

*Lüneburg, Fragment*

*Powers with integer exponents: Rules*

$$b^n \cdot b^m = b^{n+m}$$

$$\frac{b^n}{b^m} = b^{n-m}$$

$$(b^n)^m = b^{n \cdot m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$b^{-n} = \frac{1}{b^n}$$



# Power rules

## Rule 1:

Powers with the same base can be multiplied by raising the base to the power of the sum of the exponents

$$b^n \cdot b^m = b^{n+m}$$

$$b^n \cdot b^m = \underbrace{b \cdot b \cdot b \cdot \dots \cdot b}_{n \text{ factors}} \cdot \underbrace{b \cdot b \cdot b \cdot \dots \cdot b}_{m \text{ factors}} = b^{n+m}$$

## Examples:

$$a) \quad a^3 \cdot a^4 = a^{3+4} = a^7,$$

$$b) \quad c^5 \cdot c^{-2} = c^{5-2} = c^3,$$

$$c) \quad b^2 \cdot b^{3n-1} = b^{2+3n-1} = b^{1+3n},$$

$$d) \quad x^3 \cdot x^{2n+1} = x^{3+2n+1} = x^{4+2n}.$$



## Rule 1: Exercise 17

Simplify the expressions as much as possible:

$$a) \quad x^3 x^{-2}, \quad a^0 a^{-1}, \quad 4x^4 x^{-5}$$

$$b) \quad a^{n+2} \cdot a^3, \quad b^{n+1} \cdot b^2 \cdot b^{2n}, \quad x^5 \cdot x^3 \cdot x^{n+4}$$

$$c) \quad 4x^{-2n} \cdot x^{2+n} \cdot x, \quad y^2 \cdot y^{3-n} \cdot y^{-5} \cdot y^{3n+4}$$

$$d) \quad 2^3 \cdot 2^8 \cdot 2^{-7}, \quad 4 \cdot 2^{-5} \cdot 2^{n-3} \cdot 2^{1-n}$$

## Rule 1: Lösung 17

$$a) \quad x^3 x^{-2} = x, \quad a^0 a^{-1} = a^{0-1} = a^{-1} = \frac{1}{a}, \quad 4x^4 x^{-5} = \frac{4}{x}$$

$$b) \quad a^{n+2} \cdot a^3 = a^{n+2+3} = a^{n+5},$$

$$b^{n+1} \cdot b^2 \cdot b^{2n} = b^{n+1+2+2n} = b^{3n+3},$$

$$x^5 \cdot x^3 \cdot x^{n+4} = x^{5+3+n+4} = x^{n+12}$$

$$c) \quad 4x^{-2n} \cdot x^{2+n} \cdot x = 4x^{-2n+2+n+1} = 4x^{-n+3},$$

$$y^2 \cdot y^{3-n} \cdot y^{-5} \cdot y^{3n+4} = y^{2+3-n-5+3n+4} = y^{2n+4}$$

$$d) \quad 2^3 \cdot 2^8 \cdot 2^{-7} = 2^{3+8-7} = 2^4 = 16$$

$$\begin{aligned} 4 \cdot 2^{-5} \cdot 2^{n-3} \cdot 2^{1-n} &= 2^2 \cdot 2^{-5} \cdot 2^{n-3} \cdot 2^{1-n} = 2^{2-5+n-3+1-n} = \\ &= 2^{2-5+n-3+1-n} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32} \end{aligned}$$

## Rule 2:

A power can be divided by another power with same base by raising the base to the power of the difference of the exponents

$$\frac{b^n}{b^m} = b^{n-m}$$

## Rule 3:

A power can be exponentiated by giving the base an exponent which is the product of the exponents

$$(b^n)^m = b^{n \cdot m}$$



## Power rules

### Rule 4:

Powers with the same exponent can be multiplied by raising the product of the bases to the power of the same exponent:

$$a^n \cdot b^n = (a \cdot b)^n$$

### Rule 5:

Ratios of powers with the same exponent can be taken by raising the ratio of the bases to the power of the same exponent:

