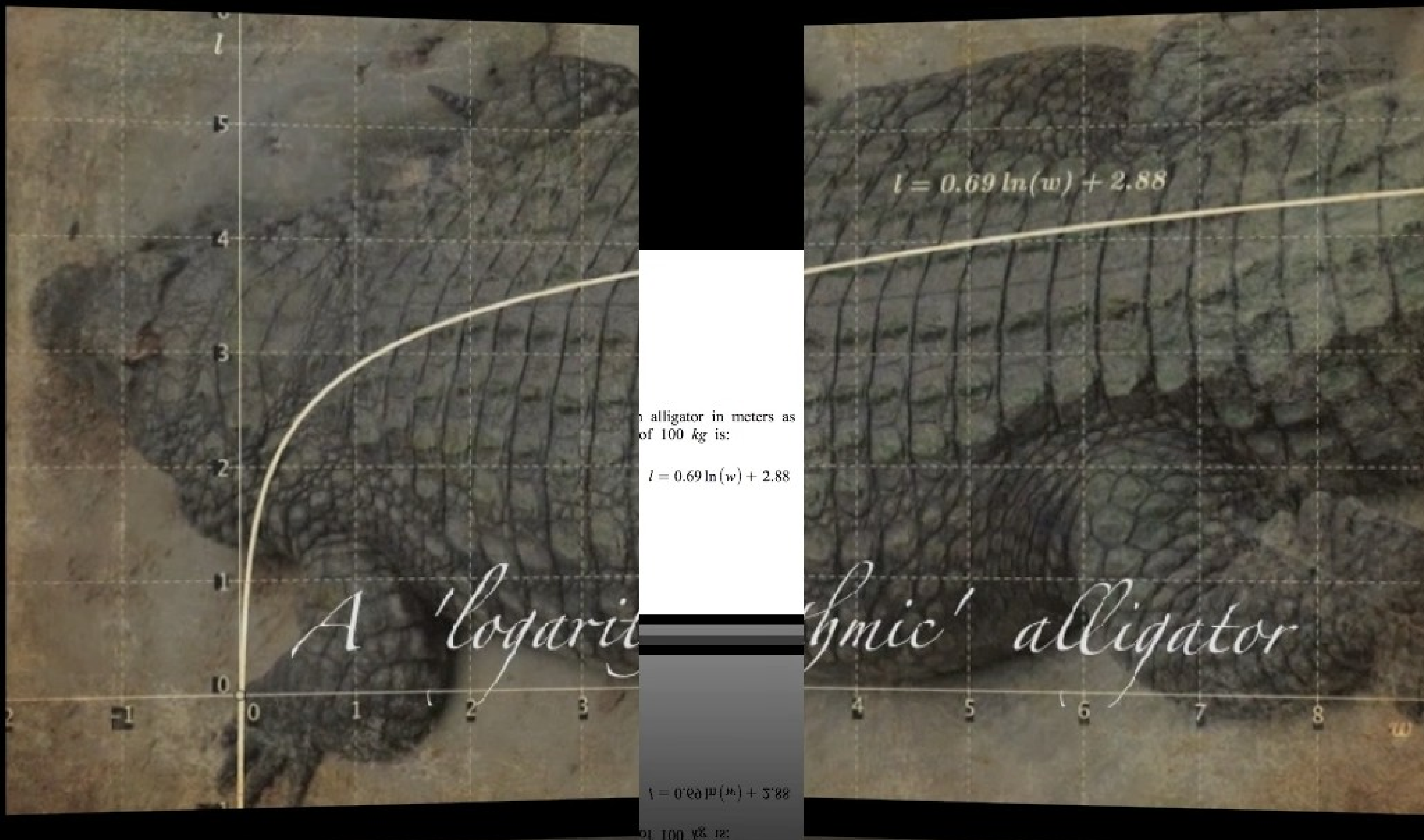

Logarithms, applied problems. Part 1

A "logarithmic" alligator

According to biologists, the length l of an alligator in meters as a function of his weight w in units of 100 kg can roughly be described by the function $l = l\,(w)$:

$$l = 0.69 \ln(w) + 2.88 \quad (1)$$

a) Evaluate the length of an alligator of weight 200, 300, 400 and 500 kg using the above formula.

