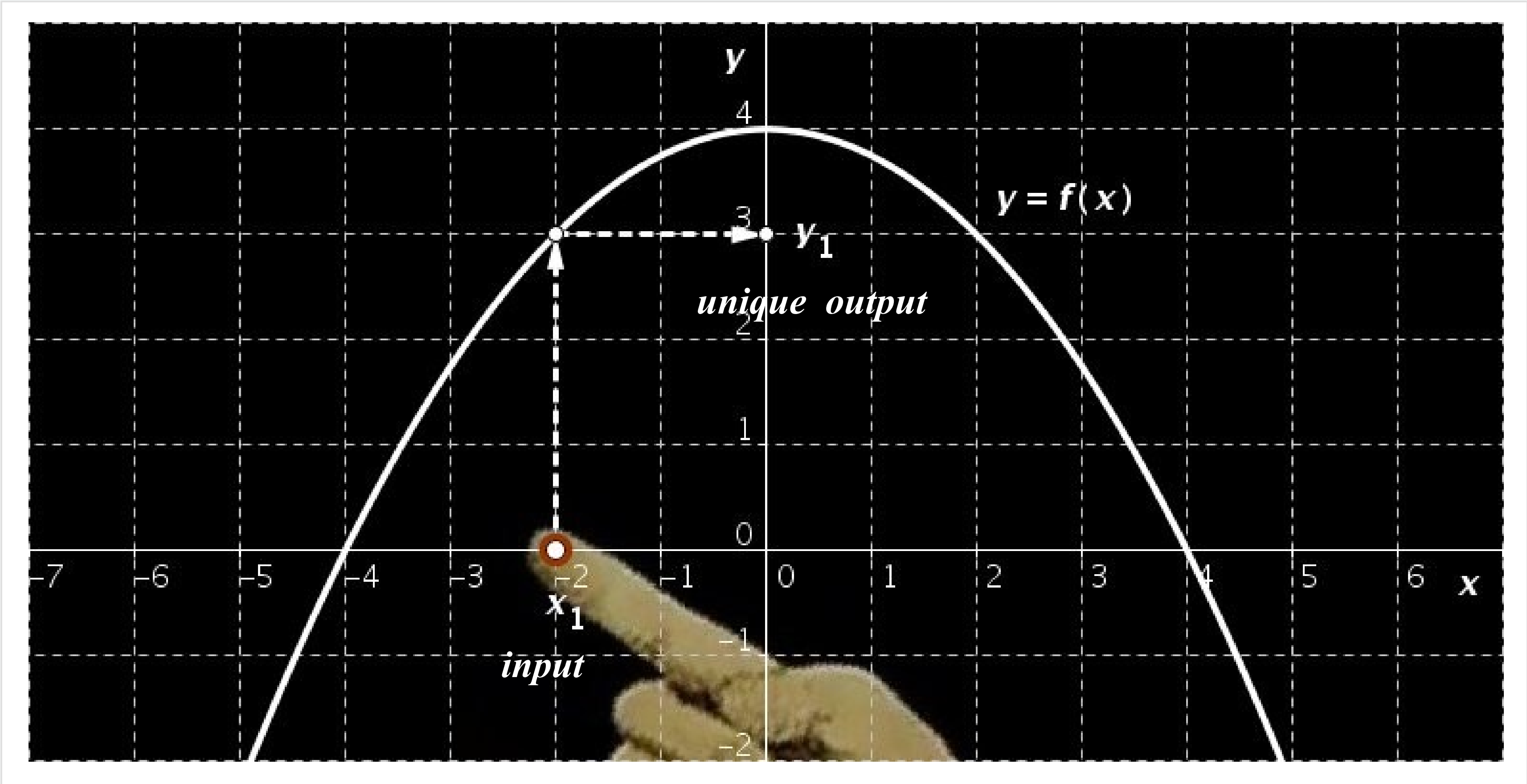


Fig. 1-3: The function assigns to each x a unique value y = f (x)

Any function answers the question:

"Which output is produced by this input?" or "Which y is assigned to that x?"