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

## Power rules

$$b^n \cdot b^m = b^{n+m}$$

$$\frac{b^n}{b^m} = b^{n-m}$$

$$(b^n)^m = b^{n \cdot m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$b^{-n} = \frac{1}{b^n}$$

$$m, n \in \mathbb{Z}, \quad b^1 = b, \quad b^0 = 1, \quad b \neq 0$$



## Power rules

### Rules:

$$1. \quad b^n \cdot b^m = b^{n+m}$$

$$2. \quad \frac{b^n}{b^m} = b^{n-m}$$

$$3. \quad (b^n)^m = b^{n \cdot m}$$

$$4. \quad a^n \cdot b^n = (a \cdot b)^n$$

$$5. \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$6. \quad b^{-n} = \frac{1}{b^n}$$

$$7. \quad b^0 = 1$$

### Examples:

$$2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 16$$

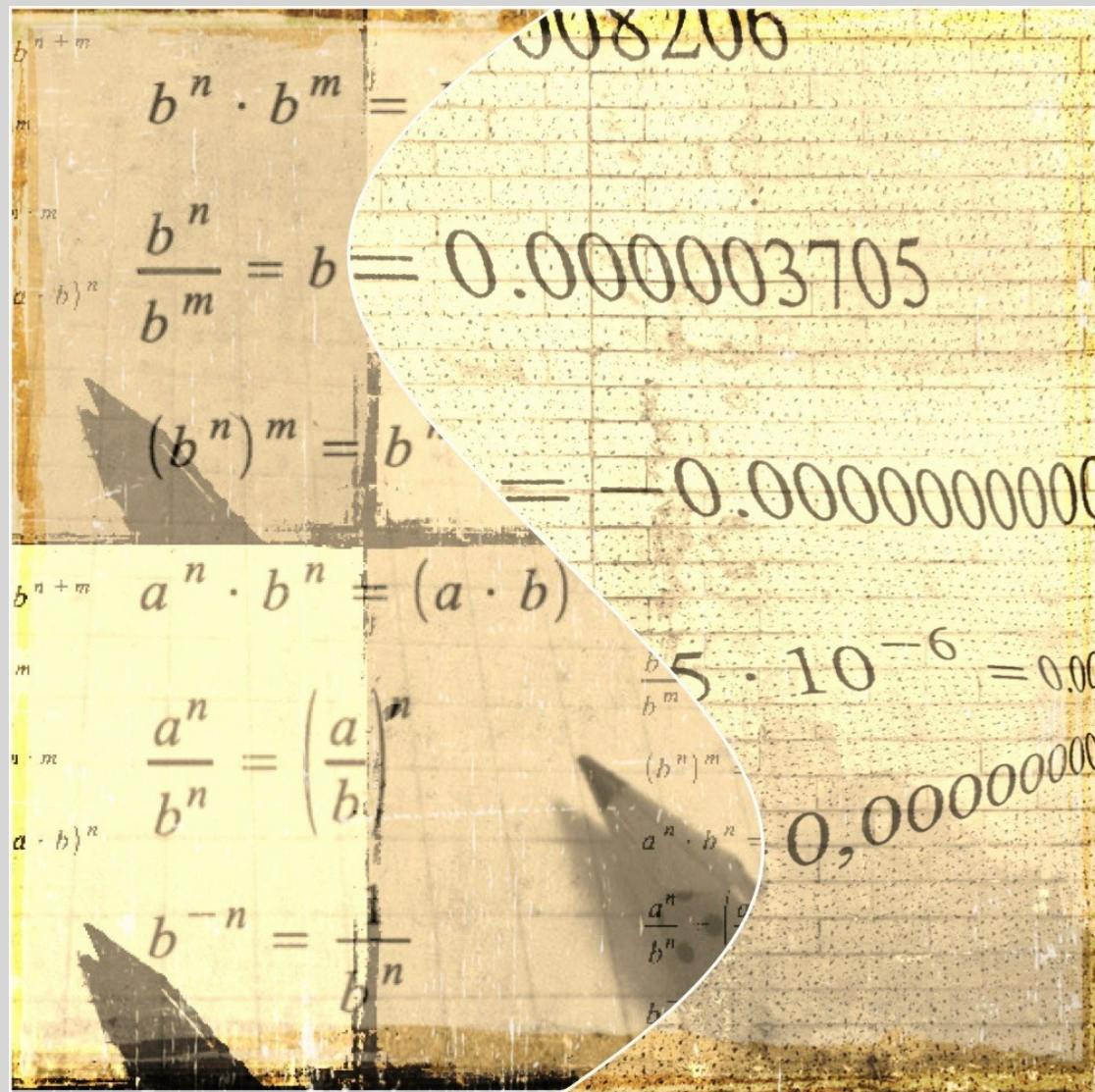
$$(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 729$$

