Simplification of Fractions

Try to get rid of the negative exponents in the fractions:

$$a)\quad \frac{a^{-m} \cdot b^{-n}}{c^{-p} \cdot d^{-q}}, \qquad b)\quad \frac{x^{-2} + y^{-3}}{x^{-3} - y^{-2}}$$

Hint:

$$\boxed{a^{-n} = \frac{1}{a^{n}}}$$

Rule, how to simplify fractions:

Powers with negative exponents which are factors in the numerator (denominator) can be moved with the corresponding positive exponent to the denominator (numerator).

There are no simple rules for powers with negative exponents when they appear as addends in fractions.

a)
$$\frac{a^{-m}\cdot b^{-n}}{c^{-p}\cdot d^{-q}} = \frac{\dfrac{1}{a^m}\cdot\dfrac{1}{b^n}}{\dfrac{1}{c^p}\cdot\dfrac{1}{d^q}} = \frac{\dfrac{1}{a^m\cdot b^n}}{\dfrac{1}{c^p\cdot d^q}} = \frac{c^p\cdot d^q}{a^m\cdot b^n}$$

Compound fractions can be removed as follows:

$$\frac{1}{\dfrac{1}{a}} = a\,, \qquad \frac{1}{\dfrac{1}{a^n}} = a^n$$

b) We first write the powers with negative exponents as fractions:

$$\frac{x^{-2}+y^{-3}}{x^{-3}-y^{-2}} = \frac{\dfrac{1}{x^2}+\dfrac{1}{y^3}}{\dfrac{1}{x^3}-\dfrac{1}{y^2}} = \frac{\dfrac{y^3+x^2}{x^2\,y^3}}{\dfrac{y^2-x^3}{x^3\,y^2}} = \frac{(y^3+x^2)}{x^2\,y^3}\cdot\frac{x^3\,y^2}{(y^2-x^3)} =$$

$$= \frac{x\,(y^3+x^2)}{y\,(y^2-x^3)}$$

a) the fraction $\left(\dfrac{2\,x^{-2} \cdot y^{3}}{3\,u^{-4} \cdot v^{n}} \right)^{-7}$

is to be simplified, such that no negative exponents remain.

b) the expression $\left[\left[(2\,a^{0} + 3\,b^{2})^{3} \right]^{0} \right]^{-6}$

should be simplified as much as possible

$$\left(\frac{2\,x^{-2}\cdot\,y^{3}}{3\,u^{-4}\cdot\,v^{n}}\right)^{-7}$$

There are several approaches:

One can first remove the negative exponent -7 by raising the inverse fraction to the seventh power. We then get

$$\left(\frac{2\cdot x^{-2}\cdot y^{3}}{3\cdot u^{-4}\cdot v^{n}}\right)^{-7} = \left(\frac{3\cdot u^{-4}\cdot v^{n}}{2\cdot x^{-2}\cdot y^{3}}\right)^{7} = \left(\frac{3\cdot x^{2}\cdot v^{n}}{2\cdot u^{4}\cdot y^{3}}\right)^{7} = \frac{3^{7}\cdot x^{14}\cdot v^{7n}}{2^{7}\cdot u^{28}\cdot y^{21}}$$

Different approach:

$$\left(\frac{2\cdot x^{-2}\cdot y^{3}}{3\cdot u^{-4}\cdot v^{n}}\right)^{-7} = \frac{2^{-7}\cdot x^{14}\cdot y^{-21}}{3^{-7}\cdot u^{28}\cdot v^{-7n}} = \frac{3^{7}\cdot x^{14}\cdot v^{7n}}{2^{7}\cdot u^{28}\cdot y^{21}}$$

Powers: solution 2b

$$\left[\left[(2\,a^0 + 3\,b^2)^3\right]^0\right]^{-6}$$

One can work through these nested brackets from inside to outside or the other way round. In the first case one works out the cube and raises the result to the power of zero. This leads to 1 independent of the previous result.

The result could have been written down immediately considering the exponent zero which leads to 1 whatever is in the bracket.