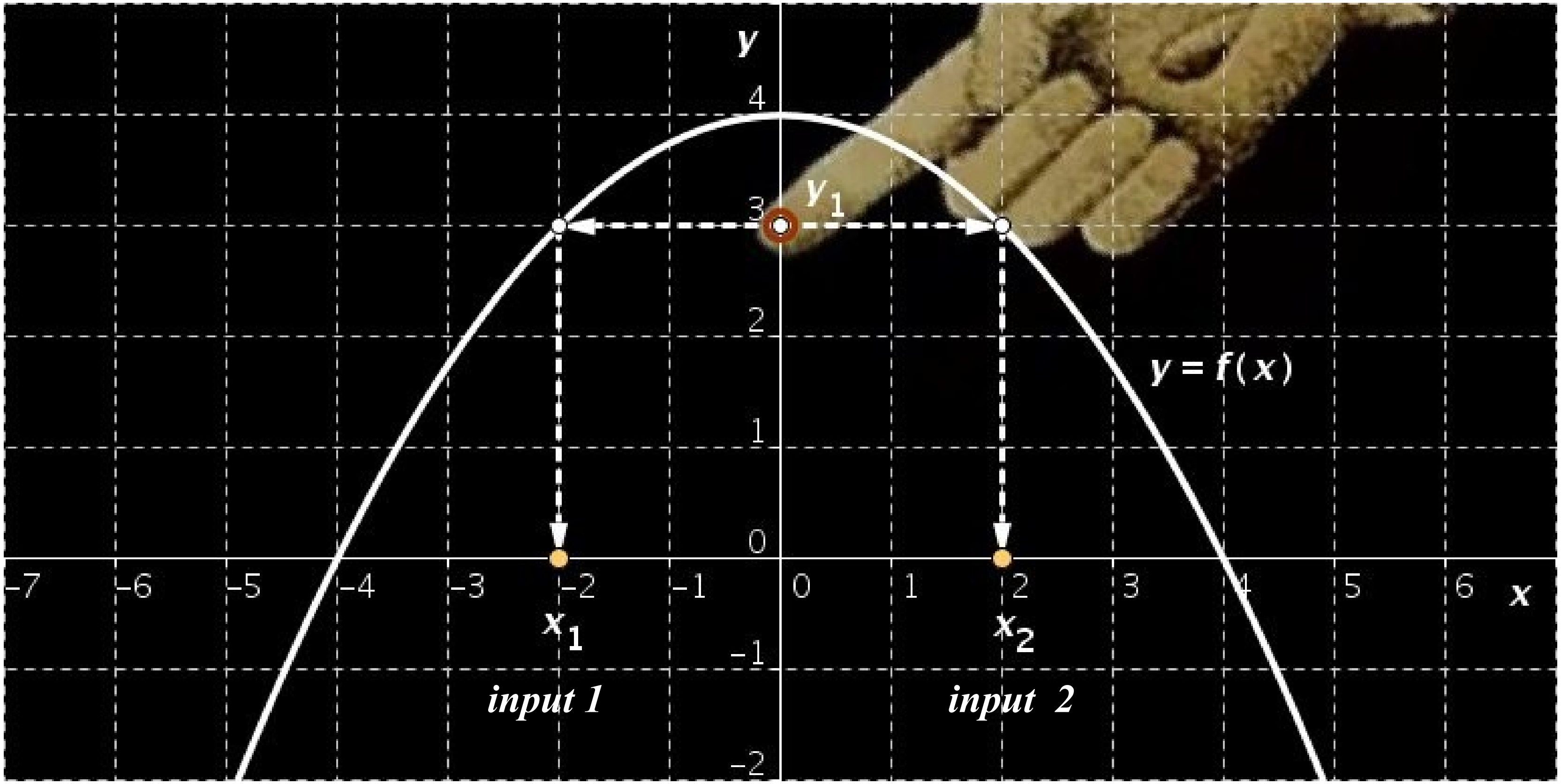


Fig. 1-4: The function output shown does not determine the input

Not each function answers a question like:
“Which input produced this output ?” or “Which x is this y assigned to ?”

