

## Radical equations: exercises 5-9

Find the solutions of the following equations:

Exercise 5:  $x + 2 - \sqrt{4 - x} = 0$

Exercise 6:  $\sqrt{2x - 3} + 5 - 3x = 0$

Exercise 7:  $\sqrt{2x + 10} - \sqrt{4x - 8} = 2$

Exercise 8:  $\sqrt{3x + 1} - x + 3 = 0$

Exercise 9:  $x = \sqrt{-x + 12}$

## Radical equations: solution 5

1.  $E: x + 2 - \sqrt{4 - x} = 0$

2. Domain of the equation:  $4 - x \geq 0 \Rightarrow D(E) = (-\infty, 4]$

3. Isolation of the root:  $x + 2 = \sqrt{4 - x}$

4. Squaring the equation:

$$\tilde{E}: (x + 2)^2 = (\sqrt{4 - x})^2 \Leftrightarrow x^2 + 4x + 4 = 4 - x \Leftrightarrow$$

$$x^2 + 5x = 0 \Leftrightarrow x(x + 5) = 0 \Rightarrow x_1 = 0, \quad x_2 = -5$$

5. The set of solutions of the transformed equation:  $S(\tilde{E}) = \{-5, 0\}$

6. Check:  $x = 0: 0 + 2 - \sqrt{4 - 0} = 0 \Leftrightarrow 2 - \sqrt{4} = 0$

$$x = -5: -5 + 2 - \sqrt{4 + 5} = 0 \Leftrightarrow -3 - \sqrt{9} \neq 0$$

$$\Rightarrow S(E) \neq S(\tilde{E})$$

7. Solutions of the equation:  $S(E) = \{0\}$

It is very important, to check the solutions!