$$2^2 \cdot 3^2 = (2 \cdot 3)^2 = 6^2 = 36$$

$$\frac{6^4}{3^4} = \left(\frac{6}{3}\right)^4 = 2^4 = 16$$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$(a^2 - 3)^0 = 1$$



*Powers with integer exponents: Exercises*

## Power rules: Exercises 18-20

Exercise 18: Simplify the expression:  $(a^3 b^2 c) \cdot (a b^3 c^5)$

Exercise 19:

Simplify the following expressions and determine their values for the given values of  $a$ ,  $b$  and  $c$ :

$$a) \left[ \frac{a^3 b^2 c^4}{a b c^{-2}} \right]_{a=1, b=-1, c=1}, \quad b) \left[ \frac{a^4 b^3 c^{-4}}{a^2 b^2 c^{-2}} \right]_{a=3, b=1/8, c=2}$$

Exercise 20:

Simplify the expressions and point to the rules you are using:

$$a) \frac{7 x^2 y^{-3}}{x^{-4} y^2}, \quad b) \left( \frac{5 x^3}{y^2} \right)^{-2}$$

## Power rules: Solution 18

First we write this expression in the form

$$(a^3 b^2 c) \cdot (a b^3 c^5) = (a^3 a) \cdot (b^2 b^3) \cdot (c c^5)$$

and multiply powers with equal bases:

$$a^3 \cdot a = a^{3+1} = a^4$$

$$b^2 \cdot b^3 = b^{2+3} = b^5$$

$$c \cdot c^5 = c^{1+5} = c^6$$

$$(a^3 b^2 c) \cdot (a b^3 c^5) = a^4 b^5 c^6$$

## Power rules: Solution 19

a)

$$\frac{a^3}{a} = a^{3-1} = a^2$$

$$\frac{b^2}{b} = b^{2-1} = b$$

$$\frac{c^4}{c^{-2}} = c^{4+2} = c^6$$

$$\frac{a^3 b^2 c^4}{a b c^{-2}} = \left( \frac{a^3}{a} \right) \left( \frac{b^2}{b} \right) \left( \frac{c^4}{c^{-2}} \right) = a^2 b c^6$$

$$\left[ \frac{a^3 b^2 c^4}{a b c^{-2}} \right]_{a=1, b=-1, c=1} = [a^2 b c^6]_{a=1, b=-1, c=1} = -1$$

$$\begin{aligned} b) \left[ \frac{a^4 b^3 c^{-4}}{a^2 b^2 c^{-2}} \right]_{a=3, b=1/8, c=2} &= \left[ \frac{a^2 b}{c^2} \right]_{a=3, b=1/8, c=2} = \\ &= 3^2 \cdot \left( \frac{1}{8} \right) \cdot \frac{1}{4} = \frac{9}{32} \simeq 0.281 \end{aligned}$$

