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## *Quadratic Functions: Exercises*

## Quadratic functions: Exercise 1



Shift the graph of the parabola  $y = x^2$

- a) upward by one unit
- b) downward by 2 units
- c) to the right by 4 units
- d) to the left by 2 units.

In each case, write down the equation of the parabola.

## Quadratic functions: Solution 1 a,b

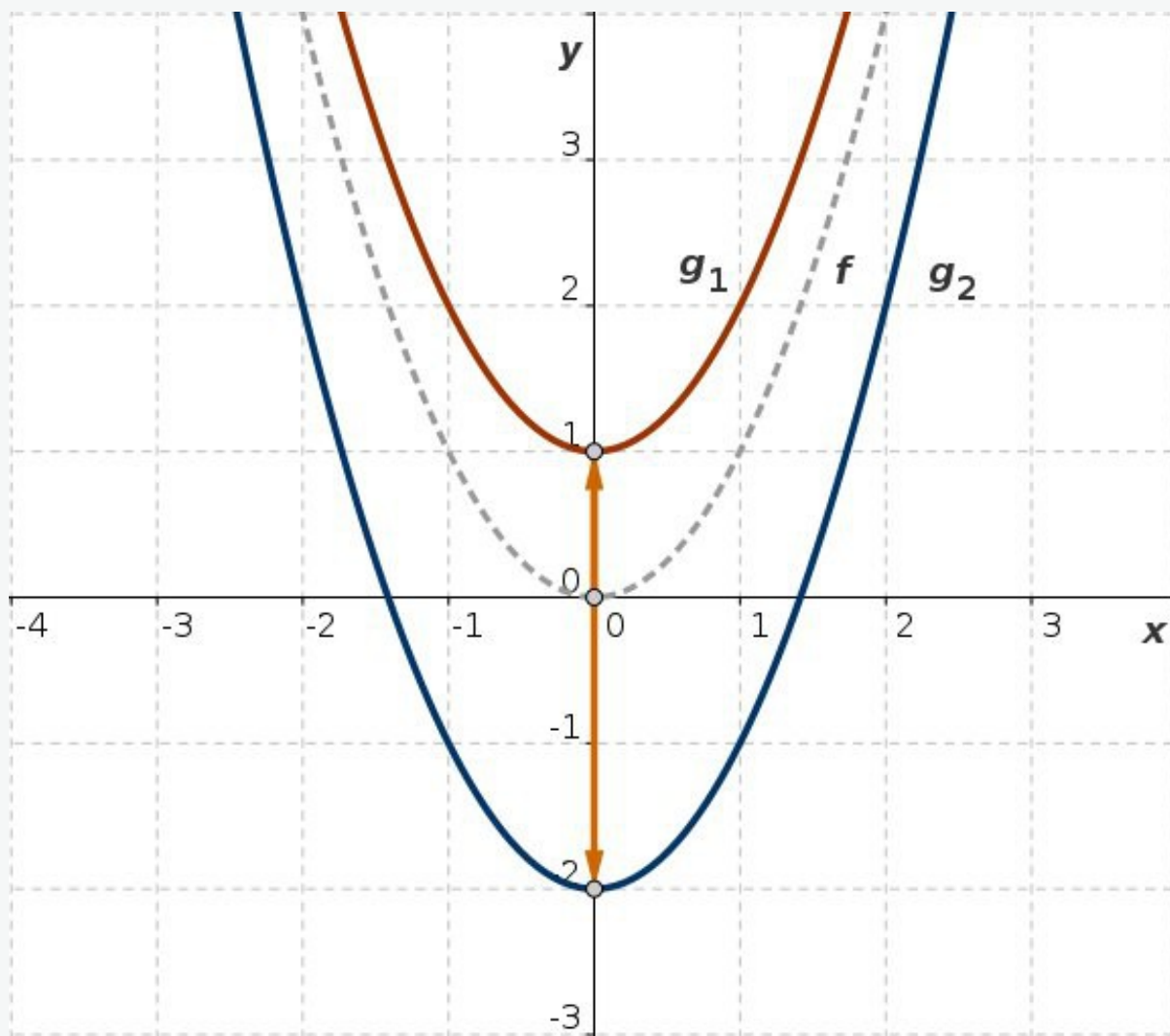


Fig. L1-1: Graphs of the functions

a) one unit upward:  $g_1(x) = x^2 + 1$

b) 2 units downward:  $g_2(x) = x^2 - 2$

## Quadratic functions: Solution 1 c,d

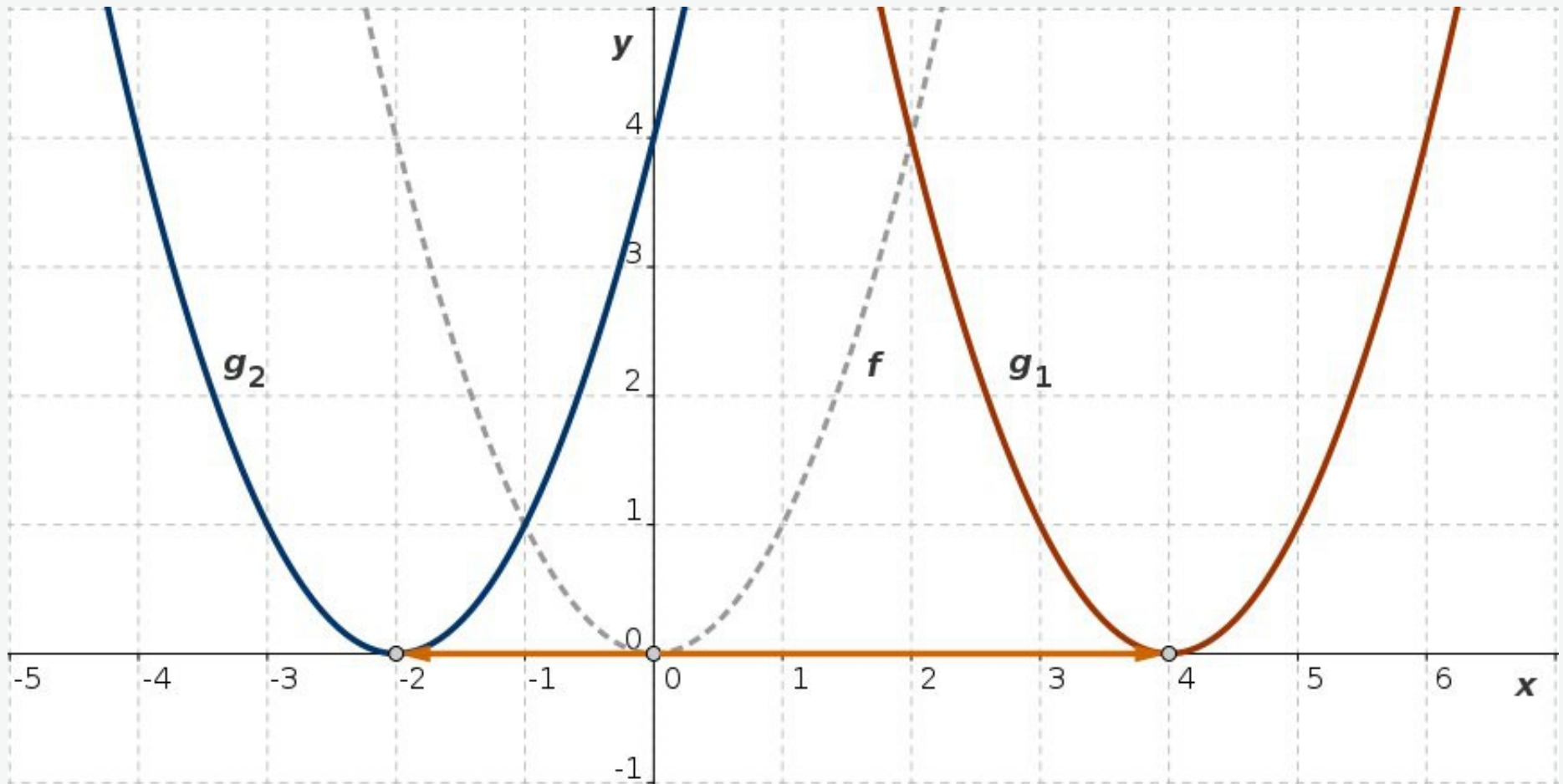


Fig. L1-2: Graphs of the functions

c) 4 units to the right:  $g_1(x) = (x - 4)^2$

d) 2 units to the left:  $g_2(x) = (x + 2)^2$

## Quadratic functions: Exercises 2,3



### Exercise 2:

Which shifts of the parabola  $y = x^2$  are the following functions corresponding to?

a)  $y = (x - 2)^2 + 1$

b)  $y = (x + 1)^2 - 4$

c)  $y = (x - 3)^2 - 4$

### Exercise 3:

Which shifts of the graph of the parabola  $y = x^2$  in the direction of the coordinate axes lead to the following equations:

a)  $y = x^2 - 8x + 7$

b)  $y = x^2 + 4x + 3$

c)  $y = x^2 - x + \frac{1}{2}$

In all cases, determine the coordinates of the vertex.

