Logarithms, logarithmic equations: Exercise 7, 8

Exercise 7:

Take the logarithm of the following functions and simplify as much as possible:

1) $f(x) = \sqrt{x \sqrt{x}}$, $\quad g(x) = \sqrt[3]{\sqrt{x} \sqrt[4]{x^2}}$

2) $f(x) = \sqrt{x \sqrt{x \sqrt{x}}}$, $\quad g(x) = \sqrt{x \sqrt{x \sqrt[3]{x^5}}}$

3) $f(x) = \dfrac{x}{x^2 - 1}$, $\quad g(x) = \dfrac{3x}{4x^2 - 1}$

Exercise 8:

Solve the given logarithmic equation:

$$\frac{lg(x + 1)}{lg\, 4 - lg\, 2} = 2$$

1) $f(x) = \sqrt{x\sqrt{x}}$, $\quad \ln f(x) = \dfrac{3}{4}\ln x$

$g(x) = \sqrt[3]{\sqrt{x}\,\sqrt[4]{x^2}}$, $\quad \ln g(x) = \dfrac{1}{3}\ln x$

2) $f(x) = \sqrt{x\sqrt{x\sqrt{x}}}$, $\quad \ln f(x) = \dfrac{7}{8}\ln x$

$g(x) = \sqrt{x\sqrt{x^3\sqrt{x^5}}}$, $\quad \ln g(x) = \dfrac{15}{8}\ln x$

3) $f(x) = \dfrac{x}{x^2 - 1}$, $\quad \ln f(x) = \ln x - \ln(x - 1) - \ln(x + 1)$

$g(x) = \dfrac{3x}{4x^2 - 1}$, $\quad \ln g(x) = \ln 3 + \ln x - \ln(2x - 1) - \ln(2x + 1)$

$$G \quad : \frac{lg\,(x + 1)}{lg\,4 - lg\,2} = 2$$

$$D\,(G) = D\left(\,lg\,(x + 1)\,\right) = (-1,\ \infty)$$

$$\frac{lg\,(x + 1)}{lg\,4 - lg\,2} = \frac{lg\,(x + 1)}{2\,lg\,2 - lg\,2} = \frac{lg\,(x + 1)}{lg\,2} = 2$$

