Intersections of Quadratic and Linear Functions

Exercises

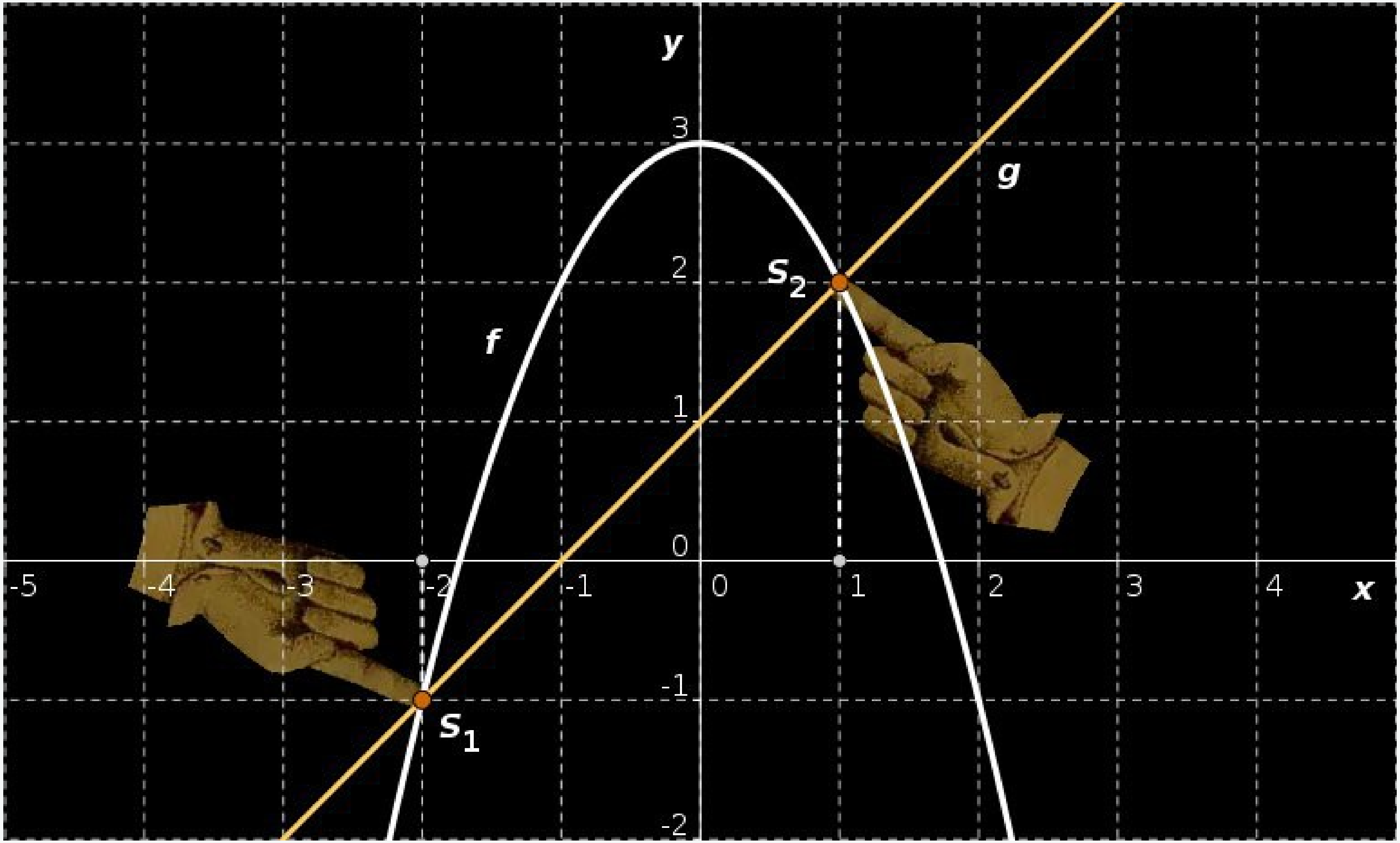

Fig. 1-1: The quadratic function y = f (x) and linear function y = g (x) have two intersections in the common domain.

$$f\,(x) = -x^2 + 3, \qquad g\,(x) = x + 1$$

Definition: An intersection (intercept) is a common point of two curves.