## Quadratic functions: Solution 2 a

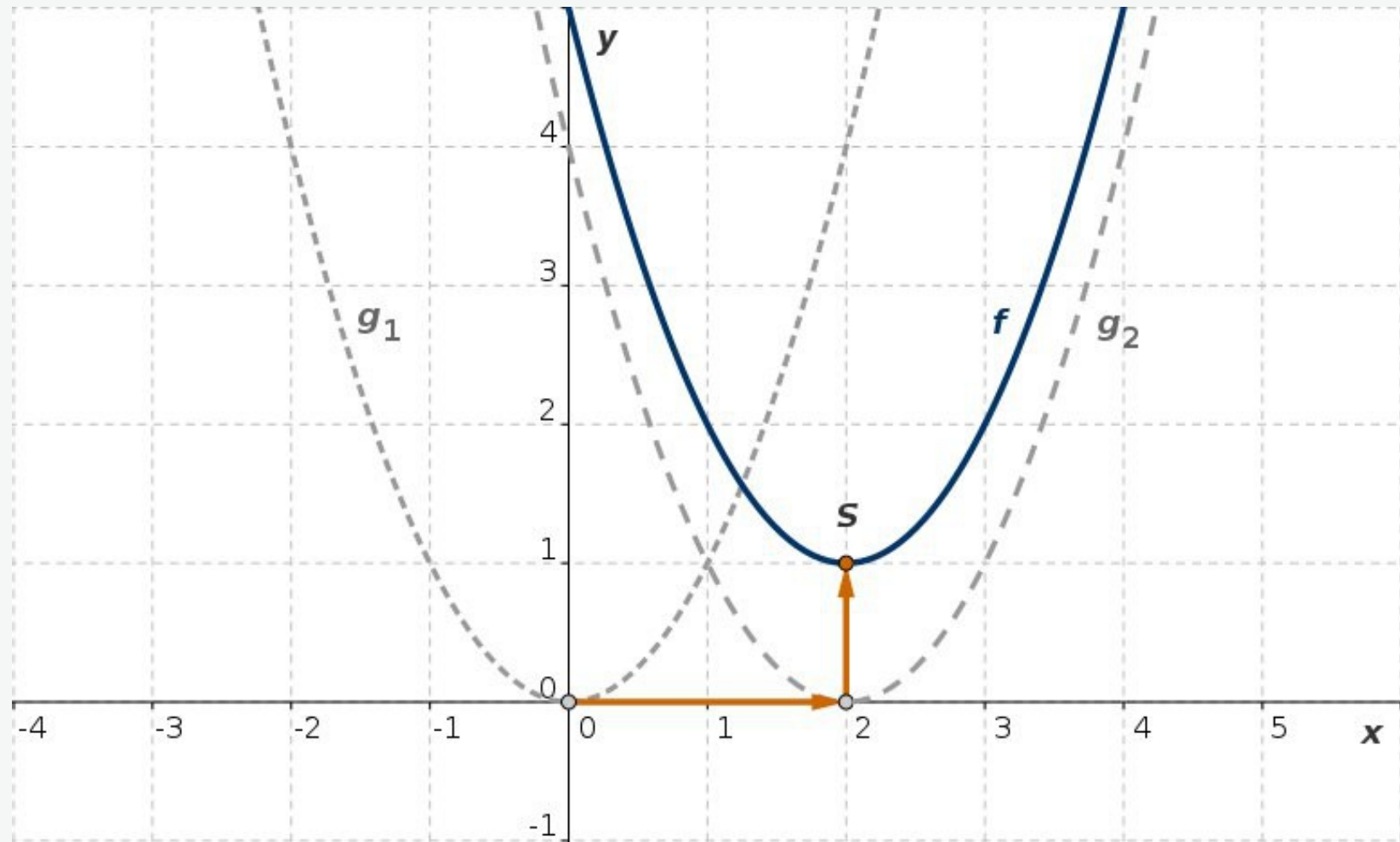


Fig. L2a: Graphs of the functions

$$g_1(x) = x^2 \quad \rightarrow \quad g_2(x) = (x - 2)^2 \quad \rightarrow \quad f(x) = (x - 2)^2 + 1$$

## Quadratic functions: Solution 2 b

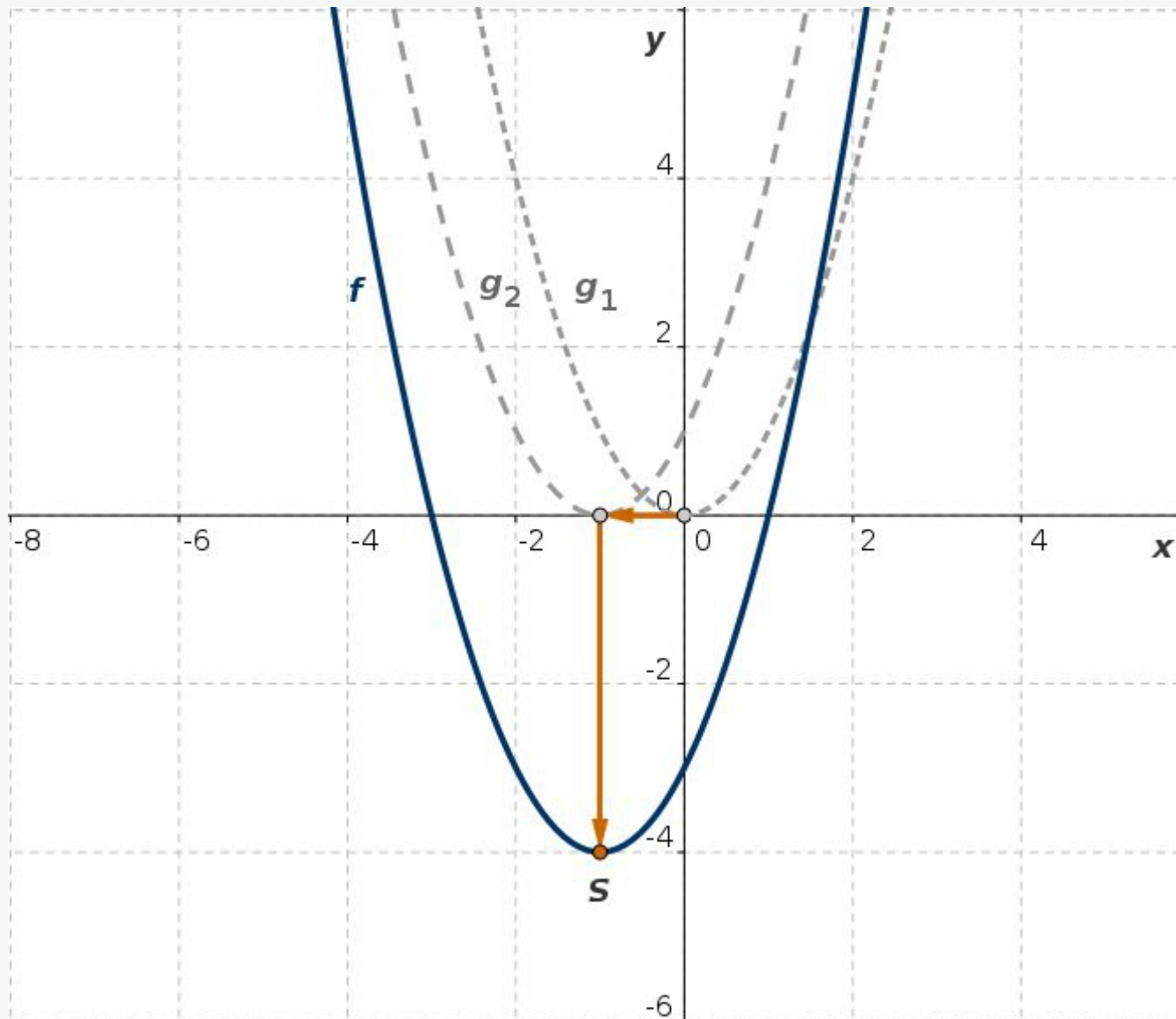


Fig. L2b: Graphs of the functions

$$g_1(x) = x^2 \quad \rightarrow \quad g_2(x) = (x + 1)^2 \quad \rightarrow \quad f(x) = (x + 1)^2 - 4$$

