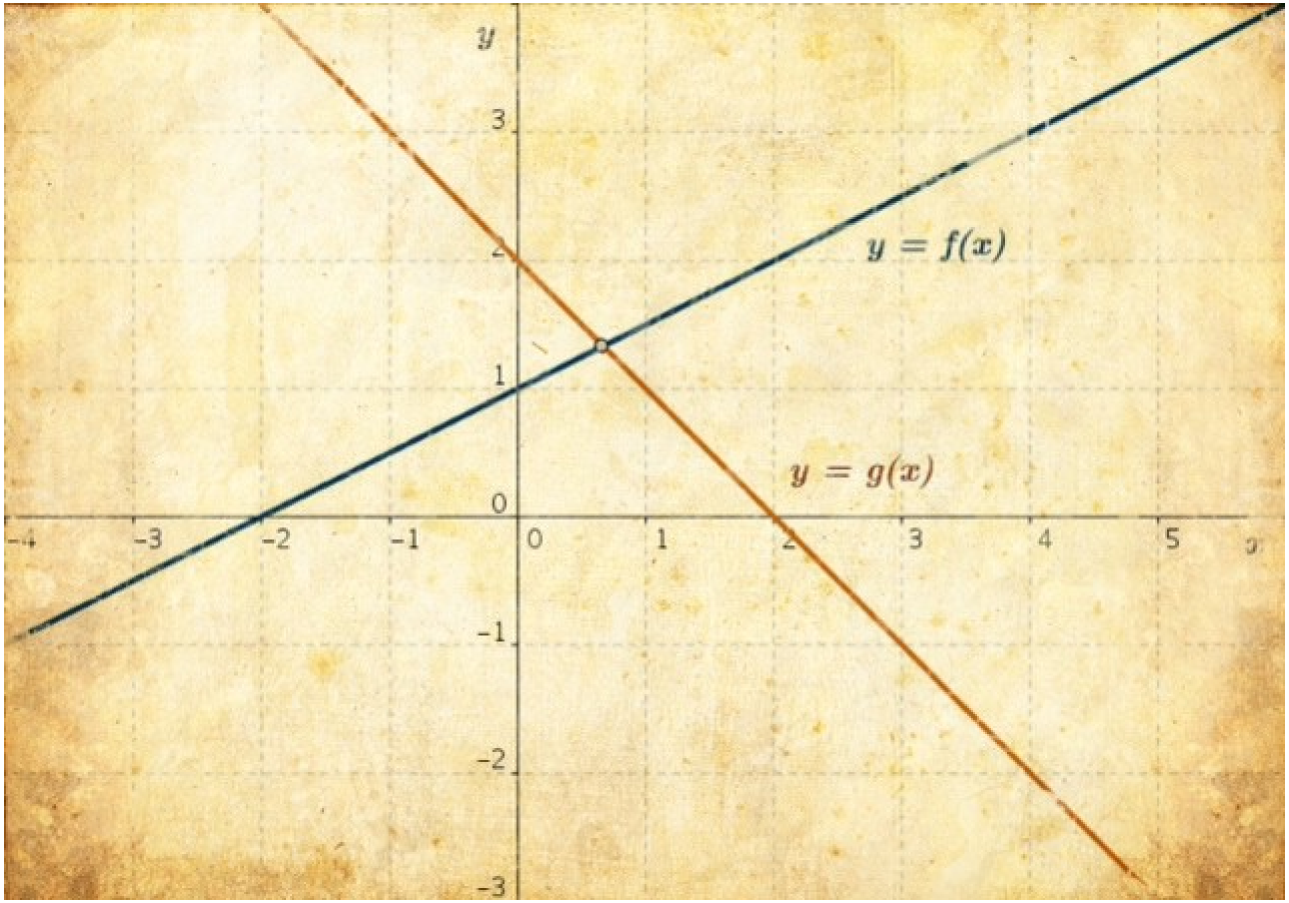

Solution of a Linear Equation

Definition:

A first order polynomial equation, that is an equation of the form

$$a\,x + b = 0\,, \qquad a \neq 0$$

is a standard form of a linear equation.

Question 1:

Which of the following equations are linear equations? Present them in the above standard form..

$a)$ $3\,x + 2 = 12$

$b)$ $3\,x + 2\,(4 - x) = 3\,(3 - x) - 5$

$c)$ $3\,x^2 + 2\,x = 4\,x^2 - 3$

$d)$ $(2\,x - 1)(2 - 3\,x) = (4 + x)(2 - x)$

$e)$ $x\,(2\,x - 1) = 2\,x^2 + 6$

a) $3x + 2 = 12$

A linear equation.