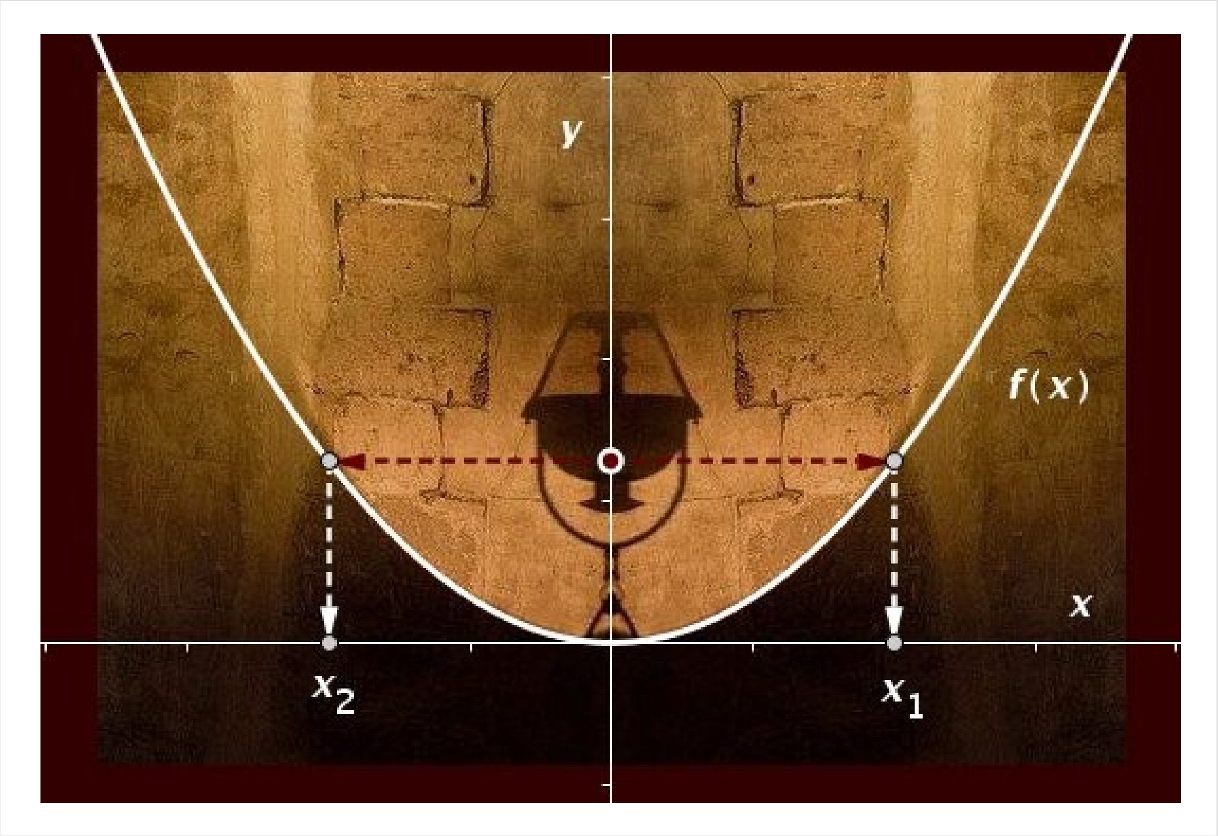


Fig. 1-5: Representation of a function $y = a\,x^2$ $(0 < a < 1)$. The question "which input generated this output?" can not be answered by this function.

For each element x of the domain holds:

$$f(-x) = f(x), \qquad f = a\,x^2, \qquad a \in \mathbb{R}$$

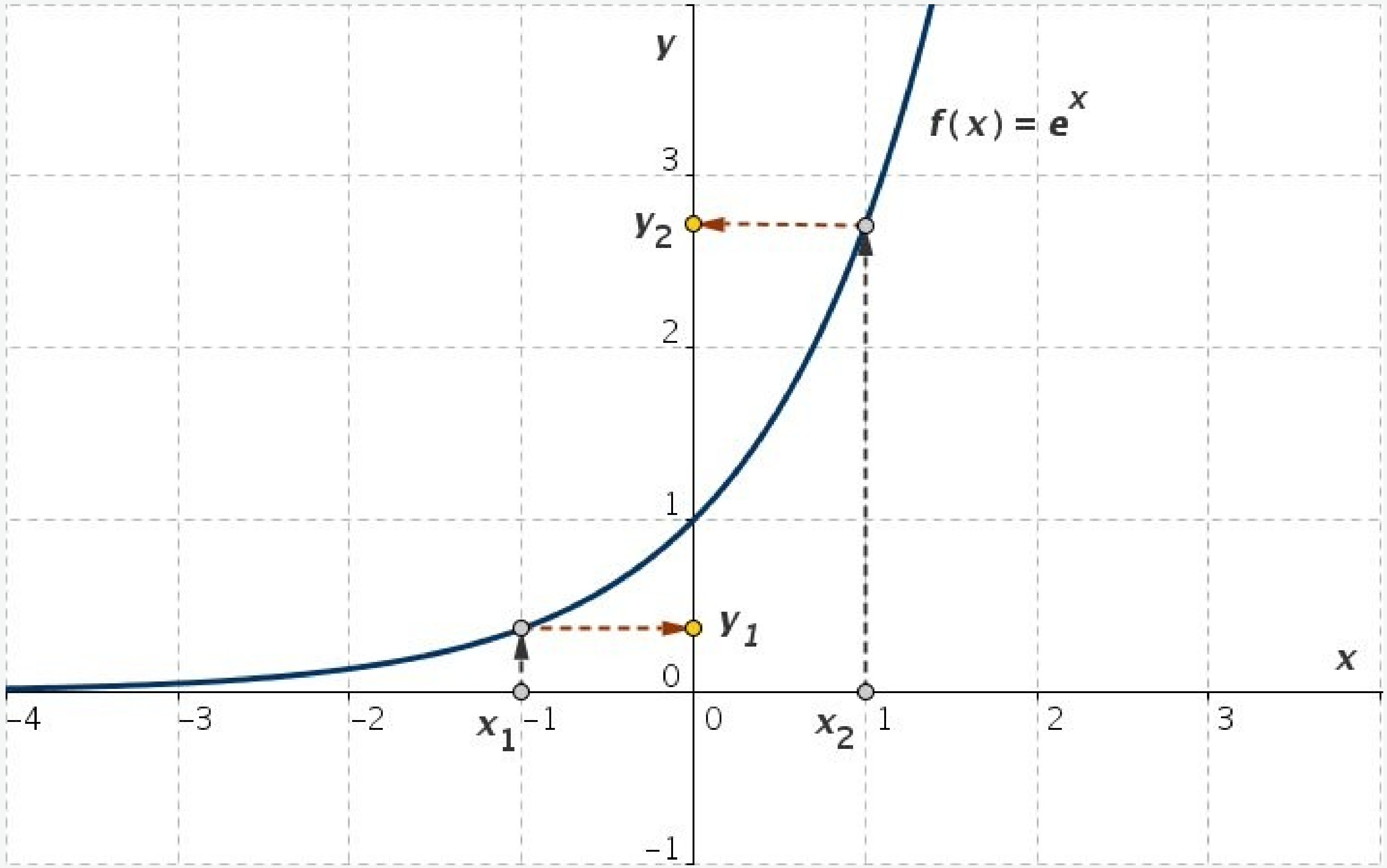


Fig. 1-6: Graph of the invertible function y = exp(x)