b) Make a graph of the function and estimate the weight of an alligator of length 3, 3.5 and 4 m. Show algebraically, that your graphic estimations are correct.

c) Transform the length-weight relation above into an equivalent relation of the form $l = a \ln(w) + b$, where the length l of the alligator is given in cm and its weight w is given in kg.

d) Show that the formula makes no sense at small weights.

e) Summarize what you have learnt solving this task.

$$\ln(x) \equiv \log_e(x)$$

To estimate the length of an alligator of weight 200, 300, 400 and 500 *kg*, we have to insert these values into expression (1), taking into account, that the weight unit is 100 *kg*. This means, that the given values of 200, 300, 400 and 500 *kg* correspond to arguments 2, 3, 4 and 5 of the logarithm.

$$200\ kg \;\Rightarrow\; w = 2: \qquad l = 0.69\ln(2) + 2.88 = 3.36\ m$$

$$300\ kg \;\Rightarrow\; w = 3: \qquad l = 0.69\ln(3) + 2.88 = 3.64\ m$$

$$400\ kg \;\Rightarrow\; w = 4: \qquad l = 0.69\ln(4) + 2.88 = 3.84\ m$$

$$500\ kg \;\Rightarrow\; w = 5: \qquad l = 0.69\ln(5) + 2.88 = 3.99\ m$$

You see, while the 4 weight values differ from each other by one unit, the corresponding length values remain in the interval from 3.36 to 3.99. This demonstrates once more a slow logarithmic dependence.

In the following figures we show on the GeoGebra applet the length of an alligator of 200 and 400 *kg*.