## Power rules: Solution 20a

$$a) \frac{7 x^2 y^{-3}}{x^{-4} y^2} =$$

$$7 \cdot \frac{x^2}{x^{-4}} \cdot \frac{y^{-3}}{y^2} =$$

combine factors with same base

$$7 \cdot (x^2 x^4) \cdot \frac{1}{y^3 y^2} =$$

rule 6:  $b^{-n} = \frac{1}{b^n}$

$$\frac{7 x^6}{y^5} =$$

rule 1:  $b^n \cdot b^m = b^{n+m}$

$$\frac{7 x^2 y^{-3}}{x^{-4} y^2} = \frac{7 x^6}{y^5}$$

## Power rules: Solution 20b

$$b) \left( \frac{5x^3}{y^2} \right)^{-2} = \frac{5^{-2}(x^3)^{-2}}{(y^2)^{-2}} =$$

rule 4:  $a^n \cdot b^n = (a \cdot b)^n$

$$\frac{5^{-2}x^{-6}}{y^{-4}} =$$

rule 3:  $(b^n)^m = b^{n \cdot m}$

$$\frac{y^4}{5^2 x^6} =$$

rule 6:  $b^{-n} = \frac{1}{b^n}$

$$\frac{y^4}{25 x^6}$$

$$\left( \frac{5x^3}{y^2} \right)^{-2} = \frac{y^4}{25 x^6}$$

## Power rules: Exercises 21, 22

Exercise 21: Write the results in scientific notation:

$$a) (3.11 \cdot 10^3) \cdot (2.02 \cdot 10^4),$$

$$e) (7.03 \cdot 10^{-5}) \cdot (1.01 \cdot 10^{-12})$$

$$b) (1.08 \cdot 10^8) \cdot (4.78 \cdot 10^{11}),$$

$$f) (6.83 \cdot 10^{-4}) \cdot (4.52 \cdot 10^7)$$

$$c) (6.98 \cdot 10^4) \cdot (7.18 \cdot 10^5),$$

$$g) (2.0 \cdot 10^4)^3, \quad (3.0 \cdot 10^6)^3$$

$$d) (3.24 \cdot 10^8) \cdot (1.25 \cdot 10^{-15}),$$

$$h) (4.1 \cdot 10^{-3})^2, \quad (5.0 \cdot 10^{-4})^3$$

Exercise 22: Write the results in scientific notation:

$$a) \frac{4.0 \cdot 10^{12}}{3.0 \cdot 10^3},$$

$$b) \frac{2.01 \cdot 10^{11}}{5.25 \cdot 10^4},$$

$$c) \frac{1.1 \cdot 10^{-4}}{4.4 \cdot 10^{-9}}$$

$$d) \frac{(2.4 \cdot 10^7) \cdot (3.4 \cdot 10^{-5})}{8.3 \cdot 10^{-6}},$$

$$e) \frac{(5.51 \cdot 10^{11}) \cdot (7.73 \cdot 10^{-13})}{2.08 \cdot 10^{-9}}$$



## *Multiplication of powers: Solution 21*

$$a) (3.11 \cdot 10^3) \cdot (2.02 \cdot 10^4) = 3.11 \cdot 2.02 \cdot 10^7 = 6.2822 \cdot 10^7 \simeq 6.28 \cdot 10^7$$

$$b) (1.08 \cdot 10^8) \cdot (4.78 \cdot 10^{11}) = 1.08 \cdot 4.78 \cdot 10^{19} = 5.1624 \cdot 10^{19} \simeq 5.16 \cdot 10^{19}$$

$$c) (6.98 \cdot 10^4) \cdot (7.18 \cdot 10^5) = 50.1164 \cdot 10^9 = 5.01164 \cdot 10^{10} \simeq 5.01 \cdot 10^{10}$$

$$d) (3.24 \cdot 10^8) \cdot (1.25 \cdot 10^{-15}) = 4.05 \cdot 10^{-7}$$

$$e) (7.03 \cdot 10^{-5}) \cdot (1.01 \cdot 10^{-12}) = 7.1003 \cdot 10^{-17} \simeq 7.1 \cdot 10^{-17}$$