Definition 1: A function $y = f(x)$ is called invertible, if the following holds:

$$x_1 \neq x_2 \quad \Rightarrow \quad f(x_1) \neq f(x_2)$$

Definition 2:

The function $f: \ x \rightarrow y$ assigns to each x a unique value y. If a function exists which in turn assigns to y a unique value x, it is called the inverse function of f:

$$f^{-1}: \quad y \rightarrow x$$

A function f with this property is said to be one-to-one.

Remark to figure 1-7:

The input x is determined by the output y

- in a unique way in the case of f, function $y = f(x)$,
- not uniquely in case of g, function $y = g(x)$

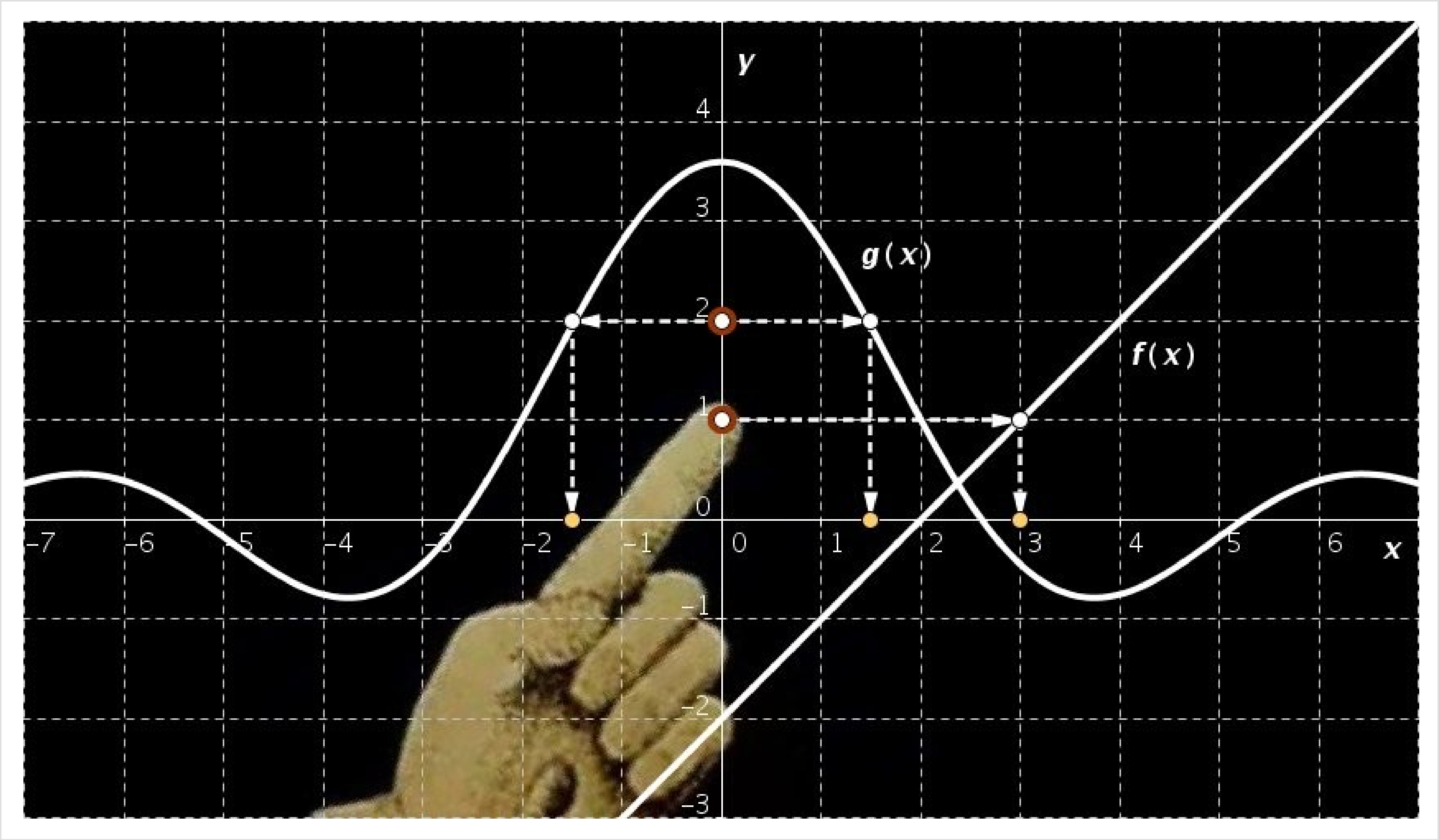


Fig. 1-7: The function y = x – 2 is invertible, the function y = 3 sin (1.2) /x is not

Inverse function: Exercise 1

A function is invertible, if and only if no line parallel to the *x*-axis intersects the function more than once.