$$lg\,(x + 1) = 2\,lg\,2 = lg\,4, \qquad x + 1 = 4, \qquad x = 3$$

$$x = 3 \in D\,(G)$$

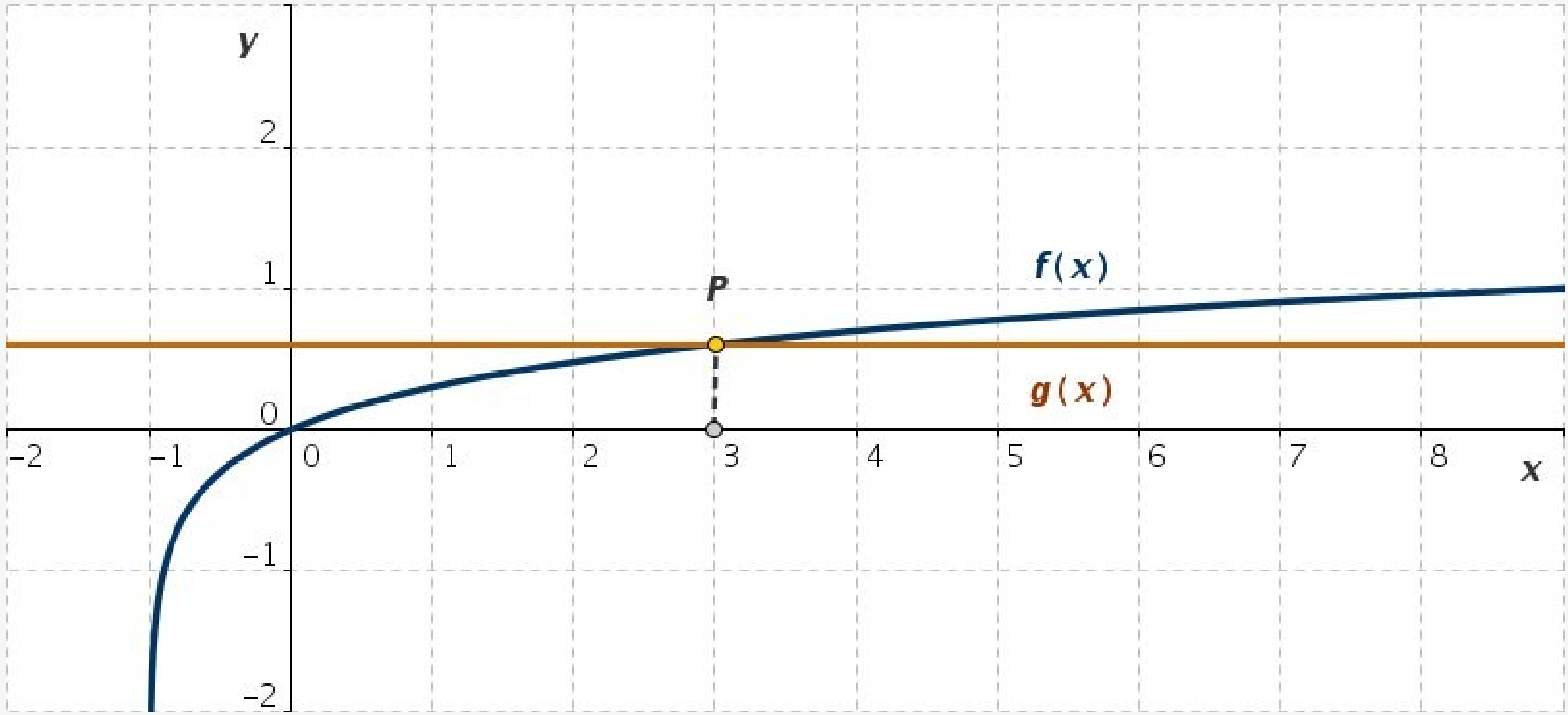

Fig. L8: Graphical solution of the logarithmic equation

$$f(x) = lg(x+1), \qquad g(x) = lg\,4$$

$$G \quad : lg(x+1) = lg\,4, \qquad D(G) = (-1, \infty)$$

The point $P = (3, lg\,4)$ is the intersection of the graphs of $f(x)$ and $g(x)$.

Solve the logarithmic equations below:

Exercise 9:

$$lg\left(\frac{1}{2}+x\right)=lg\,\frac{1}{2}-lg\,x$$

Exercise 10:

$$lg\sqrt{1+x}+3\,lg\sqrt{1-x}=lg\sqrt{1-x^2}+2$$

Exercise 11:

$$lg\sqrt{5x-4}+lg\sqrt{x+1}=2+lg\,0.18$$

$$G \quad : lg\left(\frac{1}{2} + x\right) = lg\,\frac{1}{2} - lg\,x$$

$$D\left(lg\left(\frac{1}{2} + x\right)\right) = \left(-\frac{1}{2},\ \infty\right), \qquad D\left(lg\,x\right) = (0,\ \infty) \quad \Rightarrow \quad D(G) = (0,\ \infty)$$

$$G \quad : lg\left(\frac{1}{2} + x\right) = lg\,\frac{1}{2} - lg\,x\,, \qquad lg\left(\frac{1}{2} + x\right) = lg\,\frac{1}{2\,x} \quad \Rightarrow$$

$$\frac{1}{2} + x = \frac{1}{2\,x} \quad (\times \ \ 2\,x)\,, \qquad 2\,x^2 + x - 1 = 0$$

$$a\,x^2 + b\,x + c = 0 \qquad : x_{1,\,2} = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

$$2\,x^2 + x - 1 = 0 \qquad : x_{1,\,2} = \frac{-1 \pm \sqrt{9}}{4}\,, \quad x_1 = -1, \qquad x_2 = \frac{1}{2}$$