## Quadratic functions: Solution 2 c

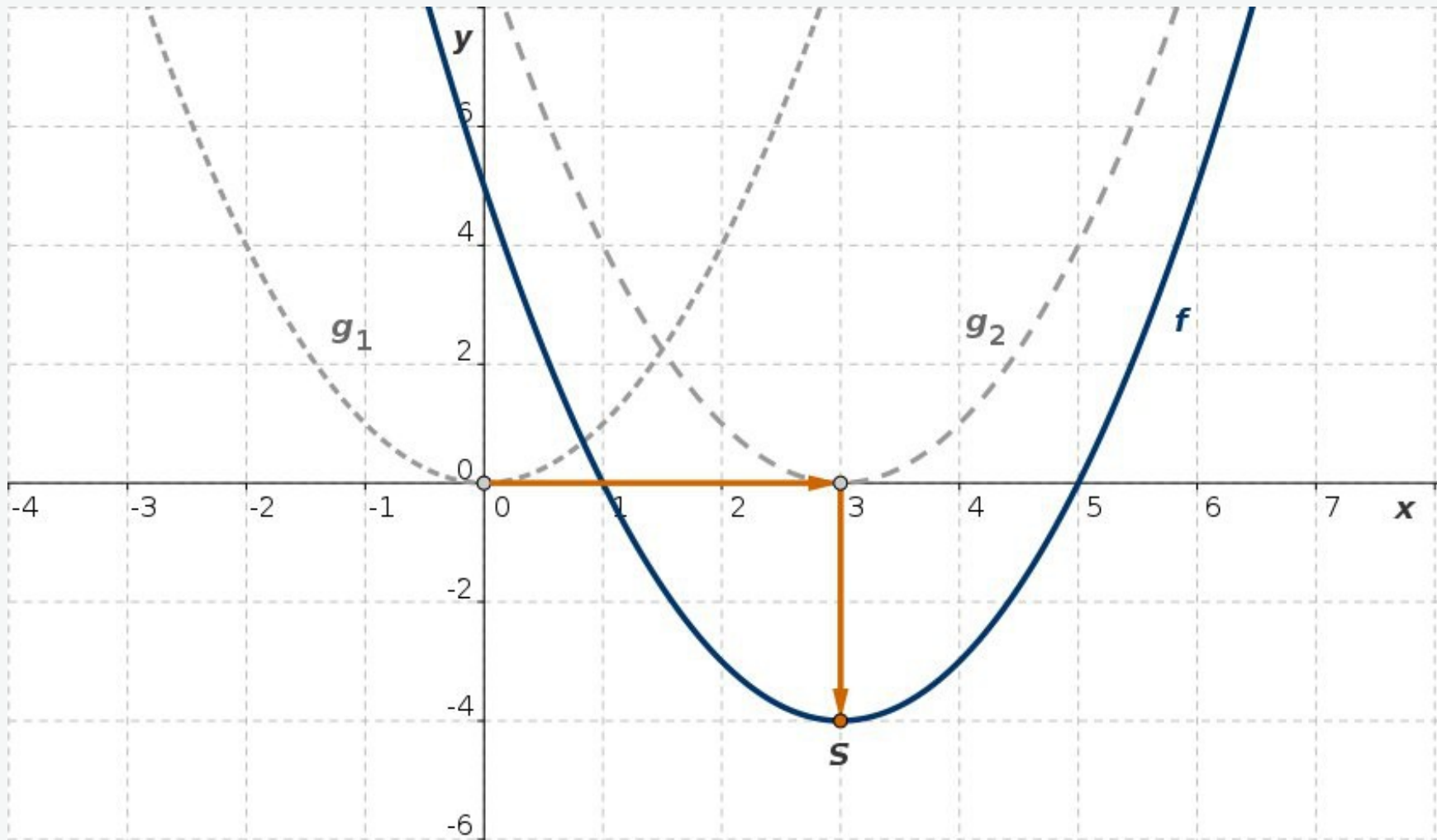


Fig. L2c: Graphs of the functions

$$g_1(x) = x^2 \quad \rightarrow \quad g_2(x) = (x - 3)^2 \quad \rightarrow \quad f(x) = (x - 3)^2 - 4$$



## Quadratic functions: Solution 3

$$\begin{aligned} a) \quad y &= x^2 - 8x + 7 = x^2 - 2 \cdot 4x + 7 = \\ &= x^2 - 2 \cdot 4x + (16 - 16) + 7 = \\ &= [x^2 - 2 \cdot 4x + 16] + [7 - 16] = (x - 4)^2 - 9 \end{aligned}$$

$$1) \quad y = x^2$$

2) 4 units to right:

$$y = (x - 4)^2$$

3) 9 units down:

$$y = (x - 4)^2 - 9, \quad S(4, -9)$$

$$b) \quad y = x^2 + 4x + 3 = (x + 2)^2 - 1, \quad S(-2, -1)$$

$$c) \quad y = x^2 - x + \frac{1}{2} = \left(x - \frac{1}{2}\right)^2 + \frac{1}{4}, \quad S\left(\frac{1}{2}, \frac{1}{4}\right)$$