## Radical equations: graphical solution 5

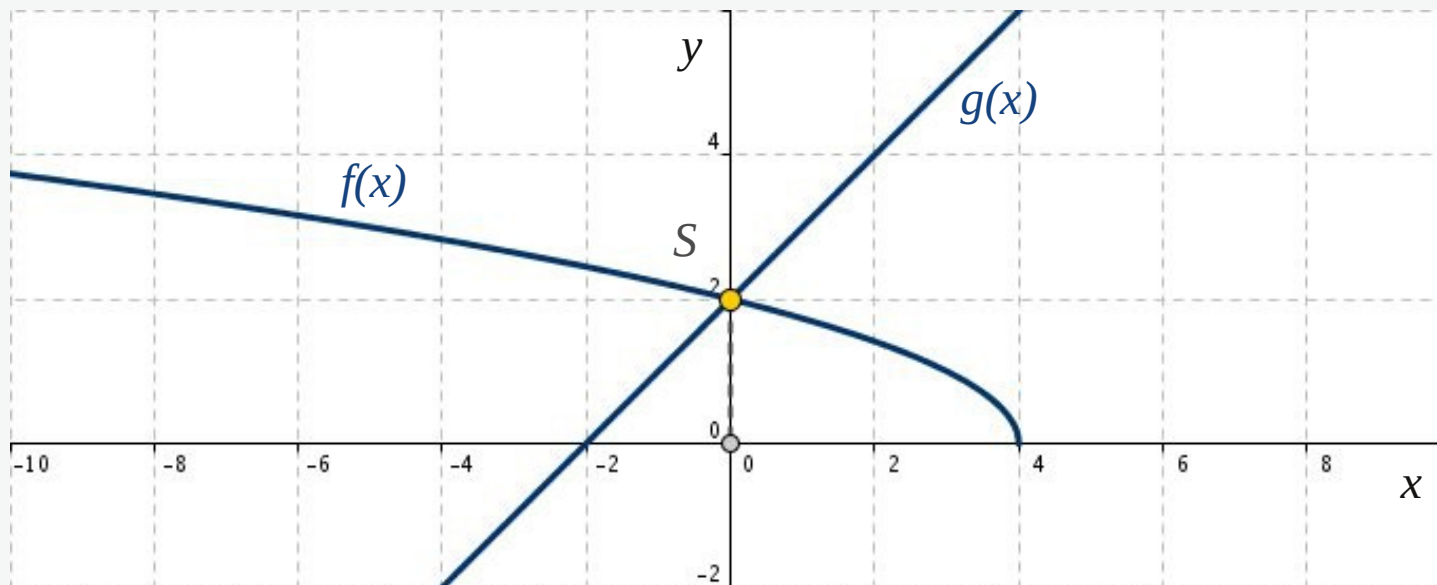


Fig. S-5a: Functions  $f(x)$  and  $g(x)$

$$f(x) = \sqrt{4 - x}, \quad g(x) = x + 2$$

The point  $S(0, 2)$  is the intercept of the functions  $f(x)$  and  $g(x)$ .