a) Alligator's length: Graphical solution

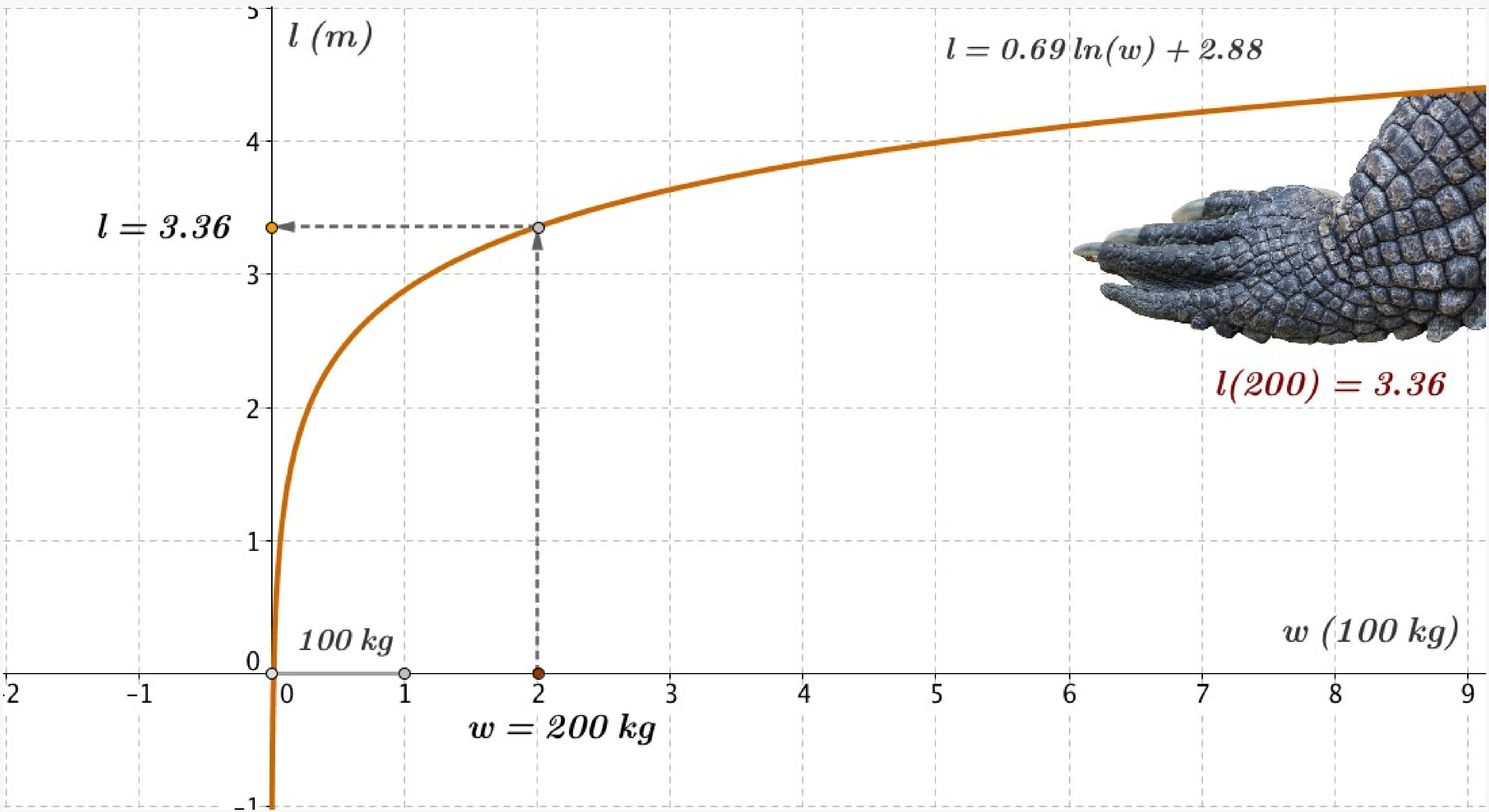

Fig. 1-1: The logarithmic curve describes roughly the length of an alligator as a function of his weight. For example, an alligator of 200 kg has a length of 3.36 m