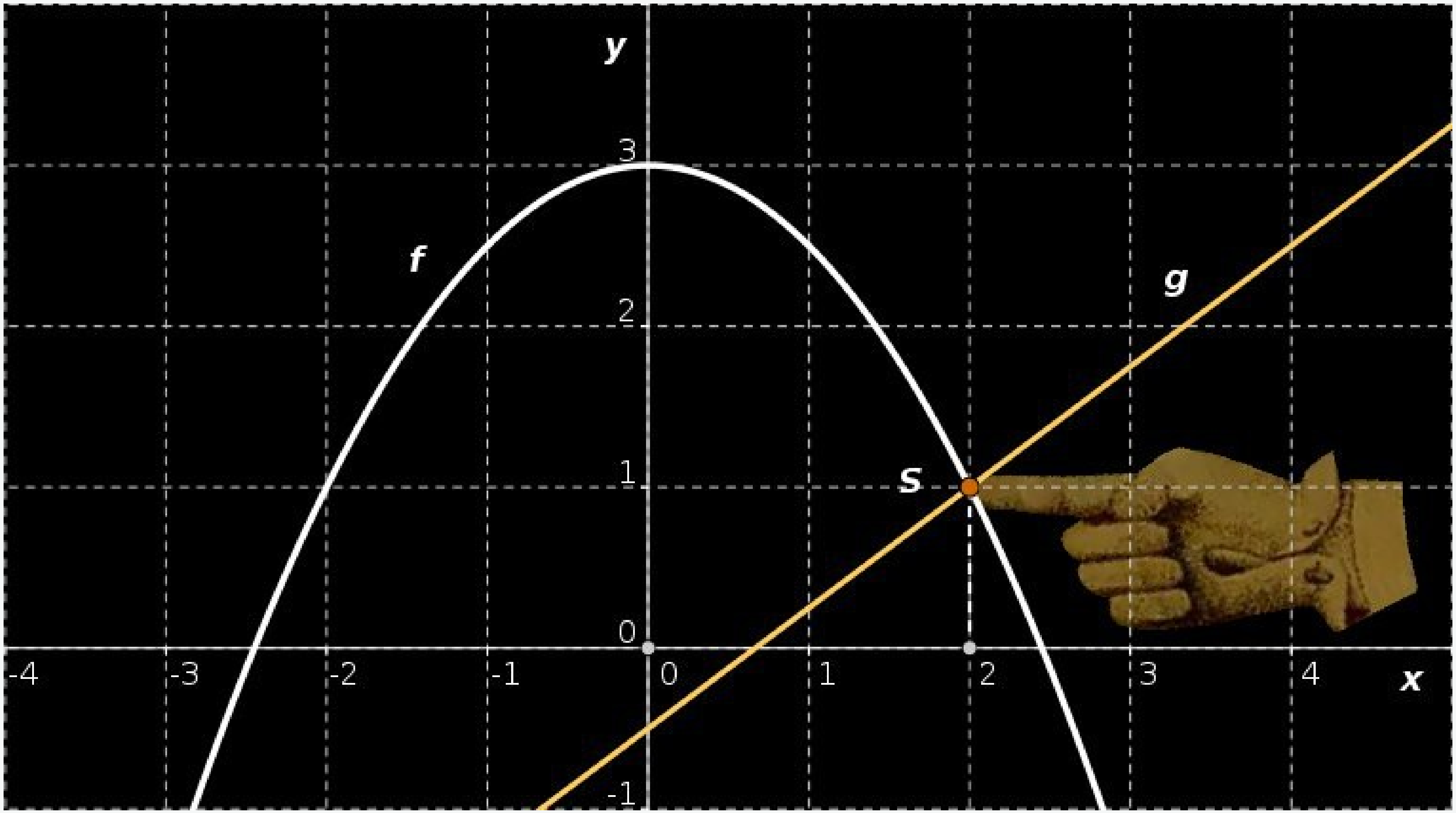

Fig. 1-2: The quadratic function $y = f(x)$ and linear function $y = g(x)$ have one intersections S in the positive domain

$$f(x) = -\frac{x^2}{2} + 3, \qquad g(x) = \frac{3}{4}\,x - \frac{1}{2}$$

Sometimes intersections of curves have to be determined in some section of the domain.