## Quadratic functions: Exercise 4



Determine the vertices of the following functions

$$a) f(x) = x^2 - 4x + 5$$

$$b) f(x) = x^2 + 6x + 7$$

$$c) f(x) = x^2 + 4x + 6$$

$$d) f(x) = \frac{x^2}{2} + x + \frac{7}{2}$$

$$e) f(x) = -2x^2 - 4x$$

$$f) f(x) = -\frac{x^2}{2} - x + \frac{1}{2}$$

by transformation to equations like

$$y = a(x - m)^2 + n$$

Draw these functions according to

$$g_1(x) = ax^2 \quad \rightarrow \quad g_2(x) = a(x - m)^2 \quad \rightarrow$$

$$\rightarrow f(x) = a(x - m)^2 + n$$

## Quadratic functions: Solution 4a

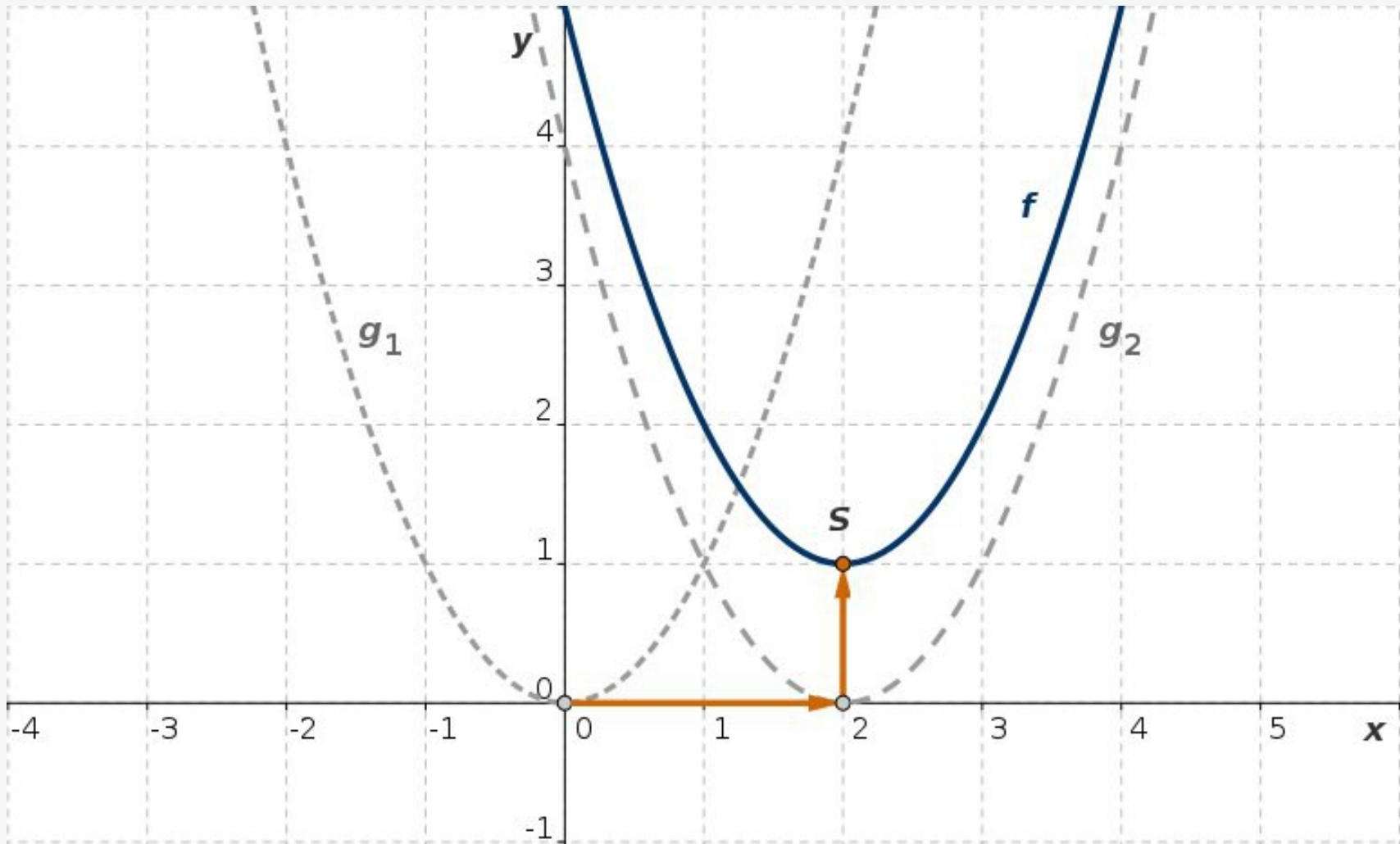


Fig. L4-1: Quadratic function  $y = f(x)$ , vertex  $S$

$$y = x^2 - 4x + 5 = (x - 2)^2 + 1, \quad S = (2, 1)$$

$$g_1(x) = x^2, \quad g_2(x) = (x - 2)^2, \quad f(x) = (x - 2)^2 + 1$$

## Quadratic functions: Solution 4b

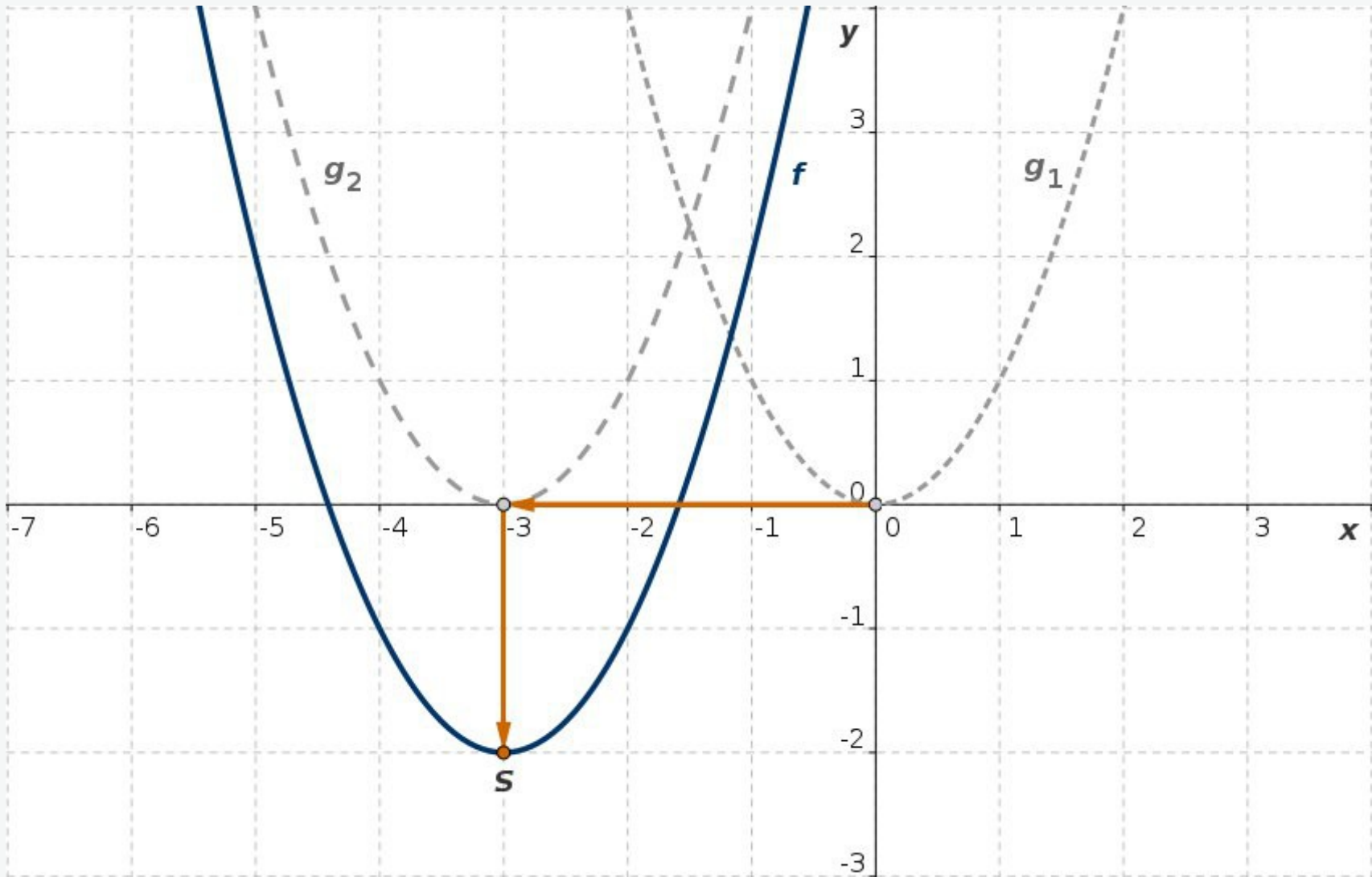


Fig. L4-2: Quadratic function  $y = f(x)$ , vertex  $S$

$$y = x^2 + 6x + 7 = (x + 3)^2 - 2, \quad S = (-3, -2)$$

$$g_1(x) = x^2, \quad g_2(x) = (x + 3)^2, \quad f(x) = (x + 3)^2 - 2$$

## Quadratic functions: Solution 4c

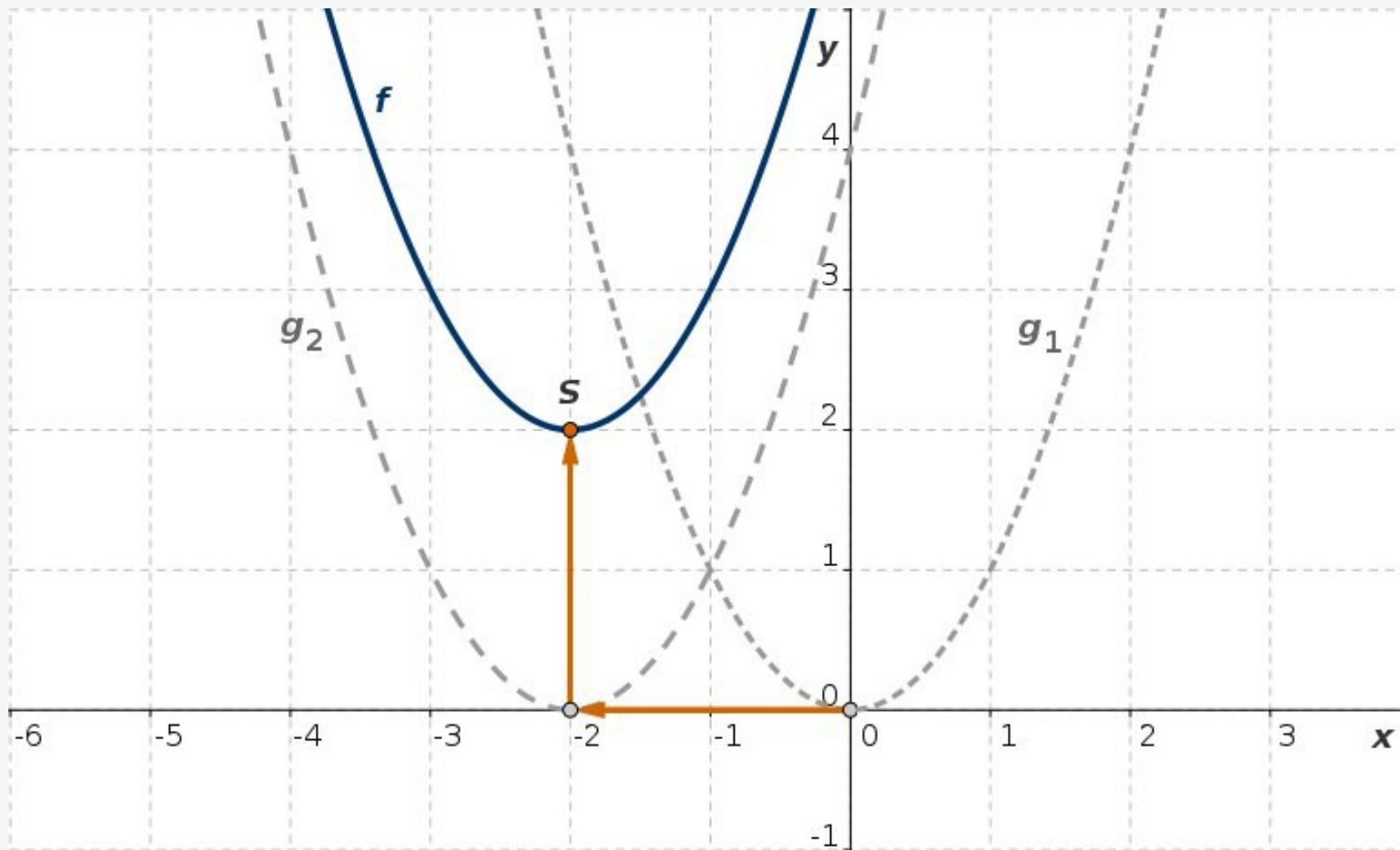