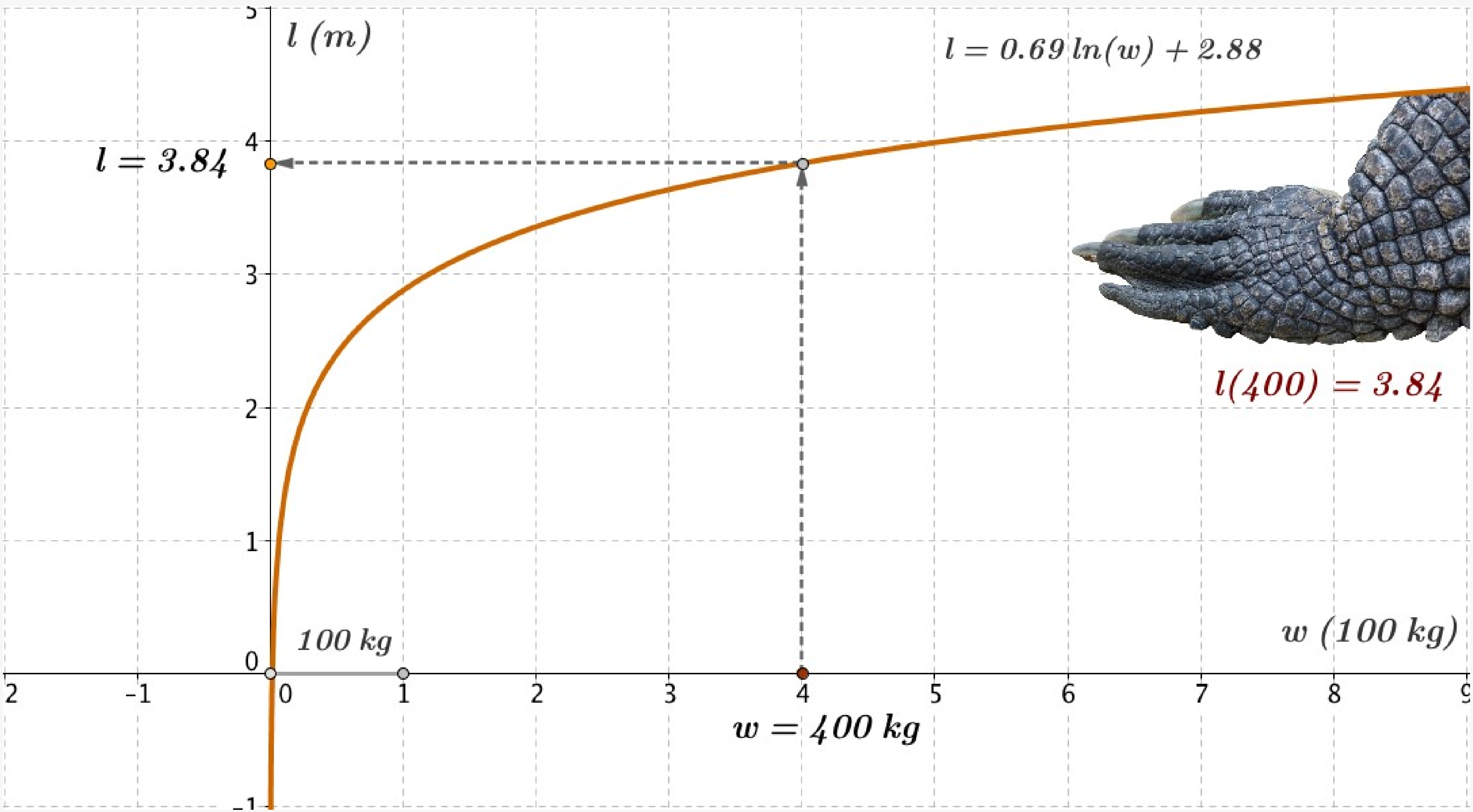

Fig. 1-2: The logarithmic curve describes roughly the length of an alligator as a function of his weight. For example, an alligator of 400 kg has a length of 3.84 m