$3x - 10 = 0$

b) $3x + 2(4 - x) = 3(3 - x) - 5$

A linear equation.

$4x + 4 = 0$

c) $3x^2 + 2x = 4x^2 - 3$

A quadratic equation.

d) $(2x - 1)(2 - 3x) = (4 + x)(2 - x)$

A quadratic equation.

e) $x(2x - 1) = 2x^2 + 6$

A linear equation. The x^2-terms cancels.

$x + 6 = 0$

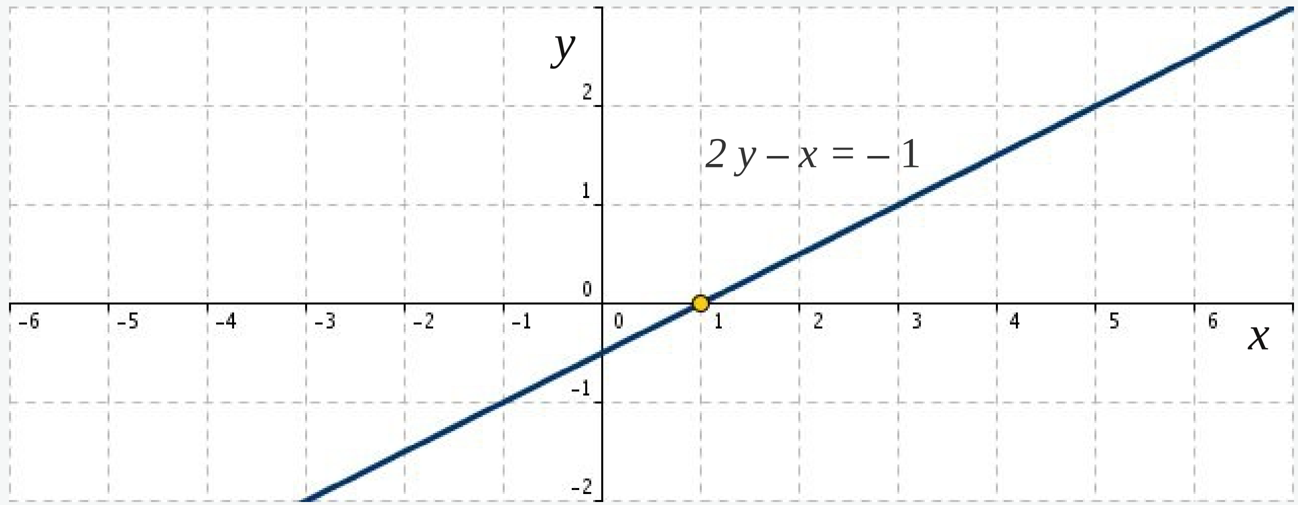

Fig. 1: Linear function $2y - x = -1$

A linear equation with one variable $ax + b = 0 \quad (a \neq 0)$, has exactly one solution $x = -\dfrac{b}{a}$

The solution of a linear equation with one variable is the root (zero) of the corresponding function $y = ax + b$.

Linear equations with two variables

A linear equation with two variables can be written the following way

$$a\,x + b\,y = 0$$

Any pair of numbers (x, y) that satisfies this equation is called a solution of it.

There is an infinite set of solutions, that satisfies one equation with two variables. Graphically the solutions may be represented by the straight line $a\,x + b\,y = 0$.

A single equation is not sufficient to fix two unknown variables. A system of two equations may determine a pair of numbers (x, y) uniquely, to satisfy both equations.

Example:

$$\begin{cases} 3x + y = 15 \\ 5x - 6y = 2 \end{cases}$$

The method to solve such a linear system is to transform the system into an equivalent one which has the same set of solutions, using the following transformations:

Transformations leading to equivalent linear systems:

- An equation may be multiplied by a number $k \neq 0$.
- Any multiple of another equation may be added to an equation.