$$f) (6.83 \cdot 10^{-4}) \cdot (4.52 \cdot 10^7) = 30.8716 \cdot 10^3 = 3.08716 \cdot 10^4 \simeq 3.09 \cdot 10^4$$

$$g) (2.0 \cdot 10^4)^3 = 8 \cdot 10^{12}, \quad (3.0 \cdot 10^6)^3 = 27 \cdot 10^{18} = 2.7 \cdot 10^{19}$$

$$h) (4.1 \cdot 10^{-3})^2 = 16.81 \cdot 10^{-6} = 1.681 \cdot 10^{-7} \simeq 1.68 \cdot 10^{-7}$$

$$(5.0 \cdot 10^{-4})^3 = 25 \cdot 10^{-12} = 2.5 \cdot 10^{-11}$$

## Multiplication of powers: Solution 22

$$a) \frac{4.0 \cdot 10^{12}}{3.0 \cdot 10^3} = \frac{4}{3} \cdot 10^9 \simeq 1.33 \cdot 10^9,$$

$$b) \frac{2.01 \cdot 10^{11}}{5.25 \cdot 10^4} \simeq 0.383 \cdot 10^7 = 3.83 \cdot 10^6$$

$$c) \frac{1.1 \cdot 10^{-4}}{4.4 \cdot 10^{-9}} = \frac{1}{4} \cdot 10^5 = 0.25 \cdot 10^5 = 2.5 \cdot 10^4$$

$$d) \frac{(2.4 \cdot 10^7) \cdot (3.4 \cdot 10^{-5})}{8.3 \cdot 10^{-6}} = \frac{2.4 \cdot 3.4}{8.3} \cdot 10^8 = 0.9831 \cdot 10^8 = 9.831 \cdot 10^7$$

$$e) \frac{(5.51 \cdot 10^{11}) \cdot (7.73 \cdot 10^{-13})}{2.08 \cdot 10^{-9}} = \frac{5.51 \cdot 7.73}{2.08} \cdot 10^7 = 20.477 \cdot 10^8 = 2.048 \cdot 10^9$$

## Multiplication of powers: Exercise 23



Simplify the expressions:

$$a) \quad 4 y^{-2} \cdot x^{2+n} \cdot y \cdot x \cdot y^{3n}$$

$$b) \quad a^2 \cdot b^{2-n} \cdot a^5 \cdot b^{3n+4}$$

$$c) \quad 2^{-3} \cdot 3^{2+n} \cdot 2^4 \cdot 3$$

$$d) \quad 7^{12} \cdot 5^3 \cdot 7^{-6} \cdot 5^{3n-4}$$

## Multiplication of powers: Solution 23

$$a) \quad 4y^{-2} \cdot x^{2+n} \cdot y \cdot x \cdot y^{3n} = 4y^{-2+1+3n} \cdot x^{2+n+1} = 4y^{3n-1} \cdot x^{n+3}$$

$$b) \quad a^2 \cdot b^{2-n} \cdot a^5 \cdot b^{3n+4} = a^{2+5} \cdot b^{2-n+3n+4} = a^7 \cdot b^{6+2n}$$

$$c) \quad 2^{-3} \cdot 3^{2+n} \cdot 2^4 \cdot 3 = 2^{-3+4} \cdot 3^{2+n+1} = 2 \cdot 3^{3+n}$$

$$d) \quad 7^{12} \cdot 5^3 \cdot 7^{-6} \cdot 5^{3n-4} = 7^{12-6} \cdot 5^{3+3n-4} = 7^6 \cdot 5^{3n-1}$$

## Powers with integer exponents: Exercise 24



The given expressions are to be presented with positive exponents

Examples:

$$\frac{1}{a^{-3}} = a^{(-3)(-1)} = a^3, \quad a^2 \cdot b^{-3} = \frac{a^2}{b^3}$$

$$a) \quad a^{-4}, \quad 3a^{-3}b, \quad 5a \cdot b^{-4}c$$

$$b) \quad 7b^{-4} \cdot d^4, \quad 2^{-1}x^{-1} \cdot y^{-1}, \quad 3a^{-1} \cdot b^{-2} \cdot c^3$$

$$c) \quad \frac{a^2}{b^{-4}}, \quad \frac{4x^4}{x^{-5}}, \quad \frac{3x^2}{y^{-3}}$$