b) Alligator's weight: Graphical solution

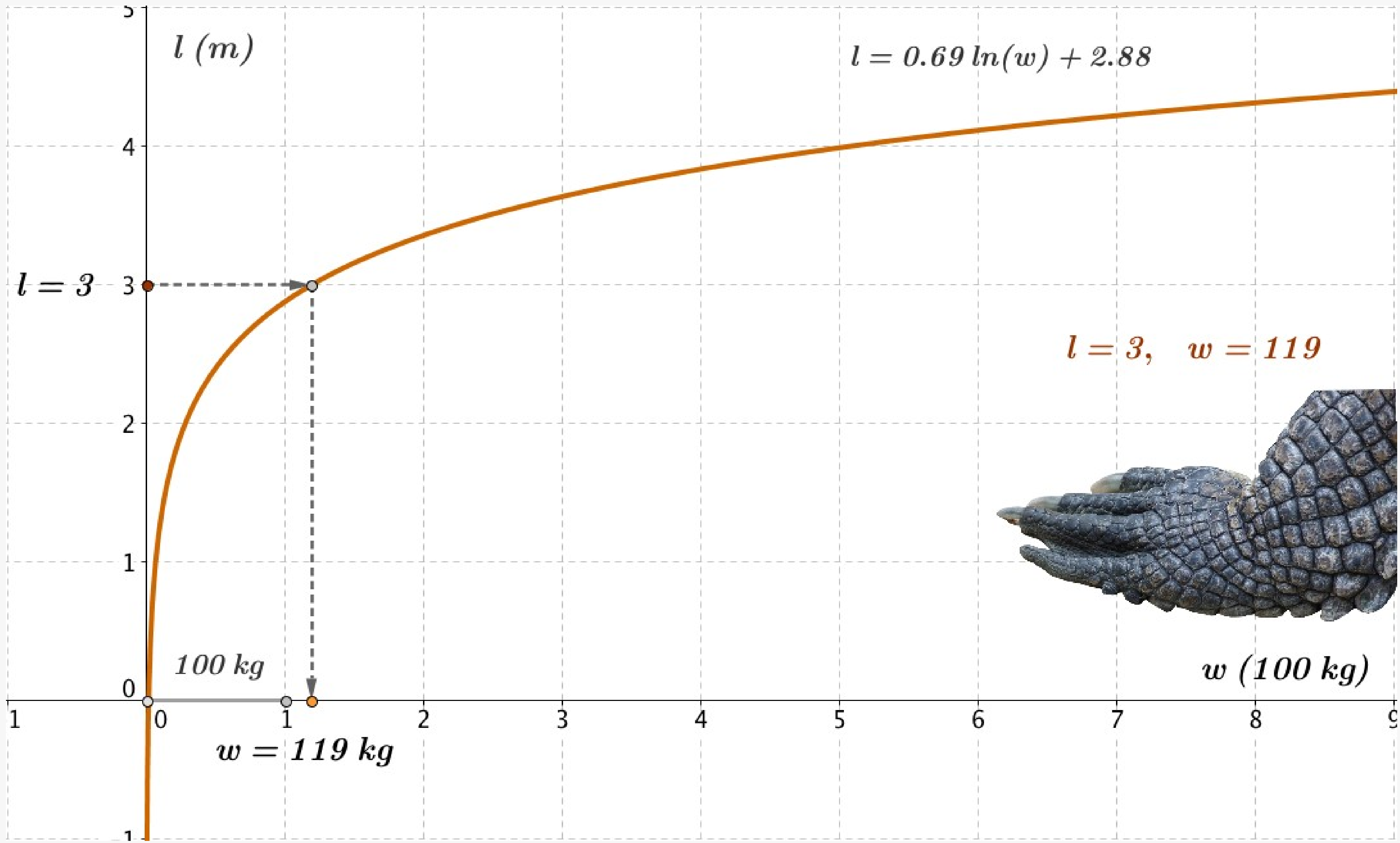

Fig. 1-3: The logarithmic curve predicts, that an alligator of 3 m length has a weight of weight of 119 kg