$$x_1 = -1 \notin D(G)\,, \qquad x_2 = \frac{1}{2} \in D(G)$$

$x_2 = \dfrac{1}{2}$ - is solution of equation G

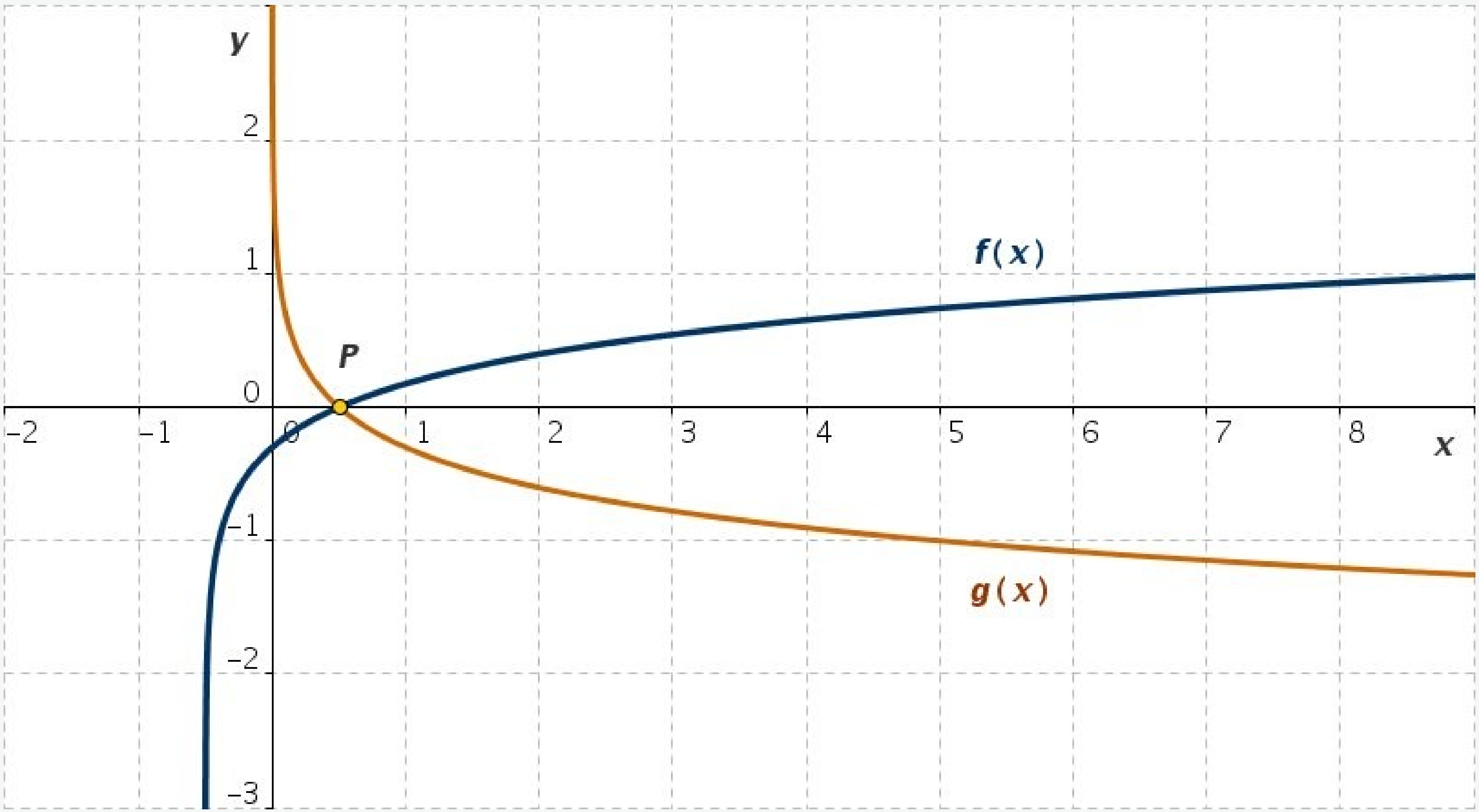

Fig. L9: Graphical solution of the logarithmic equation

$$G \quad : lg\left(\frac{1}{2}+x\right)=lg\,\frac{1}{2}-lg\,x\,, \qquad D(G)=(0,\ \infty)$$