## Powers with integer exponents: Solution 24

$$a) \quad a^{-4} = \frac{1}{a^4}, \quad 3 a^{-3} b = \frac{3 b}{a^3}, \quad 5 a \cdot b^{-4} c = \frac{5 a c}{b^4}$$

$$b) \quad 7 b^{-4} \cdot d^4 = \frac{7 d^4}{b^4}, \quad 2^{-1} x^{-1} \cdot y^{-1} = \frac{1}{2 x^1 \cdot y^1}$$

$$3 a^{-1} \cdot b^{-2} \cdot c^3 = \frac{3 c^3}{a \cdot b^2}$$

$$c) \quad \frac{a^2}{b^{-4}} = a^2 \cdot b^{(-4)(-1)} = a^2 \cdot b^4$$

$$\frac{4 x^4}{x^{-5}} = 4 x^4 \cdot x^5 = 4 x^9, \quad \frac{3 x^2}{y^{-3}} = 3 x^2 y^3$$

## Powers with integer exponents: Exercise 25



The given expressions are to be presented with positive exponents

$$a) \quad \frac{a^{-5}}{a^{-3}}, \quad \frac{a^2}{a^{-5} \cdot b^{-4}}, \quad \frac{4x^4 \cdot y^2}{x^{-5} \cdot y^{-3}}$$

$$b) \quad (a + b)^{-1}, \quad 3^{-2} (x + y)^2, \quad x^{-1} + y^{-1}$$

$$c) \quad \frac{6y^{-1}}{4x^{-2}}, \quad ab \frac{25}{5a^{-1}b^{-1}}, \quad \frac{16x^3y^3}{x^{-1}} 2^{-4} y^3$$

$$25x^0 \frac{y^{-1}}{5x^{-1}} 5^{-1} y^2$$

## Powers with integer exponents: Solution 25

$$a) \quad \frac{a^{-5}}{a^{-3}} = \frac{1}{a^{-3} \cdot a^5} = \frac{1}{a^{5-3}} = \frac{1}{a^2}$$

$$\frac{a^2}{a^{-5} \cdot b^{-4}} = a^2 \cdot a^{(-5)(-1)} \cdot b^{(-4)(-1)} = a^7 \cdot b^4$$

$$\begin{aligned} \frac{4x^4 \cdot y^2}{x^{-5} \cdot y^{-3}} &= 4x^4 \cdot y^2 \cdot x^{(-5)(-1)} \cdot y^{(-3)(-1)} = 4x^4 \cdot y^2 \cdot x^5 \cdot y^3 = \\ &= 4x^{4+5} \cdot y^{2+3} = 4x^9 \cdot y^5 \end{aligned}$$

$$b) \quad (a + b)^{-1} = \frac{1}{a + b}, \quad 3^{-2} (x + y)^2 = \frac{(x + y)^2}{3^2} = \frac{(x + y)^2}{9}$$

$$x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy}$$



$$c) \quad \frac{6 y^{-1}}{4 x^{-2}} = \frac{3 x^2}{2 y}$$

$$a b \frac{25}{5 a^{-1} b^{-1}} = a b \frac{5}{(a b)^{-1}} = 5 (a b)^2 = 5 a^2 b^2$$

$$\frac{16 x^3 y^3}{x^{-1}} 2^{-4} y^3 = \frac{16}{2^4} x^{3-(-1)} y^{3+3} = \frac{16}{2^4} x^4 y^6 = x^4 y^6$$

$$25 x^0 \frac{y^{-1}}{5 x^{-1}} 5^{-1} y^2 = 25 \frac{x^{0-(-1)} y^{2-1}}{5 \cdot 5} = 25 \frac{x y}{5 \cdot 5} = x y$$

