



Factorisation of Polynomials: Exercises

Factoring Polynomials: Exercises 1-6



Factorise the following expressions:

Exercise 1: $a^2 - 2a^3b - 2ab^3 + b^2$

Exercise 2: $a^4 - 5a^3 - 8a + 40$

Exercise 3: $a^4 - 10a^2 + 169$

Exercise 4: $2x^4 - 2x^2 - 24$

Exercise 5: $-x^6 - 4x^3 + 21$

Exercise 6: $(x^2 - x - 1)^2 - 3(x^2 - x - 1) + 2$

Factoring Polynomials: Solutions 1-3

Solution 1:

$$\begin{aligned}a^2 - 2a^3b - 2ab^3 + b^2 &= a^2(1 - 2ab) + b^2(1 - 2ab) = \\ &= (1 - 2ab)(a^2 + b^2)\end{aligned}$$

Solution 2:

$$\begin{aligned}a^4 - 5a^3 - 8a + 40 &= a^3(a - 5) - 8(a - 5) = (a - 5)(a^3 - 8) = \\ &= (a - 5)(a^3 - 2^3) = (a - 5)(a - 2)(a^2 + 2a + 4)\end{aligned}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Solution 3:

$$\begin{aligned}a^4 - 10a^2 + 169 : a^4 + 169 &= (a^2)^2 + (13)^2 \\ a^4 + 169 &= (a^2)^2 + (13)^2 + 2 \cdot 13 a^2 - 2 \cdot 13 a^2 = (a^2 + 13)^2 - 26a^2 \\ a^4 - 10a^2 + 169 &= (a^2 + 13)^2 - 26a^2 - 10a^2 = (a^2 + 13)^2 - 36a^2 = \\ &= (a^2 - 6a + 13)(a^2 + 6a + 13)\end{aligned}$$

Factoring Polynomials: Solution 4-5

Solution 4:

$$2x^4 - 2x^2 - 24 = 2y^2 - 2y - 24, \quad y = x^2$$

$$2y^2 - 2y - 24 = 2(y - y_1)(y - y_2) = 2(y - 4)(y + 3)$$

$$y_1 = 4, \quad y_2 = -3 \quad \text{-- solutions of the equation}$$

$$2y^2 - 2y - 24 = 0$$

$$\begin{aligned} 2x^4 - 2x^2 - 24 &= 2(y - 4)(y + 3) = 2(x^2 - 4)(x^2 + 3) = \\ &= 2(x - 2)(x + 2)(x^2 + 3) \end{aligned}$$

$$a^2 - b^2 = (a - b)(a + b)$$

Solution 5:

$$\begin{aligned} -x^6 - 4x^3 + 21 &= -y^2 - 4y + 21 = \quad (y = x^3) \\ &= -(y - y_1)(y - y_2) = -(y - 3)(y + 7) = \\ &= -(x^3 - 3)(x^3 + 7) \end{aligned}$$

Factoring Polynomials: Solution 6

$$(x^2 - x - 1)^2 - 3(x^2 - x - 1) + 2 \equiv y^2 - 3y + 2 =$$

$$y = x^2 - x - 1$$

$$= (y - y_1)(y - y_2) = (y - 1)(y - 2) =$$

$$= (x^2 - x - 1 - 1)(x^2 - x - 1 - 2) = (x^2 - x - 2)(x^2 - x - 3)$$

$$x^2 - x - 2 = (x - 2)(x + 1)$$

$$x^2 - x - 3 = (x - x_1)(x + x_2)$$

$$x_1 = \frac{1}{2} + \frac{\sqrt{13}}{2}, \quad x_2 = \frac{1}{2} - \frac{\sqrt{13}}{2}$$

$$(x^2 - x - 1)^2 - 3(x^2 - x - 1) + 2 =$$

$$= (x - 2)(x + 1)(x^2 - x - 3)$$

Factoring Polynomials: Exercises 7-10

Exercise 7: $x^4 - a^2 x^2 - x^2 + a^2$

Exercise 8: $x^3 - (a - 1)x + a$

Exercise 9: $a^5 - 4a^3 + 8a^2 - 32$

Exercise 10: $x^5 + x^3 + x$

Factoring Polynomials: Solutions 7-9

solution 7:

$$\begin{aligned}x^4 - a^2 x^2 - x^2 + a^2 &= x^2 (x^2 - 1) - a^2 (x^2 - 1) = (x^2 - 1) (x^2 - a^2) = \\ &= (x - 1) (x + 1) (x - a) (x + a)\end{aligned}$$

solution 8:

$$\begin{aligned}x^3 - (a - 1)x + a &= x^3 - ax - x + a = x(x^2 - 1) - a(x - 1) = \\ &= x(x - 1)(x + 1) - a(x - 1) = (x - 1)(x(x - 1) - a)\end{aligned}$$

solution 9:

$$\begin{aligned}a^5 - 4a^3 + 8a^2 - 32 &= a^3(a^2 - 4) + 8(a^2 - 4) = (a^2 - 4)(a^3 + 8) = \\ &= (a^2 - 4)(a^3 + 8) = (a - 2)(a + 2)(a^3 + 8) =\end{aligned}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\begin{aligned}a^3 + 8 &= a^3 + 2^3 = (a + 2)(a^2 - 2a + 4) \\ &= (a - 2)(a + 2)^2(a^2 - 2a + 4)\end{aligned}$$

Factoring Polynomials: Solution 10

Solution 10: $x^5 + x^3 + x = x(x^4 + x^2 + 1)$

$$x^4 + x^2 + 1 = x^2(x^2 + 1) + 1 = x^2(x^2 + 1) + 1 + x^2 - x^2 =$$

$$= (x^2 + 1)^2 - x^2 = (x^2 + 1 - x)(x^2 + 1 + x) =$$

$$= (x^2 - x + 1)(x^2 + x + 1)$$

$$x^5 + x^3 + x = x(x^2 - x + 1)(x^2 + x + 1)$$