Exercise 1:

Decide whether the functions shown in the following figures are invertible.

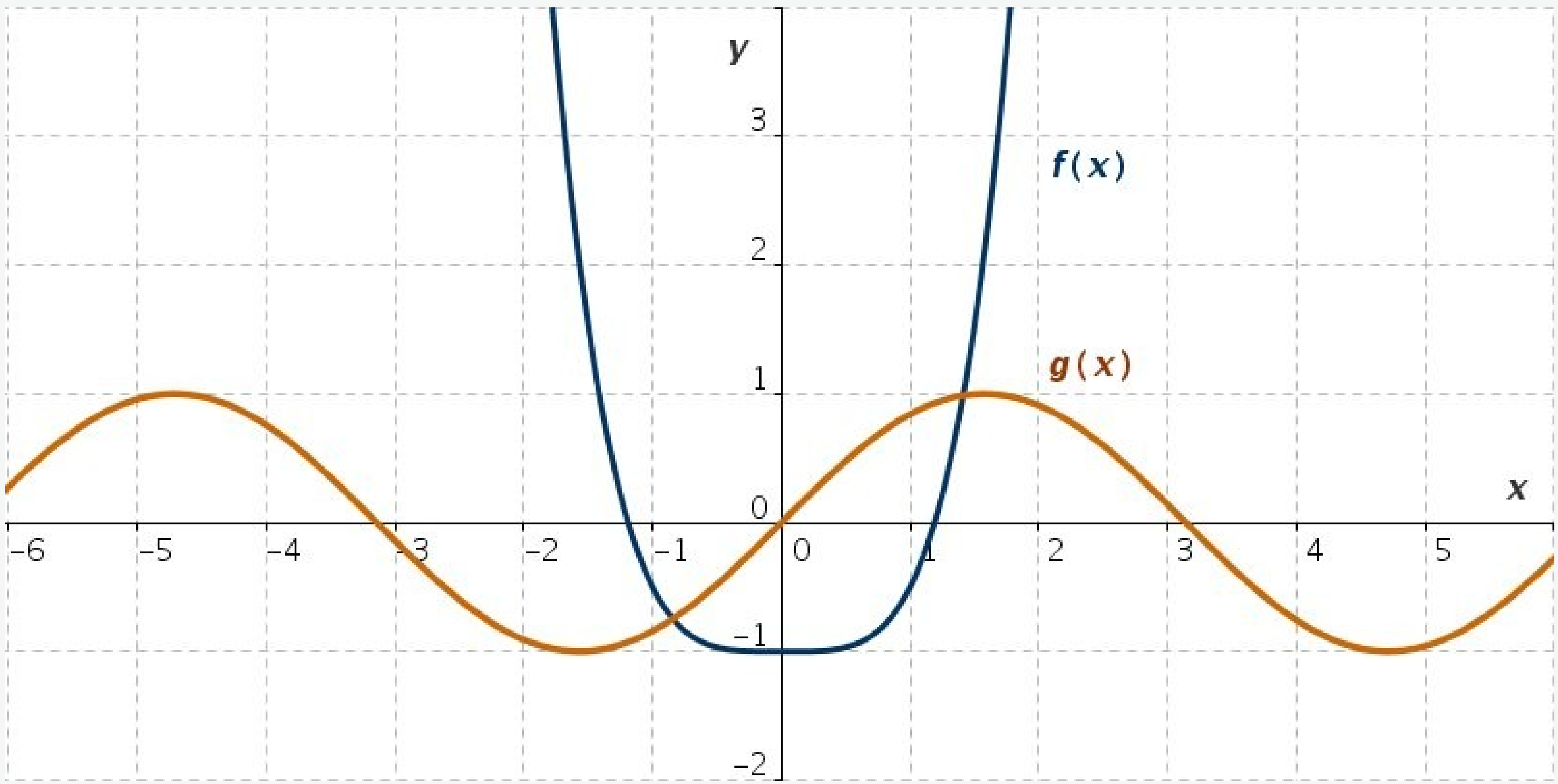


Fig. 3-1: Functions y = f (x) and y = g (x)