## Powers with integer exponents: Exercise 26



The given expressions are to be presented with positive exponents

$$a) \frac{x^3 y^3 z^4}{x^{-1} y^3 z^{-4}}$$

$$b) \frac{a^{-3} b^{-2} c}{a^{-6} b^{-4} c^{-3}}$$

$$c) \frac{x^{-3} y^0 z^2}{x^4 y^{-3} z^{-2}}$$

$$d) \frac{3^5 2^{-3} 10^{-2}}{4^{-6} 5^3 6^{-1}}$$

$$e) \frac{5^3 (-3)^5 2^2}{(-6)^{-2} 2^{-3}}$$

## Powers with integer exponents: Solution 26

$$a) \frac{x^3 y^3 z^4}{x^{-1} y^3 z^{-4}} = x^{3-(-1)} y^{3-3} z^{4-(-4)} = x^4 z^8$$

$$b) \frac{a^{-3} b^{-2} c}{a^{-6} b^{-4} c^{-3}} = a^{-3-(-6)} b^{-2-(-4)} c^{1-(-3)} = a^3 b^2 c^4$$

$$c) \frac{x^{-3} y^0 z^2}{x^4 y^{-3} z^{-2}} = x^{-3-4} y^{0-(-3)} z^{2-(-2)} = \frac{y^3 z^4}{x^7}$$

$$d) \frac{3^5 2^{-3} 10^{-2}}{4^{-6} 5^3 6^{-1}} = \frac{3^5 2^{-3} (2 \cdot 5)^{-2}}{(2^2)^{-6} 5^3 (2 \cdot 3)^{-1}} = \frac{3^5 2^{-5} 5^{-2}}{2^{-13} 5^3 3^{-1}} = \frac{3^6 2^8}{5^5} = \frac{186\,624}{3125}$$

$$\begin{aligned} e) \frac{5^3 (-3)^5 2^2}{(-6)^{-2} 2^{-3}} &= 5^3 (-3)^5 2^2 (-6)^2 2^3 = -5^3 3^5 2^2 6^2 2^3 = \\ &= -5^3 3^5 2^2 (2 \cdot 3)^2 2^3 = -2^7 3^7 5^3 = \\ &= -34\,992\,000 \end{aligned}$$



The given expressions are to be presented with positive exponents

$$a) \frac{2 a^2 b^{-2}}{3 b} \div \frac{a^{-2}}{3^{-2} b^3}$$

$$b) \frac{2^2 a^3}{b x} \div \frac{4 x^5 b^{-1} a}{a^{-4} x^6}$$

$$c) \frac{2^3 3^2}{5 x y^{-2}} \div \frac{9 x^{-2} y 5^{-1}}{8^{-1} y^{-1}}$$

$$d) \left( \frac{25 x y}{36 y^{-4} z^3} \div \frac{x^3 y^3}{81 z} \right) \cdot \frac{4 x^2 z^2}{225 y^2}$$

## Powers with integer exponents: Solution 27

$$\begin{aligned} a) \quad \frac{2a^2b^{-2}}{3b} \div \frac{a^{-2}}{3^{-2}b^3} &= \frac{2a^2b^{-2}}{3b} \cdot \left( \frac{a^{-2}}{3^{-2}b^3} \right)^{-1} = \frac{2a^2b^{-2}}{3b} \cdot \frac{a^2}{3^2b^{-3}} = \\ &= \frac{2a^4}{3^3} = \frac{2a^4}{27} \end{aligned}$$

$$\begin{aligned} b) \quad \frac{2^2a^3}{bx} \div \frac{4x^5b^{-1}a}{a^{-4}x^6} &= \frac{2^2a^3}{bx} \cdot \left( \frac{4x^5b^{-1}a}{a^{-4}x^6} \right)^{-1} = \frac{2^2a^3}{bx} \cdot \left( \frac{2^2a^5}{bx} \right)^{-1} = \\ &= \frac{2^2a^3}{bx} \cdot \frac{bx}{2^2a^5} = \frac{1}{a^2} \end{aligned}$$

$$c) \quad \frac{2^33^2}{5xy^{-2}} \div \frac{9x^{-2}y5^{-1}}{8^{-1}y^{-1}} = \frac{2^33^2}{5xy^{-2}} \cdot \frac{9^{-1}x^2y^{-1}5}{8y} = x$$

$$d) \quad \left( \frac{25xy}{36y^{-4}z^3} \div \frac{x^3y^3}{81z} \right) \cdot \frac{4x^2z^2}{225y^2} = 1$$