Fig. L4-3: Quadratic function  $y = f(x)$ , vertex  $S$

$$y = x^2 + 4x + 6 = (x + 2)^2 + 2, \quad S = (-2, 2)$$

$$g_1(x) = x^2, \quad g_2(x) = (x + 2)^2, \quad f(x) = (x + 2)^2 + 2$$

## Quadratic functions: Solution 4d

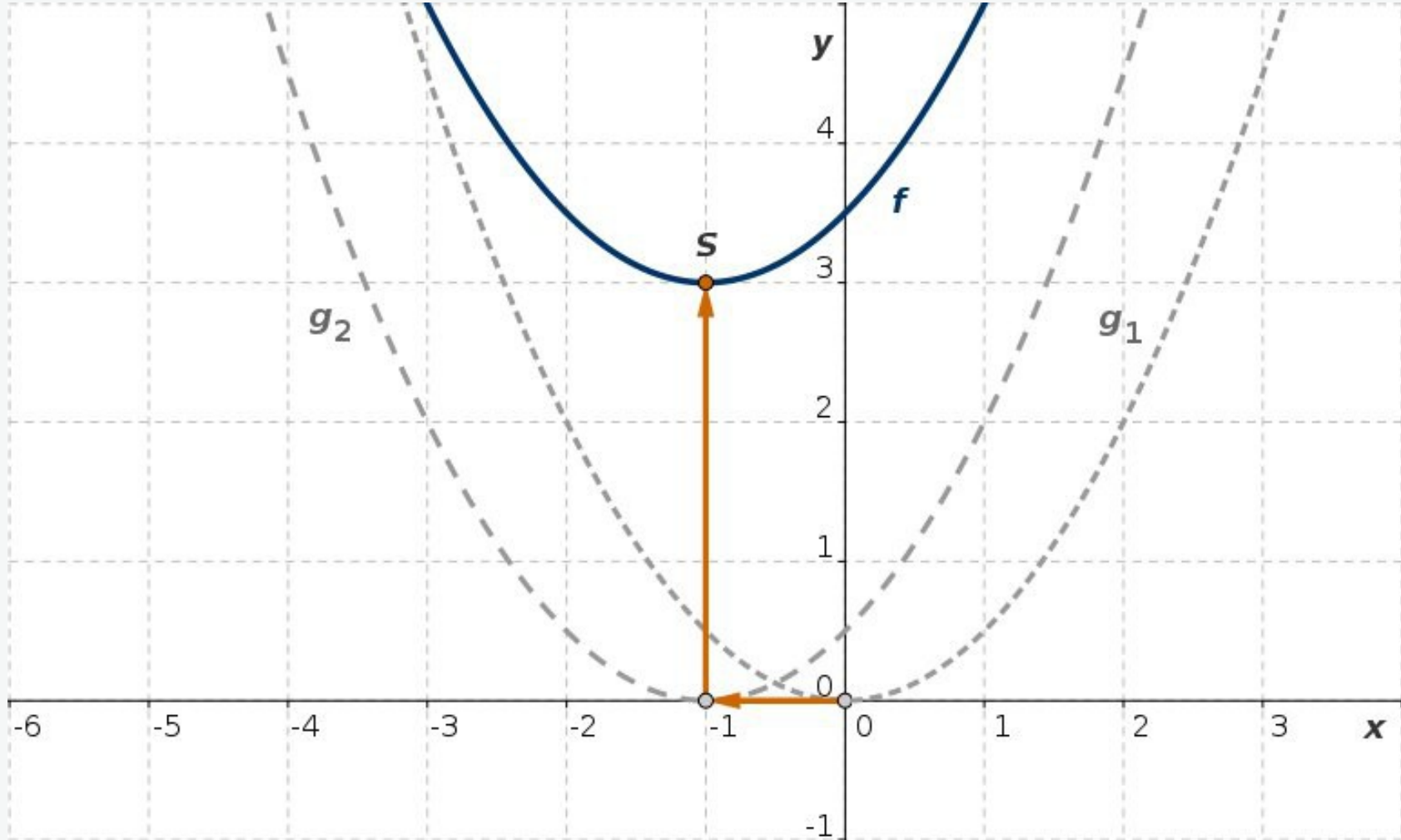


Fig. 14-4: Quadratic function  $y = f(x)$ , vertex  $S$

$$y = \frac{x^2}{2} + x + \frac{7}{2} = \frac{1}{2} (x + 1)^2 + 3, \quad S = (-1, 3)$$

$$g_1(x) = \frac{x^2}{2}, \quad g_2(x) = \frac{1}{2} (x + 1)^2, \quad f(x) = \frac{1}{2} (x + 1)^2 + 3$$