$$f(x) = \frac{x^4}{2} - 1, \qquad g(x) = \sin x$$

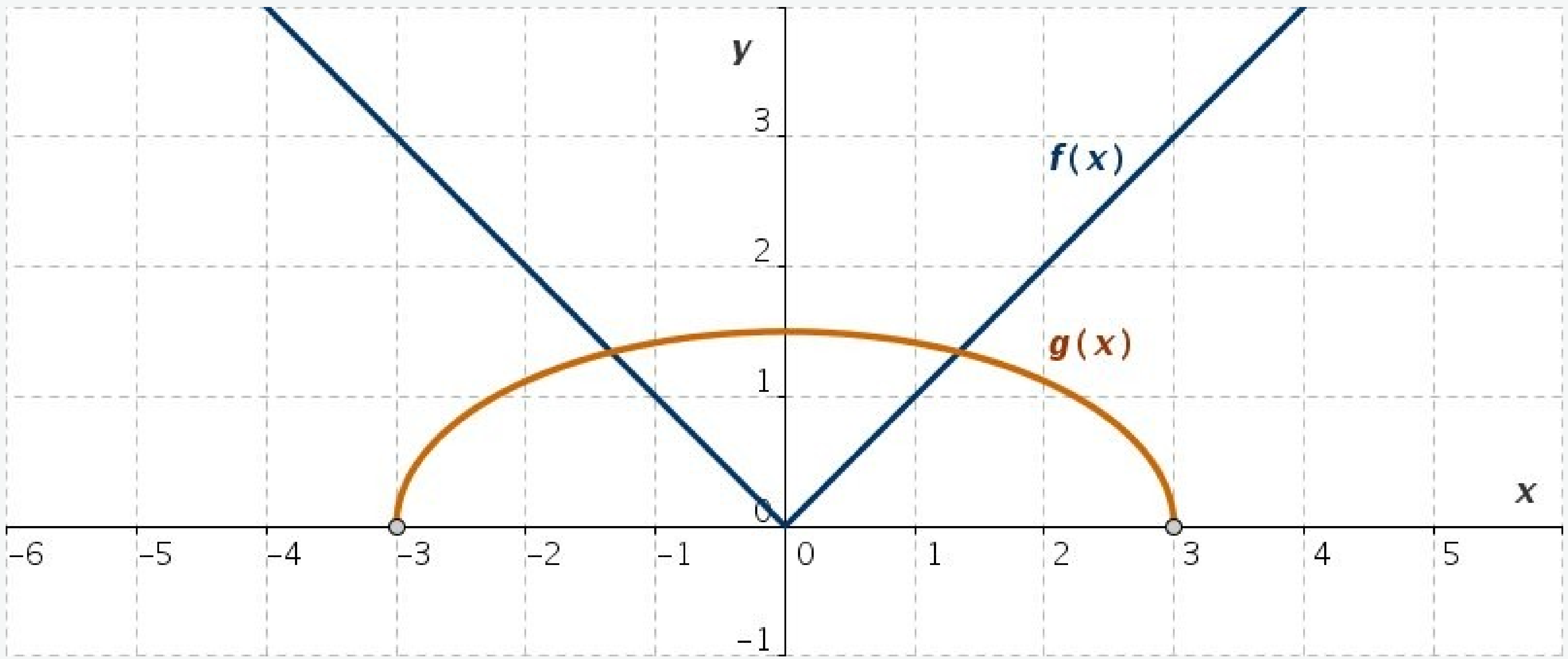


Fig. 3-2: Functions y = f (x) and y = g (x)