$$f(x)=lg\left(\frac{1}{2}+x\right), \qquad g(x)=lg\,\frac{1}{2}-lg\,x$$

The functions *f (x)* and *g (x)* have a common point $P = (1/2, 0)$.

$$G \quad : lg\sqrt{1+x} + 3\,lg\sqrt{1-x} = lg\sqrt{1-x^2} + 2$$

$$D\left(lg\sqrt{1+x}\right) = (-1,\ \infty), \qquad D\left(lg\sqrt{1-x}\right) = (-\infty,\ 1)$$

$$D\left(lg\sqrt{1-x^2}\right) = (-1,\ 1)$$

$$D(G) = (-1,\ \infty) \cap (-\infty,\ 1) \cap (-1,\ 1) = (-1,\ 1)$$

$$G \quad : \frac{1}{2}\,lg(1+x) + \frac{3}{2}\,lg(1-x) = \frac{1}{2}\,lg(1-x^2) + 2$$

$$lg(1-x^2) = lg\,(1-x)(1+x) = lg\,(1-x) + lg(1+x)$$

$$G \quad : lg(1-x) = 2 = lg\,100 \quad \Rightarrow \quad 1-x = 100 \quad \Rightarrow \quad x = -99$$

$$x = -99 \ \notin \ D(G)$$

This equation has no real solution

Logarithmic Equation: Solution 10

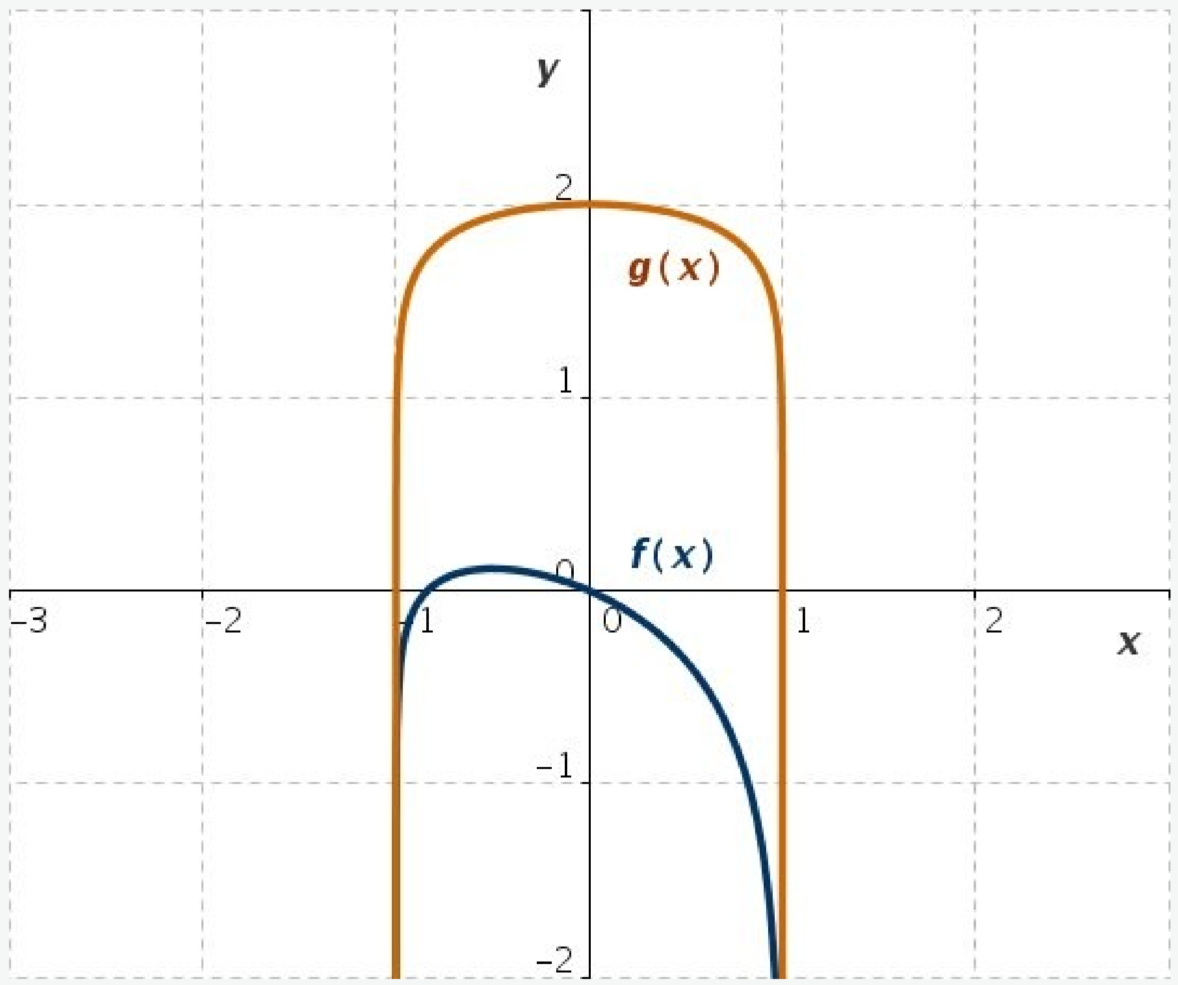

Fig. L10: The functions f (x) and g (x) have no intersection

$$G \quad : lg\sqrt{1+x} + 3\,lg\sqrt{1-x} = lg\sqrt{1-x^2} + 2$$

$$f(x) = lg\sqrt{1+x} + 3\,lg\sqrt{1-x}\,, \qquad g(x) = lg\sqrt{1-x^2} + 2$$

$$G \quad : lg\sqrt{5x-4} + lg\sqrt{x+1} = 2 + lg\,0.18$$

$$D\left(lg\sqrt{5x-x}\right) = \left(\frac{4}{5},\ \infty\right), \qquad D\left(lg\sqrt{x+1}\right) = (-1,\ \infty)$$

$$D(G) = \left(\frac{4}{5},\ \infty\right) \cap (-1,\ \infty) = \left(\frac{4}{5},\ \infty\right)$$