Write the following expressions with positive exponents only:

a) $\left(\frac{a}{b}\right)^{-1}, \quad \left(\frac{1}{2}\right)^{-3}, \quad \left(\frac{3}{4}\right)^{-2}, \quad \left(\frac{3\,x^{-1}}{y}\right)^{-3}$

b) $\left(\frac{2}{3}\right)^{-2}\left(\frac{9}{4}\right)^{-3}, \quad \left(-\frac{2}{3}\right)^{-2}\left(\frac{4}{9}\right)^{-1}, \quad \left(\frac{1}{6}\right)^{-4}\left(\frac{2}{5}\right)^{5}$

c) $\left(\frac{x}{y}\right)^{-5}\left(\frac{y}{x}\right)^{-5}, \quad \left(\frac{2\,a^2}{3}\right)^{-3}\left(\frac{6}{5\,a^3}\right)^{2}$

d) $\left(\frac{3\,x^{-2}}{2\,y}\right)^{3}\left(\frac{3\,x}{y}\right)^{-1}, \quad \left(-\frac{4\,x^3}{3^2}\right)^{-3}\left(\frac{2\,x^2}{3}\right)^{5}$

e) $\left(\frac{3^2\,x}{4\,a}\right)^{-4} : \left(\frac{16 \cdot 3^{-1}\,x^{-2}}{-a^{-2}}\right)^{2}$

f) $\frac{(a+b)^0}{(a+b)^{-1}}, \quad \frac{a+b}{a^{-1}+b^{-1}}$

a) $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}, \qquad \left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^3 = 8$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{16}{9}, \qquad \left(\frac{3\,x^{-1}}{y}\right)^{-3} = \left(\frac{3}{x\,y}\right)^{-3} = \left(\frac{x\,y}{3}\right)^{3} = \frac{x^3\,y^3}{27}$$

b) $\left(\frac{2}{3}\right)^{-2}\left(\frac{9}{4}\right)^{-3} = \left(\frac{3}{2}\right)^{2}\left(\frac{2}{3}\right)^{6} = \frac{16}{81}$

$$\left(-\frac{2}{3}\right)^{-2}\left(\frac{4}{9}\right)^{-1} = \frac{3^4}{2^4} = \frac{81}{16}, \qquad \left(\frac{1}{6}\right)^{-4}\left(\frac{2}{5}\right)^{5} = \frac{2^9 \cdot 3^4}{5^5} = \frac{41472}{3125}$$

c) $\left(\frac{x}{y}\right)^{-5}\left(\frac{y}{x}\right)^{-5} = \left(\frac{x}{y}\cdot\frac{y}{x}\right)^{-5} = 1^{-5} = 1$

$$\left(\frac{2\,a^2}{3}\right)^{-3}\left(\frac{6}{5\,a^3}\right)^{2} = \left(\frac{3}{2\,a^2}\right)^{3}\left(\frac{6}{5\,a^3}\right)^{2} = \frac{3^3\,(2\cdot 3)^2}{2^3\,5^2\,a^{12}} = \frac{3^5\,2^2}{2^3\,5^2\,a^{12}} =$$

$$= \frac{3^5}{2\cdot 5^2\,a^{12}} = \frac{243}{50\,a^{12}}$$

$$d)\quad \left(\frac{3\,x^{-2}}{2\,y}\right)^{3}\left(\frac{3\,x}{y}\right)^{-1} = \frac{3^2}{2^3\;y^2\;x^7} = \frac{9}{8\;x^7\;y^2}$$

$$\left(-\frac{4\,x^3}{3^2}\right)^{-3}\left(\frac{2\,x^2}{3}\right)^{5} = -\frac{3\,x}{2}$$

$$e)\quad \left(\frac{3^2\,x}{4\,a}\right)^{-4}\cdot\left(\frac{16\cdot 3^{-1}\;x^{-2}}{-a^{-2}}\right)^{-2} = \left(\frac{4\,a}{3^2\,x}\right)^{4}\cdot\left(-\frac{a^{-2}}{16\cdot 3^{-1}\;x^{-2}}\right)^{2} = \frac{1}{3^6} = \frac{1}{729}$$

$$f)\quad \frac{(a+b)^0}{(a+b)^{-1}} = \frac{1}{(a+b)^{-1}} = a+b$$

$$\frac{a+b}{a^{-1}+b^{-1}} = \frac{a+b}{\frac{1}{a}+\frac{1}{b}} = \frac{a+b}{\frac{a+b}{a\,b}} = a\,b$$