Determine the intersections of a quadratic function $y = f(x)$ and a linear function $y = g(x)$ or a function $x = c$ $(c = const)$

Exercise 1: $f(x) = -x^2 + 2, \quad g(x) = 1$

Exercise 2: $f(x) = -x^2 + 2, \quad g(x) = -1$

Exercise 3: $f(x) = -x^2 + 2, \quad g(x) = x$

Exercise 4: $f(x) = -x^2 + 2, \quad x = 1$

Exercise 5: $f(x) = -x^2 + 2, \quad g(x) = 3$

Intersections of quadratic and linear functions: Solution 1

The functions of the parabola and of the straight line are:

$$f(x) = -x^2 + 2, \qquad g(x) = 1$$

The intersections are obtained by demanding equal output of the functions for some input x

$$f(x) = g(x) \qquad \Leftrightarrow \qquad -x^2 + 2 = 1$$

and by solving the resulting quadratic equation:

$$-x^2 + 1 = 0, \qquad x^2 - 1 = 0, \qquad (x - 1)\cdot(x + 1) = 0$$

$$x_1 = -1, \quad x_2 = 1 \qquad \Rightarrow \qquad S_1 = (-1,\ 1), \quad S_2 = (1,\ 1)$$

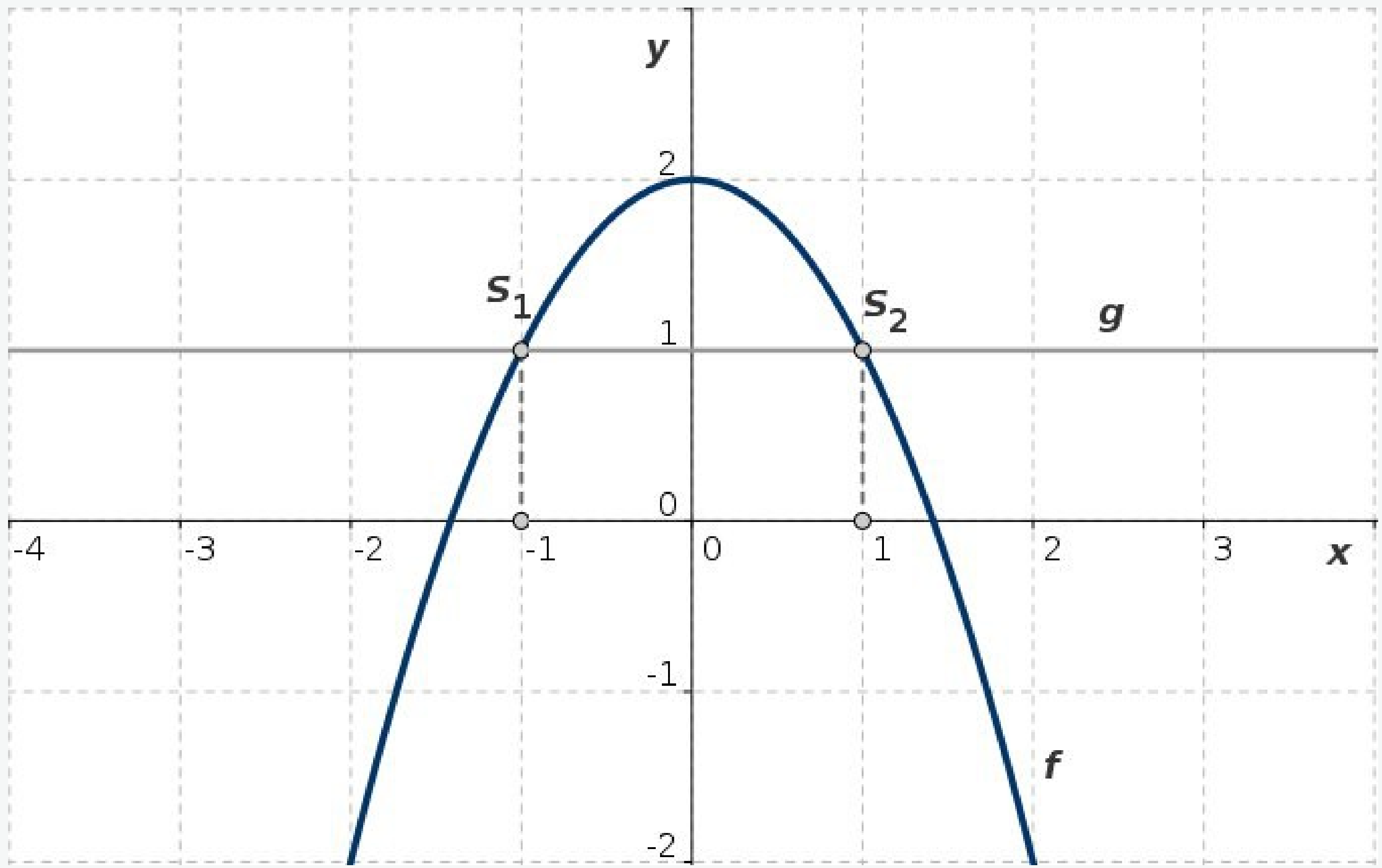

Fig. L1: Functions $y = f(x)$, $y = g(x)$ and intersections

$$f(x) = -x^2 + 2, \quad g(x) = 1, \quad S_1 = (-1, 1), \quad S_2 = (1, 1)$$

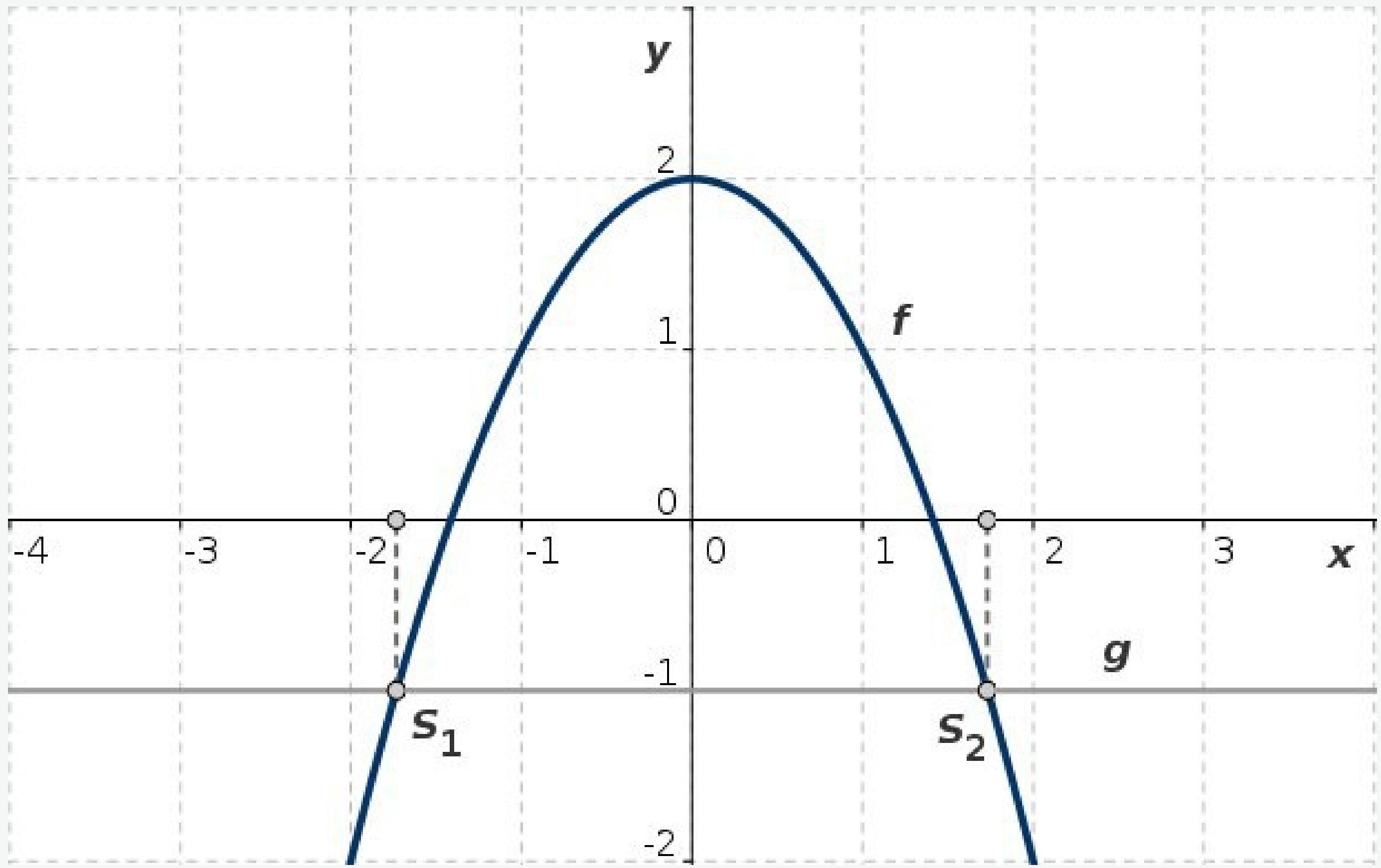

Fig. L2: Functions $y = f(x)$, $y = g(x)$ and intersections

$$f(x) = g(x) \quad \Leftrightarrow \quad -x^2 + 2 = -1 \quad \Leftrightarrow \quad x^2 - 3 = 0$$