$$f(x) = |x|, \qquad g(x) = \frac{1}{2}\sqrt{9 - x^2}$$

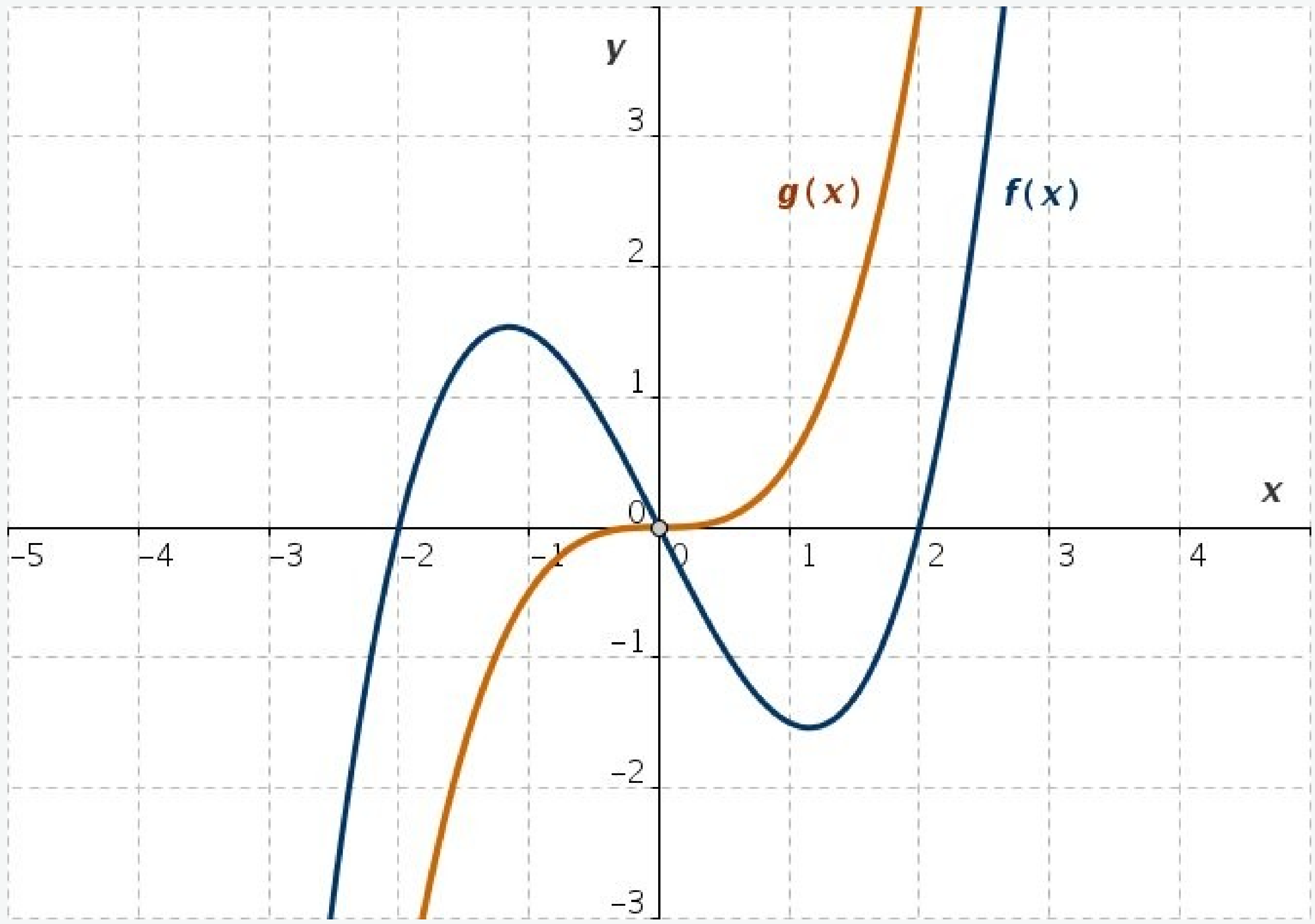


Fig. 3-3: Functions $y = f(x)$ and $y = g(x)$

$$f(x) = \frac{x^3}{2} - 2x, \qquad g(x) = \frac{x^3}{2}$$

Inverse function: Exercise 1d

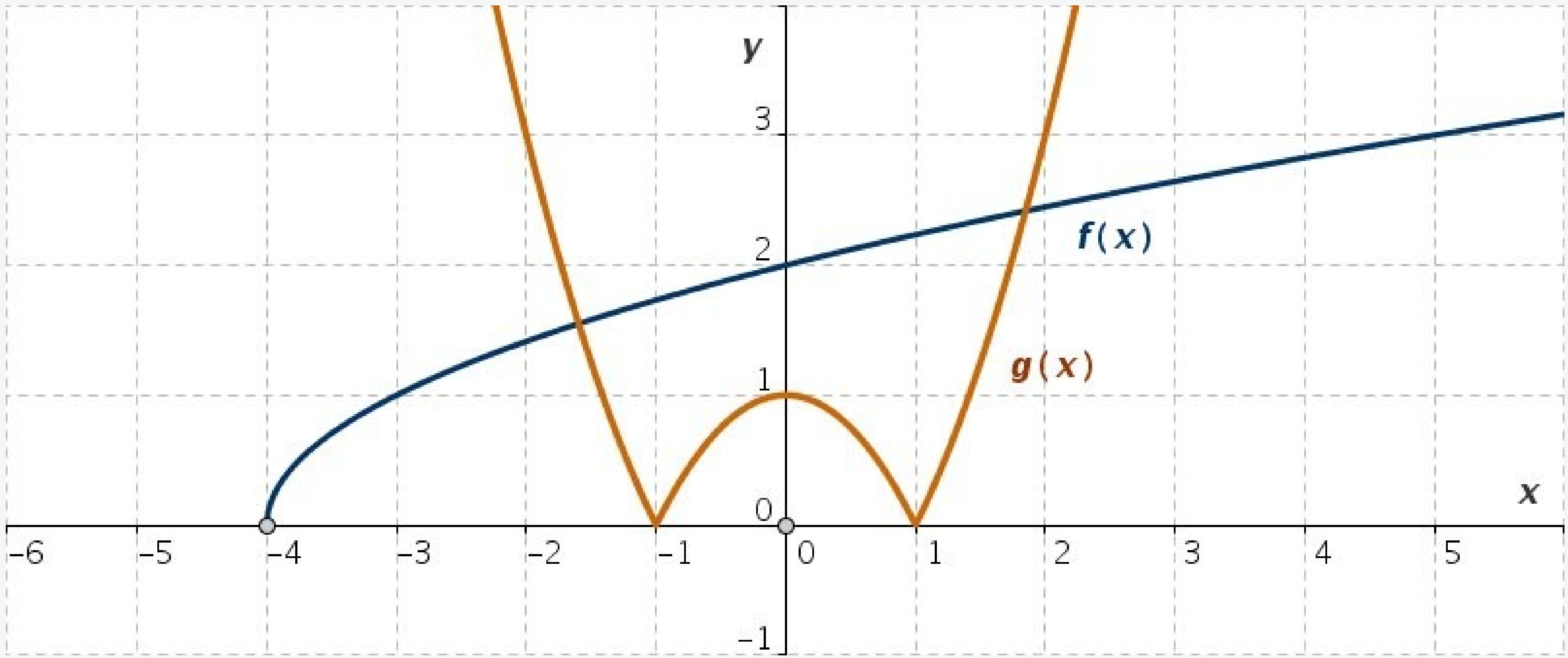


Fig. 3-4: Functions y = f (x) and y = g (x)