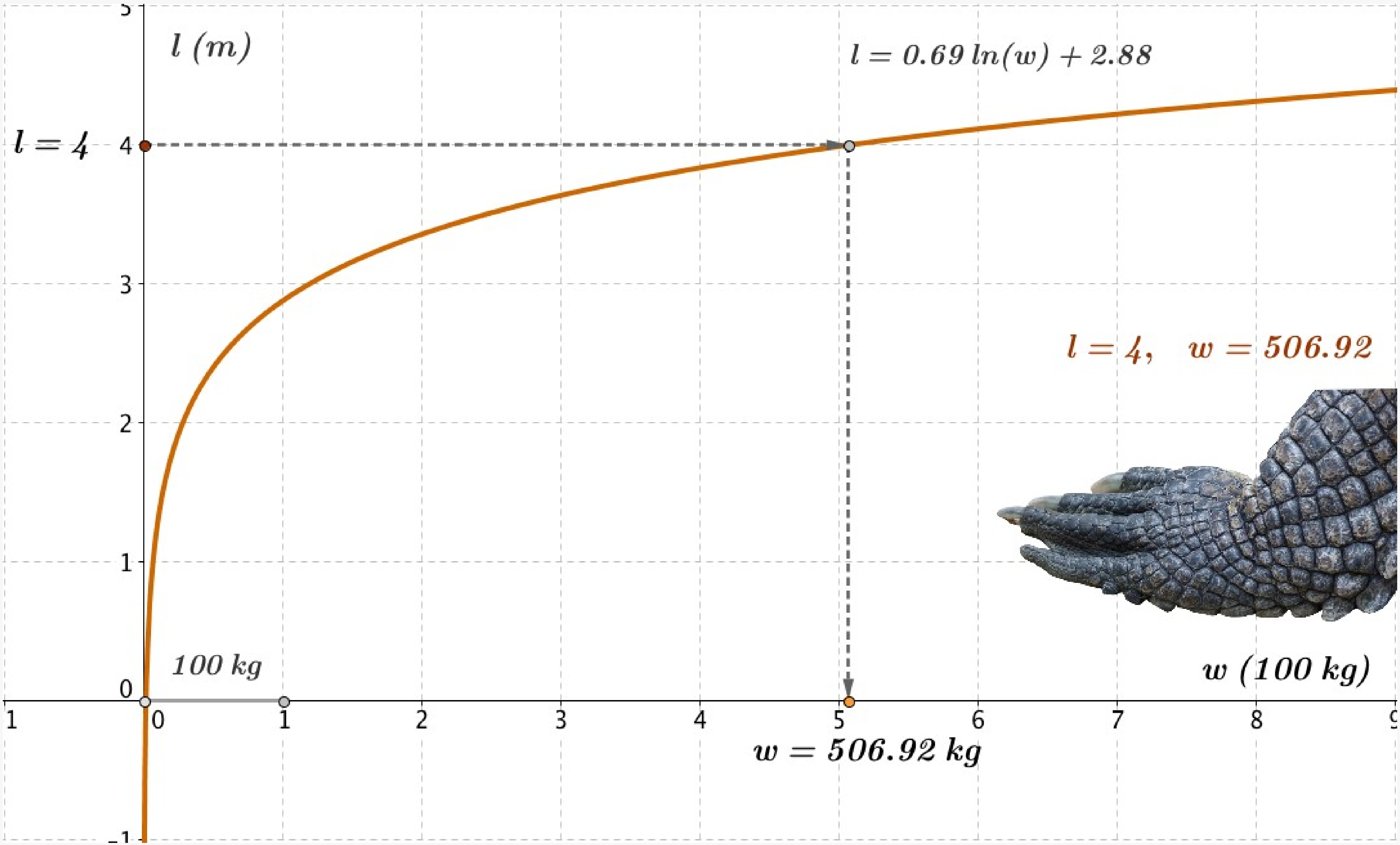

Fig. 1-4: The logarithmic curve predicts, that an alligator of 4 m length has a weight of weight of 507 kg

b) Alligator's weight: Analytic solution

To show algebraically, that our graphic estimations are correct, we have to invert equation (1) to obtain the weight as a function of the length.

$$l = 0.69 \ln(w) + 2.88 \quad (1)$$

$$l - 2.88 = 0.69 \ln(w), \qquad \ln(w) = \frac{l - 2.88}{0.69}$$

$$w = e^{\frac{l-2.88}{0.69}} \qquad \text{(in units 100 kg)}$$

$$w = 100\, e^{\frac{l-2.88}{0.69}} \qquad \text{(in units 1 kg)}$$

$$l = 3\ m: \quad w = 100\, e^{\frac{3-2.88}{0.69}} = 100\, e^{\frac{0.12}{0.69}} \simeq 119\, kg$$

$$l = 3.5\ m: \quad w = 100\, e^{\frac{3.5-2.88}{0.69}} = 100\, e^{\frac{0.62}{0.69}} \simeq 245.6\, kg$$

$$l = 4\ m: \quad w = 100\, e^{\frac{4-2.88}{0.69}} = 100\, e^{\frac{1.12}{0.69}} \simeq 506.9\, kg$$

c) Length-weight relation: scale transformation

$$l = 0.69 \ln(w) + 2.88$$

Equation (1) gives the length l in m and weights w in units of 100 kg. We write equation (1) again, indicating the units used for l and w:

$$l_m = 0.69 \ln(w_{100\,kg}) + 2.88$$

The length l in cm is then given by:

$$l_{cm} = 100\,(0.69 \ln(w_{100\,kg}) + 2.88) = 69 \ln(w_{100\,kg}) + 288$$

If we express w in kg, instead of units of 100 kg, we obtain:

$$l_{cm} = 69 \ln\left(\frac{w_{kg}}{100}\right) + 288$$

Note: numerically the argument of the logarithm should be unchanged.

To arrive at the desired form, we write:

$$l_{cm} = 69\,(\ln(w_{kg}) - \ln(100)) + 288 \simeq 69\,(\ln(w_{kg}) - 4.61) + 288$$

i.e.

$$l_{cm} \simeq 69 \ln(w_{kg}) - 30$$

d) Negative alligator's length or the interval of application

The equation obtained for the length l in cm and the weight w in kg

$$l_{cm} \simeq 69 \ln\left(w_{kg}\right) - 30$$

demonstrates, that there is only a limited range of application. For example, for the weight of 1 kg, the alligator length is negative:

$$l_{cm} \simeq 69 \ln\left(1\right) - 30 = -30$$

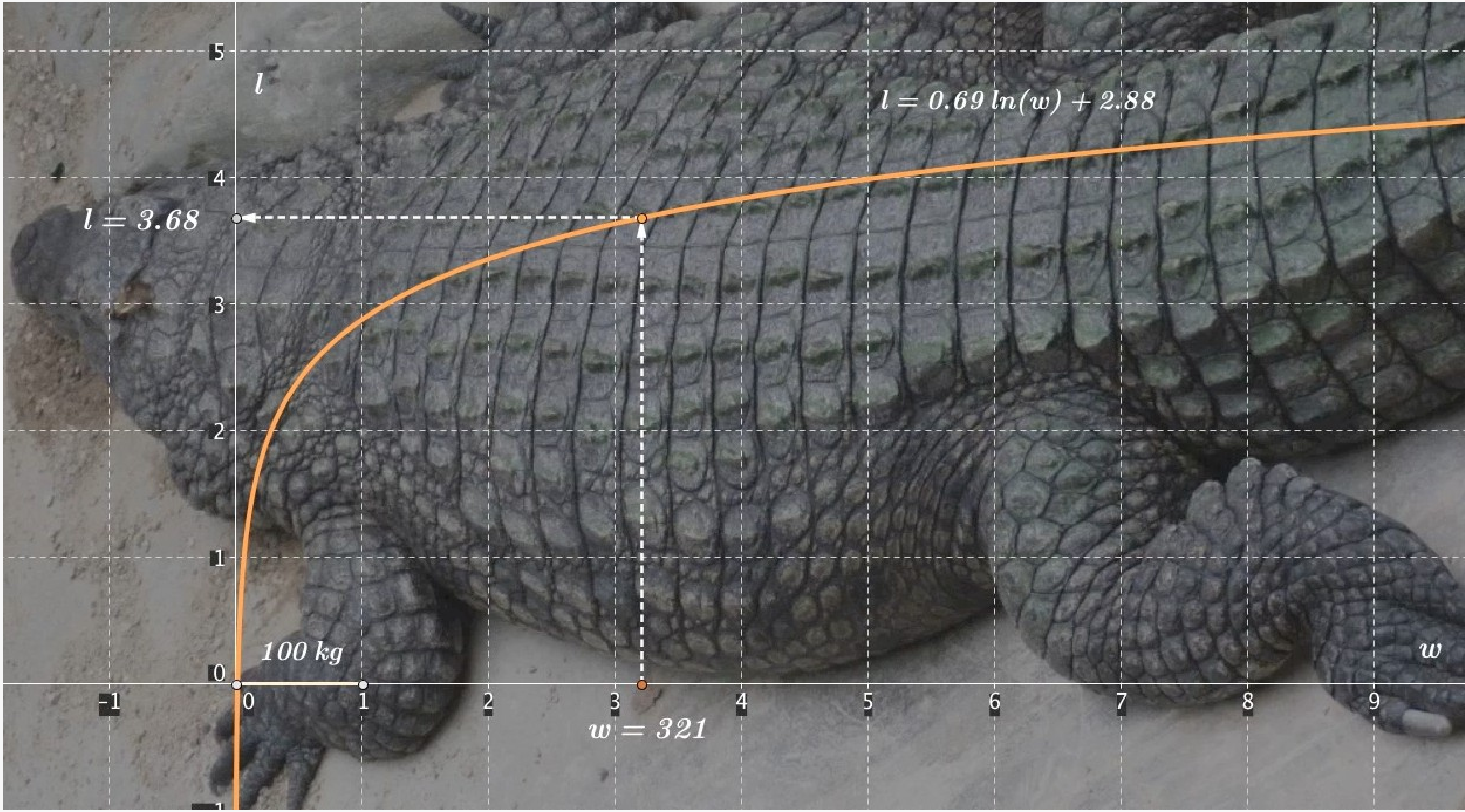

Fig. 1-5: The picture from the following video presentation showing the logarithmic dependence of the alligator length from his weight

Being not so 'friendly' the alligator happened to be very useful demonstrating how laws of nature with logarithmic functions can be used to estimate values.

e) A 'useful' alligator or what have we learnt?

- We have used the equation $l = f\,(w)$ (1) on page 1-A to estimate the length l of an alligator as function of his weight w algebraically by inserting the values of w into Eq. (1) .

- We have also shown, that the length can be estimated using the function graph (obtained with GeoGebra) without evaluating the logarithm values numerically.

- To calculate the weight as a function of length, we had to find the inverse function of $l = f(w)$ and estimate the numerical values after that.

- The numerical weight estimations were confirmed using the graph of $l = f\,(w)$.

- We have shown that Eq. (1) can be applied in a limited w-interval only. For example, the resulting length gets negative at small weights.