The given expressions are to be presented with positive exponents

a)  $(a^{-1})^0$ ,  $(a^0)^{-1}$

b)  $(b^{-2})^3$ ,  $(a b^{-3})^2$

c)  $(-x^2)^{-3}$ ,  $(-2 y)^4$

d)  $(-2^2 x)^{-1}$ ,  $(-3^{-1})^2$

## *Powers with integer exponents: Solution 28*

$$a) \quad (a^{-1})^0 = 1, \quad (a^0)^{-1} = 1$$

$$b) \quad (b^{-2})^3 = \frac{1}{b^6}, \quad (a b^{-3})^2 = \frac{a^2}{b^6}$$

$$c) \quad (-x^2)^{-3} = -\frac{1}{x^6}, \quad (-2y)^4 = 16y^4$$

$$d) \quad (-2^2 x)^{-1} = \frac{1}{(-2^2 x)} = -\frac{1}{4x}$$

$$(-3^{-1})^2 = (3^{-1})^2 = 3^{-2} = \frac{1}{9}$$

## Powers with integer exponents: Exercise 29



Calculate  $A$ , that makes an expression true.

$$a) \quad x^7 y^3 z^2 = x^2 y^2 z^3 A$$

$$b) \quad x^2 y^{-1} z^4 = x^2 y^{-3} z^{-3} A$$

$$c) \quad 3x^2 y^2 = \frac{x^3 y^5 z^2}{A}$$

$$d) \quad 27 \frac{a^2}{b^2 c} = \frac{a^4 b^{-3} c^2}{A}$$

$$e) \quad (x^2 y^{-2})^3 = x^{-3} y^{-3} z^{-3} A$$

$$f) \quad (s^{-1} u^{-2})^{-2} = s^{-2} u^3 \frac{A}{t}$$



## Powers with integer exponents: Solution 29

$$a) \quad x^7 y^3 z^2 = x^2 y^2 z^3 A, \quad A = \frac{x^7 y^3 z^2}{x^2 y^2 z^3} = \frac{x^5 y}{z}$$

$$b) \quad x^2 y^{-1} z^4 = x^2 y^{-3} z^{-3} A, \quad A = \frac{x^2 y^{-1} z^4}{x^2 y^{-3} z^{-3}} = y^2 z^7$$

$$c) \quad 3x^2 y^2 = \frac{x^3 y^5 z^2}{A}, \quad A = \frac{x^3 y^5 z^2}{3x^2 y^2} = \frac{1}{3} x y^3 z^2$$

$$d) \quad 27 \frac{a^2}{b^2 c} = \frac{a^4 b^{-3} c^2}{A}, \quad A = a^4 b^{-3} c^2 \frac{b^2 c}{27 a^2} = \frac{a^2 c^3}{27 b}$$

$$e) \quad (x^2 y^{-2})^3 = x^{-3} y^{-3} z^{-3} A, \quad x^6 y^{-6} = x^{-3} y^{-3} z^{-3} A$$

$$A = \frac{x^6 y^{-6}}{x^{-3} y^{-3} z^{-3}} = \frac{x^9 z^3}{y^3}$$

$$f) \quad (s^{-1} u^{-2})^{-2} = s^{-2} u^3 \frac{A}{t}, \quad s^2 u^4 = s^{-2} u^3 t^{-1} A$$

$$A = \frac{s^2 u^4}{s^{-2} u^3 t^{-1}} = s^4 u t$$