$$G \quad : \frac{1}{2}\,lg(5x-4) + \frac{1}{2}\,lg(x+1) = 2 + lg\,0.18$$

$$lg\,0.18 = lg\left(18\cdot 10^{-2}\right) = lg(18) - 2\,lg\,10 = lg(18) - 2$$

$$G \quad : lg\left[(5x-4)(x+1)\right] = 2\,lg\,18 = lg\,18^2$$

$$(5x-4)(x+1) = 18^2 \quad \Leftrightarrow \quad 5x^2 + x - 328 = 0$$

$$x_{1,\,2} = \frac{-1 \pm 81}{10}, \quad x_1 = -\frac{41}{5}, \qquad x_2 = 8$$

$$x_1 = -\frac{41}{5} \notin D(G), \qquad x_2 = 8 \in D(G)$$

$x_2 = 8$ - is the solution of equation G

Logarithmic Equation: Solution 11

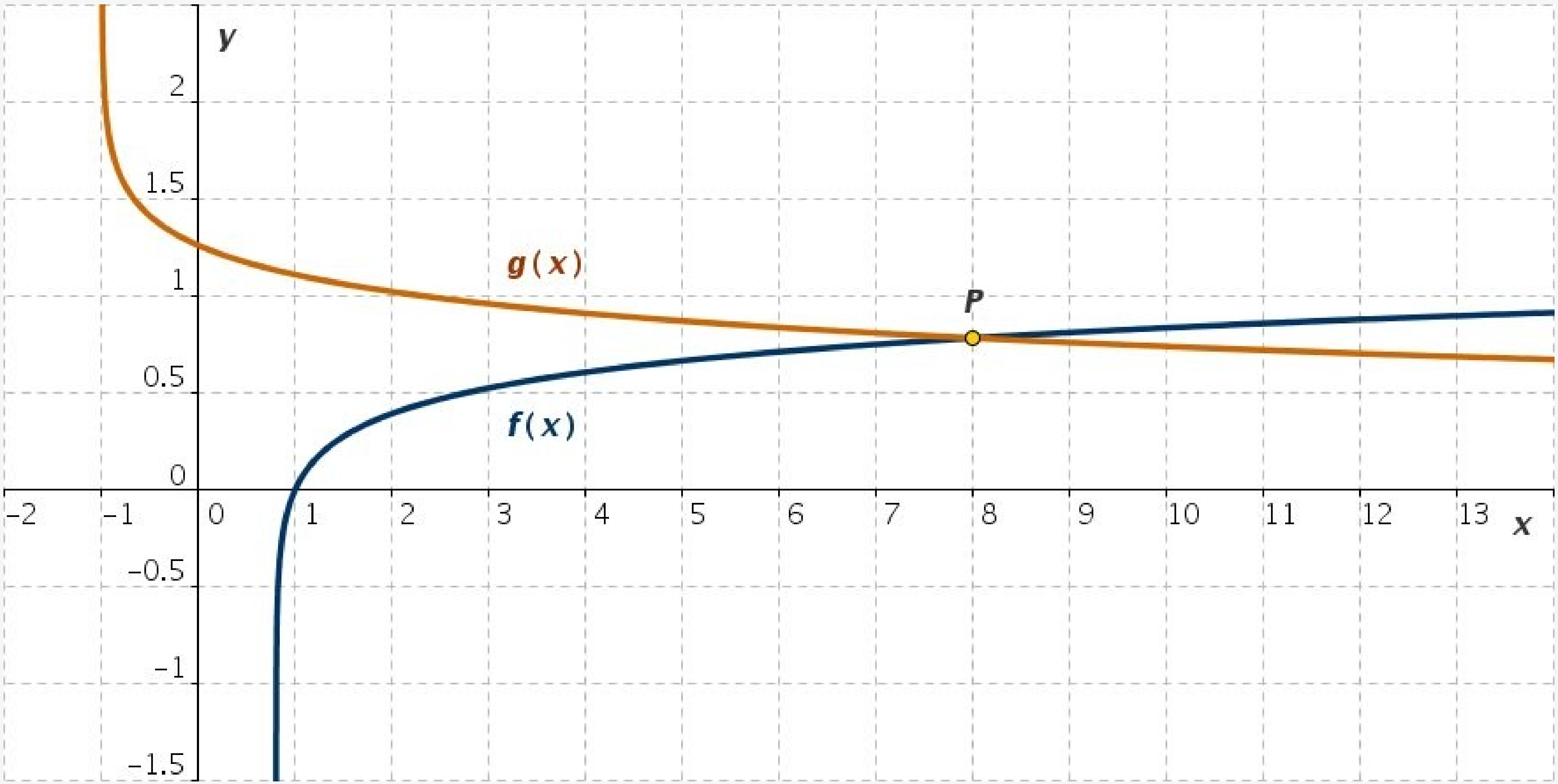

Fig. L11: Graphical solution of the logarithmic equation

$$G \quad : lg\sqrt{5x-4} + lg\sqrt{x+1} = 2 + lg\, 0.18, \qquad D(G) = \left(\frac{4}{5},\ \infty\right)$$

$$f(x) = lg\sqrt{5x-4}, \qquad g(x) = 2 + lg\, 0.18 - lg\sqrt{x+1}$$

The functions $f(x)$ and $g(x)$ intersect in point $P = (8, \lg 6)$.