$$(x - \sqrt{3})\cdot(x + \sqrt{3}) = 0, \quad x_1 = -\sqrt{3}, \quad x_2 = \sqrt{3}$$

$$S_1 = (-\sqrt{3}, -1) \qquad S_2 = (\sqrt{3}, -1)$$

$$f(x) = -x^2 + 2, \qquad g(x) = x$$

$$f(x) = g(x), \qquad -x^2 + 2 = x, \qquad -x^2 - x + 2 = 0,$$

$$x^2 + x - 2 = 0, \qquad x^2 + px + q = 0, \qquad p = 1, \qquad q = -2$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x_{1,2} = -\frac{1}{2} \pm \frac{3}{2}, \qquad x_1 = -2, \qquad x_2 = 1$$

$$S_1 = (-2, -2), \qquad S_2 = (1, 1)$$

Here and in similar problems, a quadratic solution is transformed into an equivalent equation. The graphical solution of the transformed equation $x^2 + x - 2 = 0$ is the determination of the intersections of the quadratic function

$$h(x) = x^2 + x - 2$$

and the x-axis (x-intercepts).

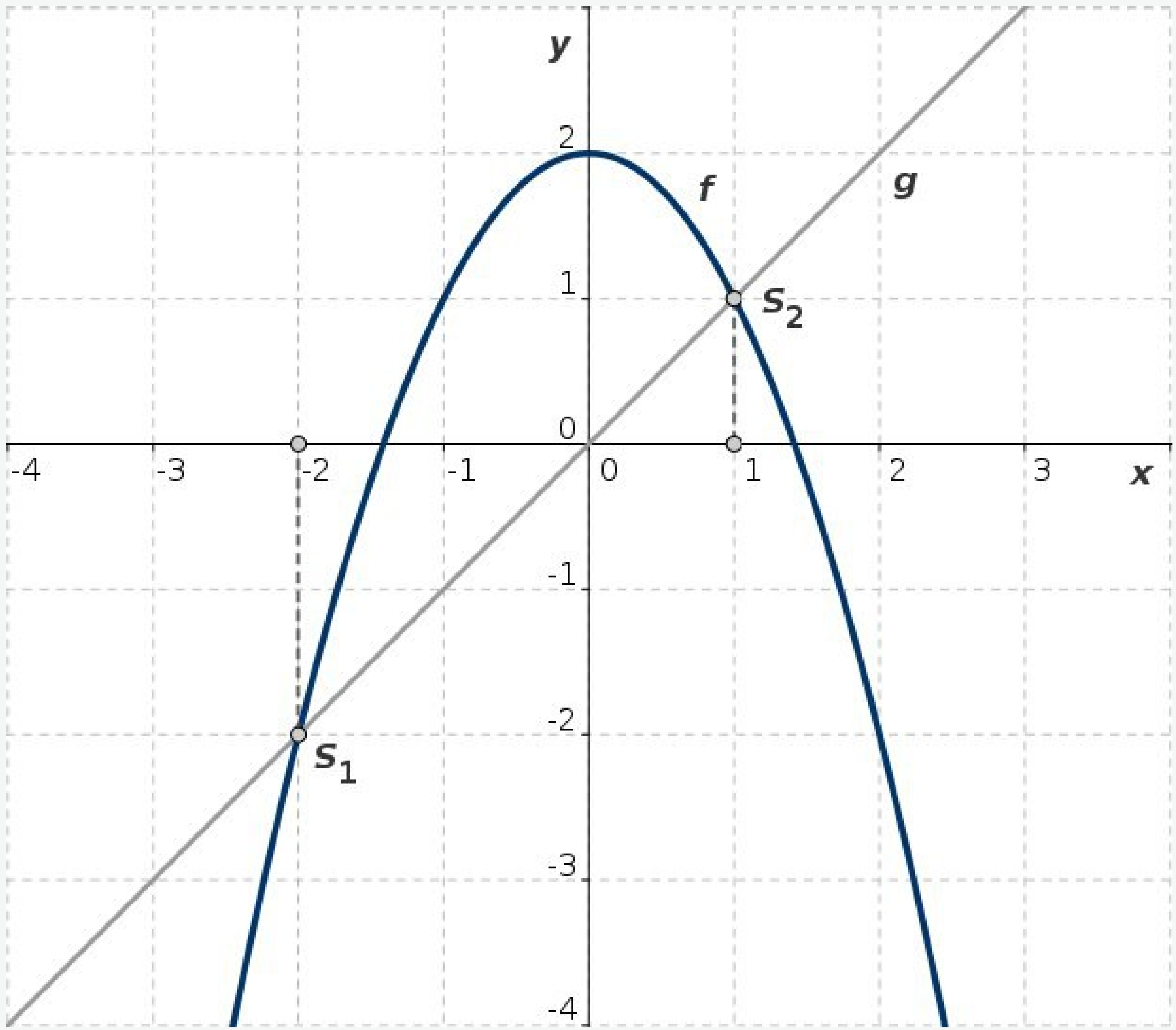

Fig. L3-1: Functions $y = f(x)$, $y = g(x)$ and intersections

$$f(x) = -x^2 + 2, \quad g(x) = x, \quad S_1 = (-2, -2), \quad S_2 = (1, 1)$$

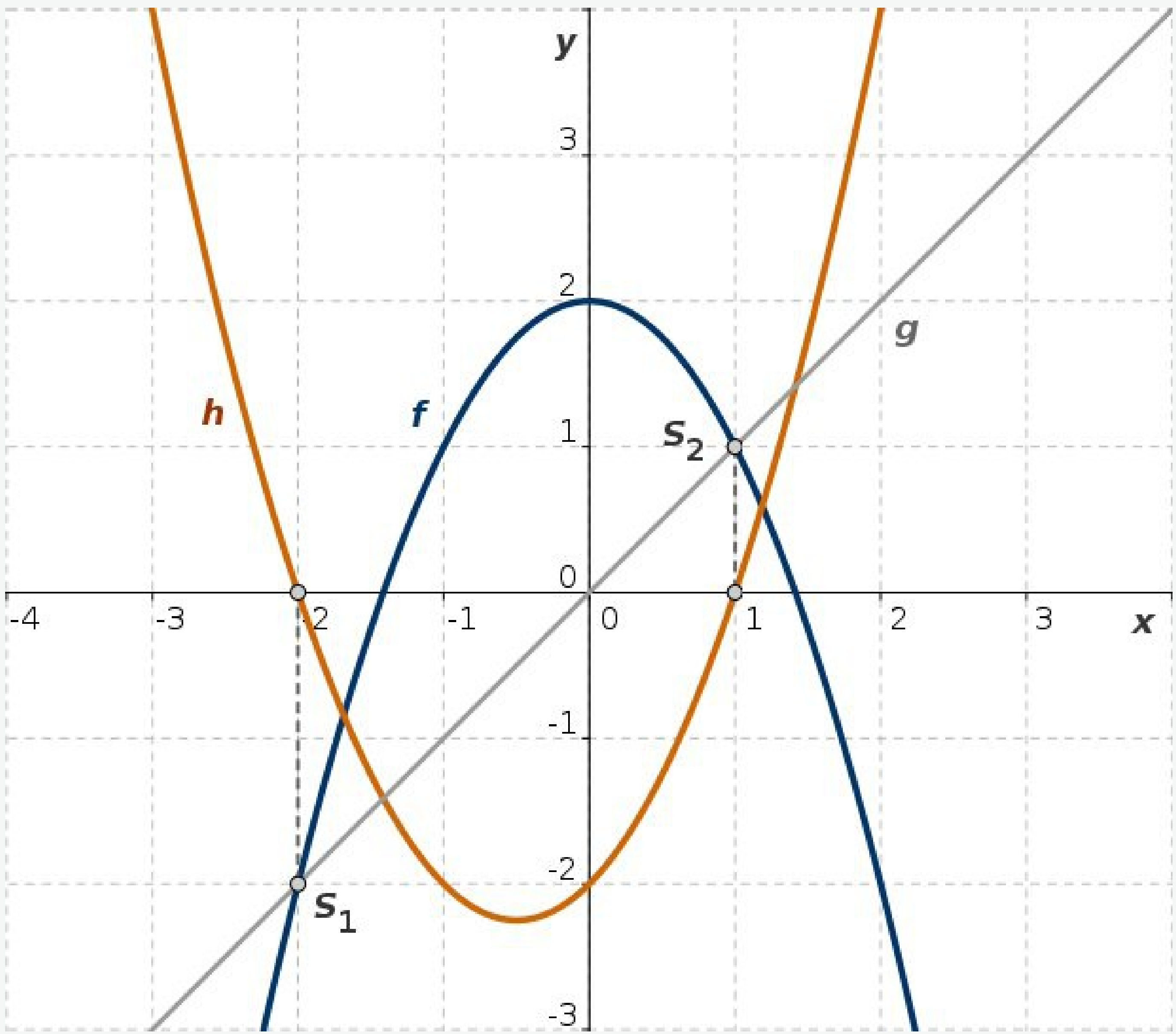

Fig. L3-2: Functions $y = f(x)$, $y = g(x)$ and $y = h(x)$

$$f(x) = -x^2 + 2, \quad g(x) = x, \quad h(x) = x^2 + x - 2$$