$$f(x) = \sqrt{x+4}\,, \qquad g(x) = |\,x^2 - 1\,|$$

Inverse function: Exercise 1e

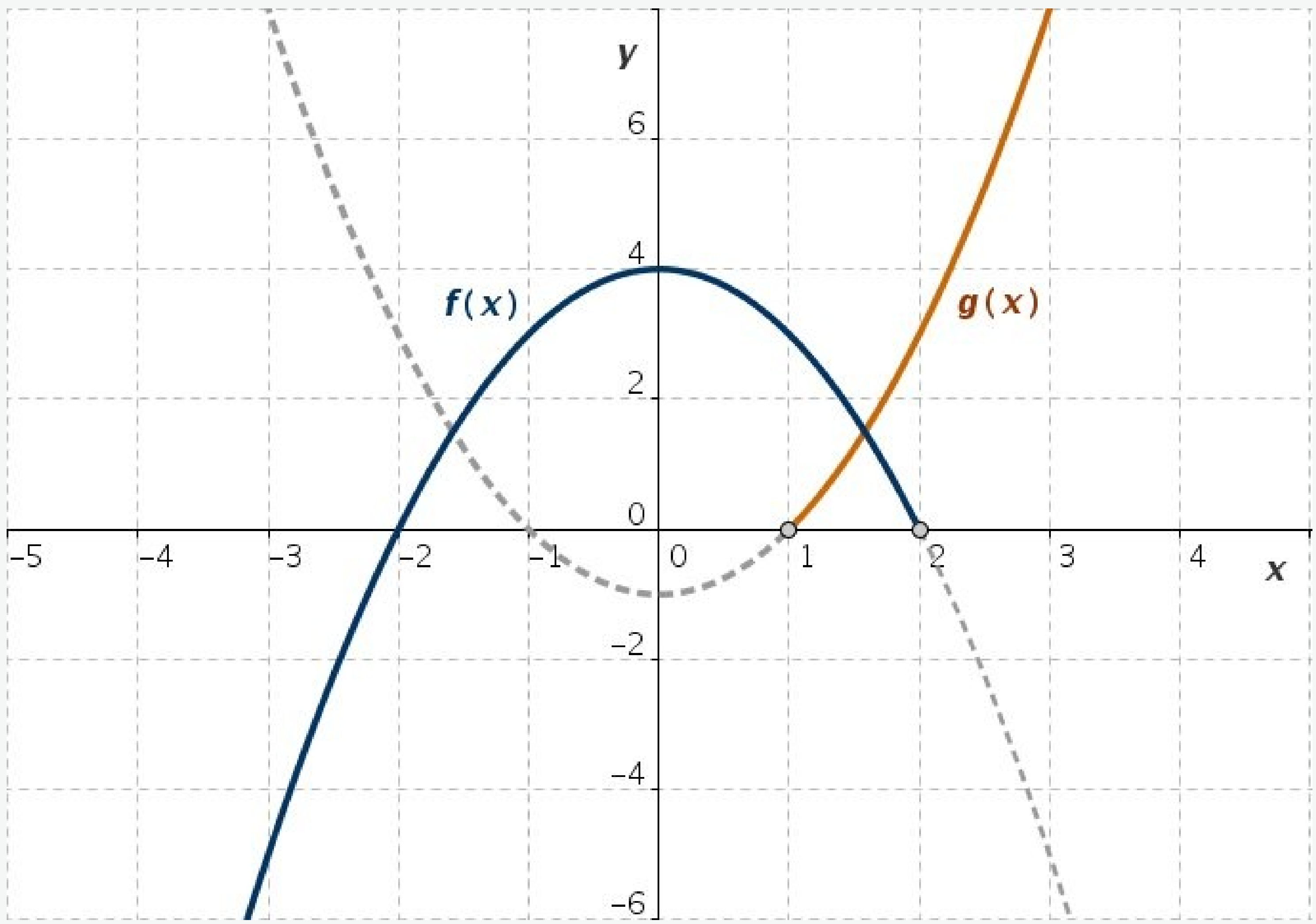


Fig. 3-5: Functions y = f (x) and y = g (x)

$$f(x) = -x^2 + 4, \qquad D(f) = (-\infty,\ 2\,]$$

$$g(x) = x^2 - 1, \qquad D(g) = [\,1,\ \infty)$$

Invertible functions are:

$$c)\quad g(x) = \frac{x^3}{2}$$

$$d)\quad f(x) = \sqrt{x + 4}$$

$$e)\quad g(x) = x^2 - 1, \qquad D(g) = [1, \infty)$$