$$\begin{cases} E_1: & 3x + y = 15 \\ E_2: & 5x - 6y = 2 \end{cases} \Leftrightarrow$$

$$\begin{cases} 6E_1: & 18x + 6y = 90 \\ E_2: & 5x - 6y = 2 \end{cases} \Leftrightarrow$$

$$\begin{cases} 6E_1 + E_2: & 23x = 92 \\ E_2: & 5x - 6y = 2 \end{cases} \Leftrightarrow$$

$$\begin{cases} \frac{1}{23}(6E_1 + E_2): & x = 4 \\ E_2: & 5x - 6y = 2 \end{cases}$$

$$S = \{ (4,3) \}$$

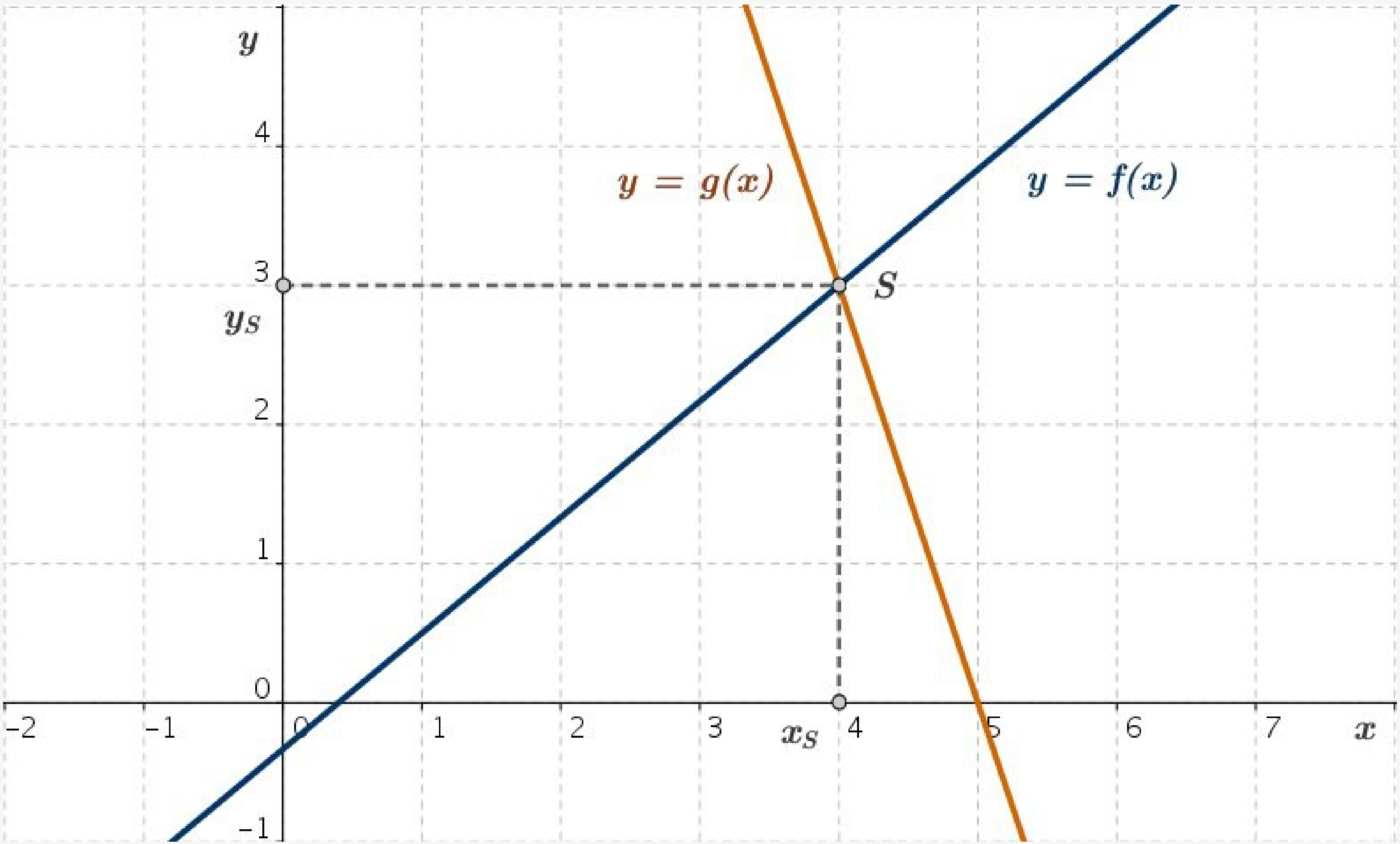

Fig. 2: Graphical solution of a system of linear equations $y = f(x)$ and $y = g(x)$

$$f(x) = \frac{5}{6}x - \frac{1}{3}, \qquad g(x) = -3x + 15$$

System of linear equations which corresponds to Fig. 2:

$$\begin{cases} 3x + y = 15 \\ 5x - 6y = 2 \end{cases}$$