## Quadratic functions: Solution 4e

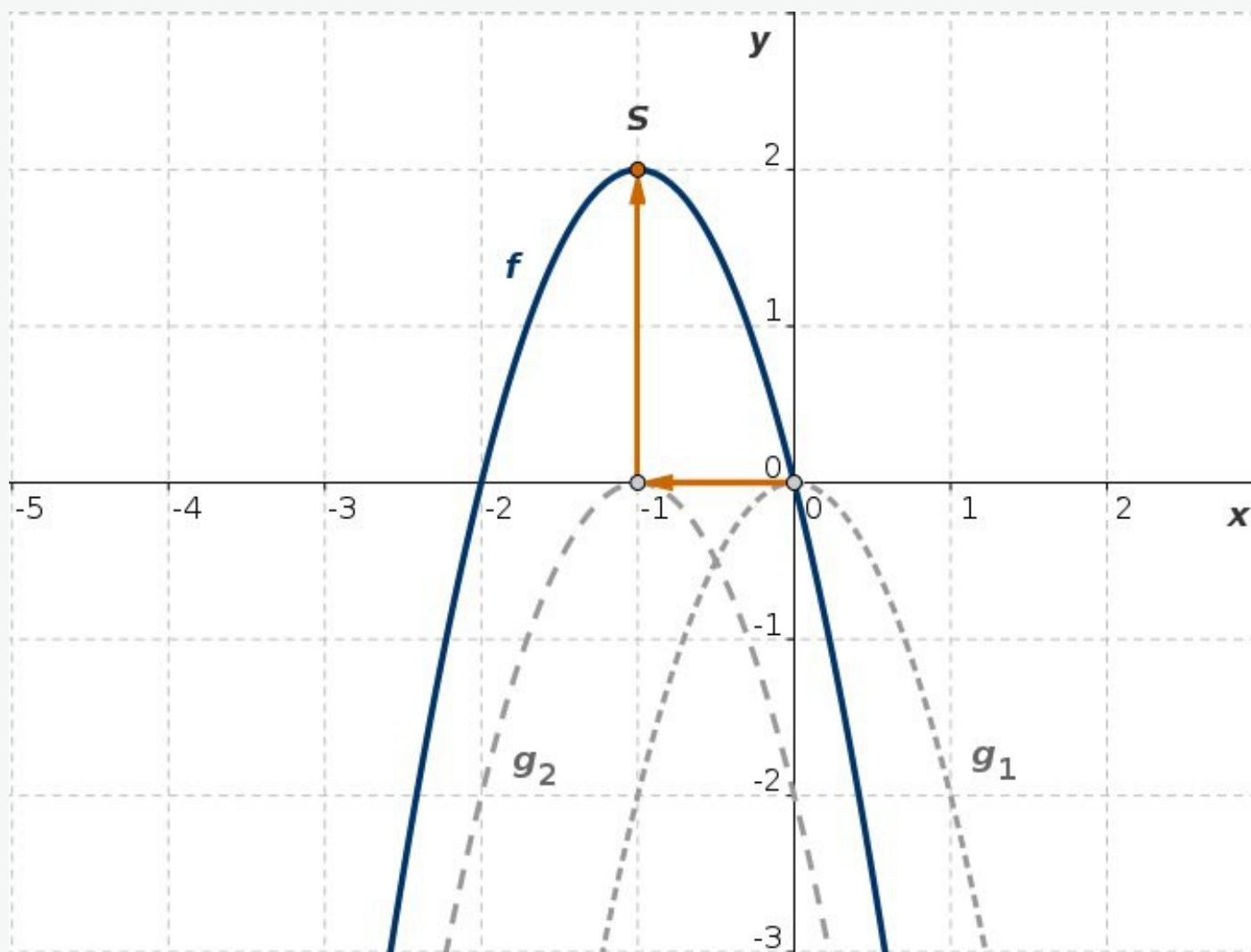


Fig. L4-5: Quadratic function  $y = f(x)$ , vertex  $S$

$$y = -2x^2 - 4x = -2(x + 1)^2 + 2, \quad S = (-1, 2)$$

$$g_1(x) = -2x^2, \quad g_2(x) = -2(x + 1)^2, \quad f(x) = -2(x + 1)^2 + 2$$

## Quadratic functions: Solution 4f

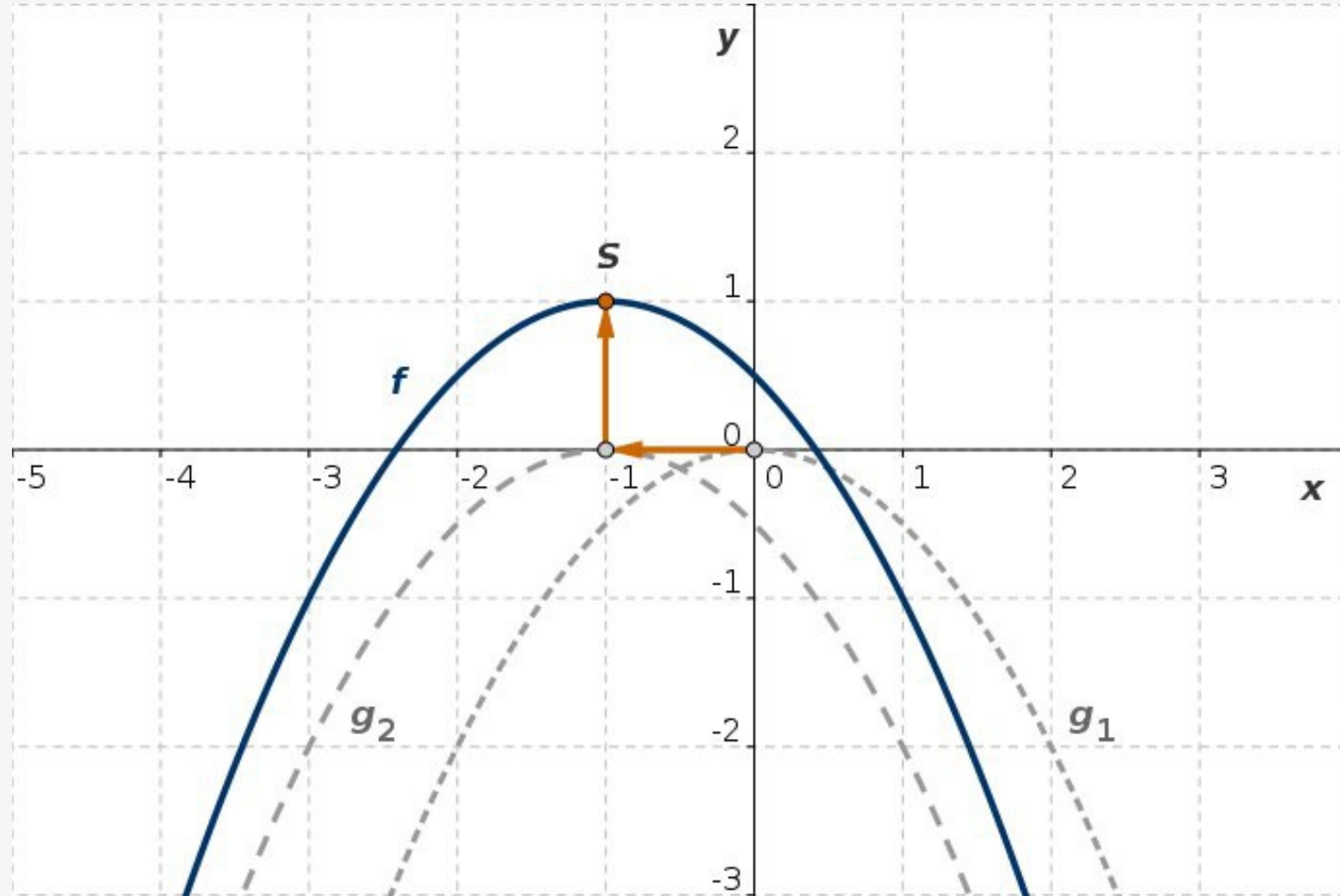


Fig. L4-6: Quadratic function  $y = f(x)$ , vertex  $S$

$$y = -\frac{x^2}{2} - x + \frac{1}{2} = -\frac{1}{2}(x + 1)^2 + 1, \quad S = (-1, 1)$$

$$g_1(x) = -\frac{x^2}{2}, \quad g_2(x) = -\frac{1}{2}(x + 1)^2, \quad f(x) = -\frac{1}{2}(x + 1)^2 + 1$$