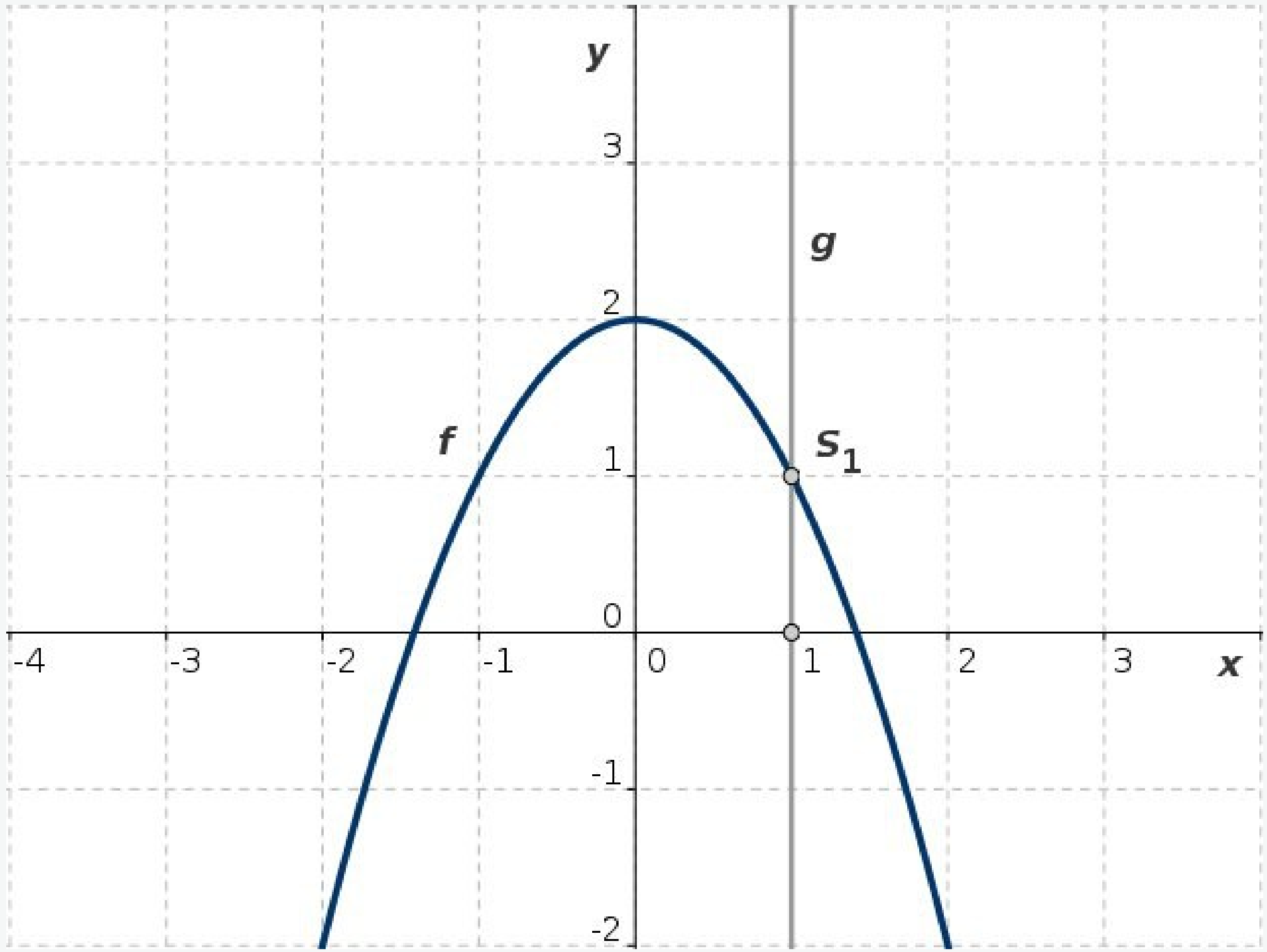

Fig. L4: Functions $y = f(x)$, $x = 1$ and intercept

$$f(x) = -x^2 + 2, \qquad x = 1, \qquad S_1 = (1,\ 1)$$

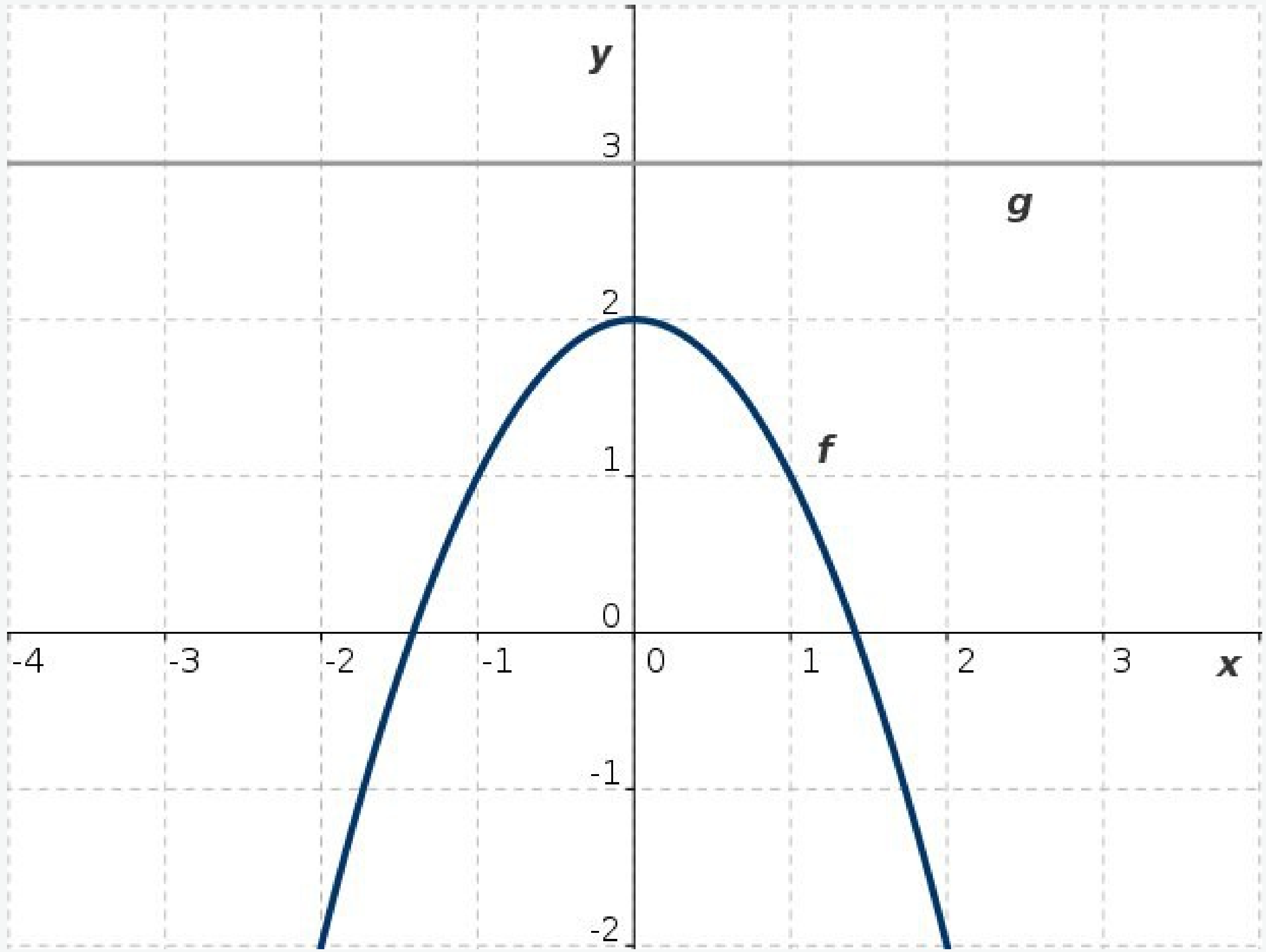

Fig. L5: functions $y = f(x)$, $y = g(x)$

$$f(x) = -x^2 + 2, \quad g(x) = 3, \quad f(x) = g(x) \iff x^2 = -1$$

There is no real solution.