## Radical equations: graphical solution 5

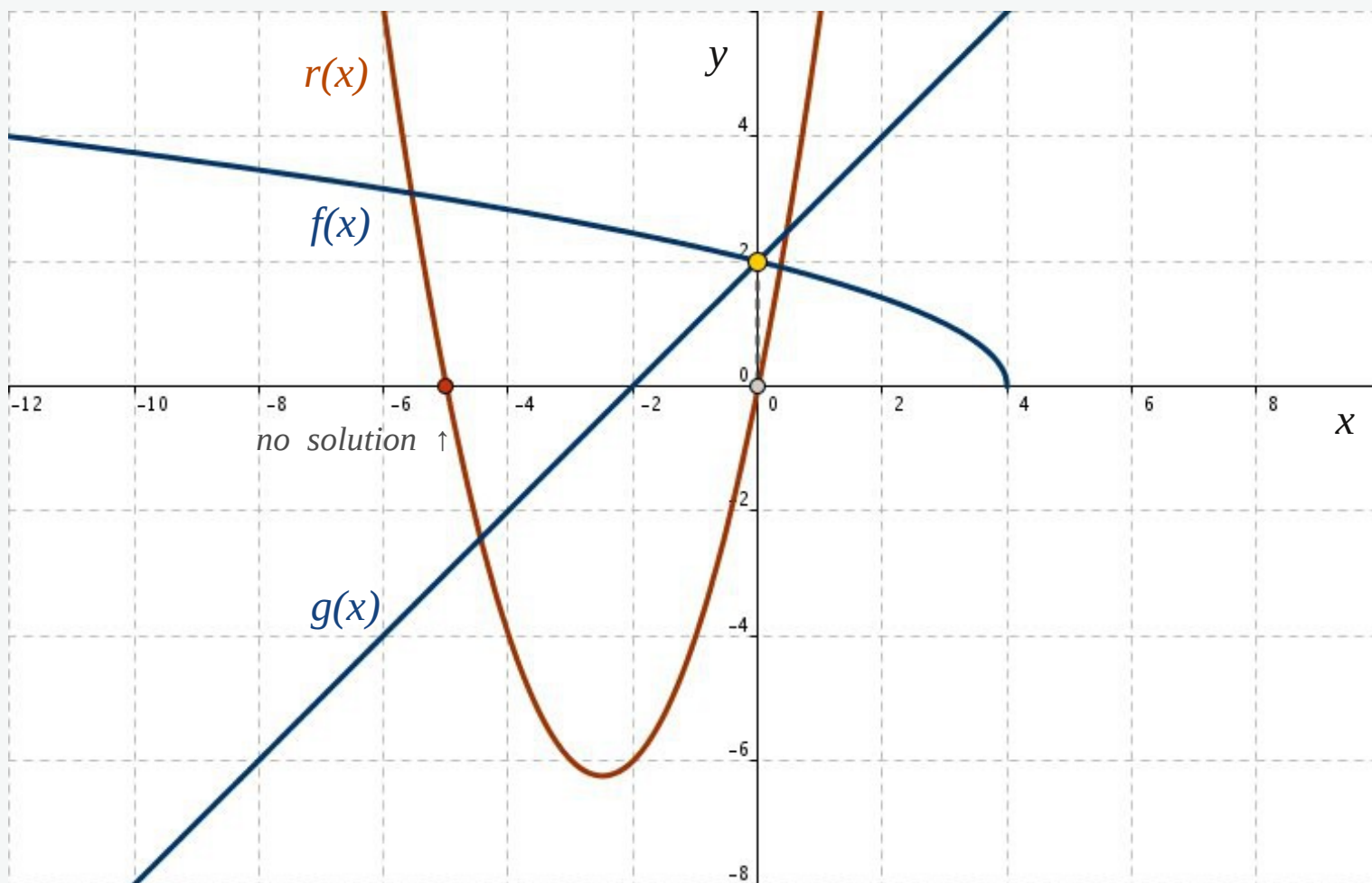


Fig. S-5: Functions  $f(x)$ ,  $g(x)$  and  $r(x)$

$$f(x) = \sqrt{4 - x}, \quad g(x) = x + 2, \quad r(x) = x^2 + 5x$$

## Radical equations: solution 6

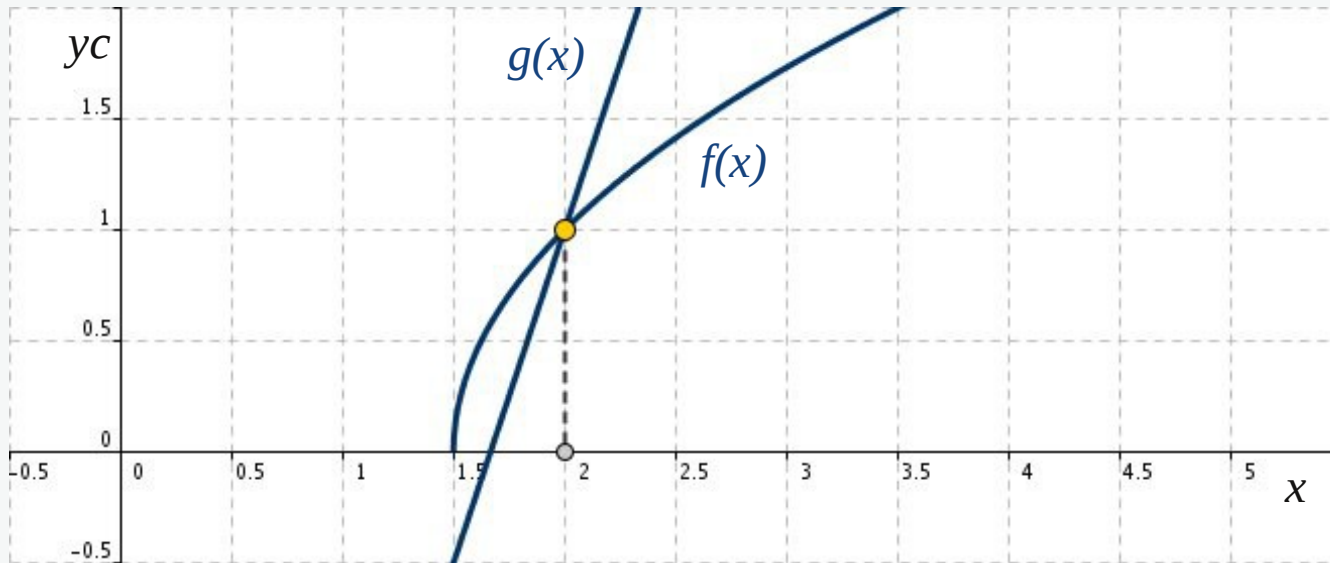


Fig. S-6a: Functions  $f(x)$  and  $g(x)$

$$f(x) = \sqrt{2x - 3}, \quad g = 3x - 5$$

$$E : \sqrt{2x - 3} + 5 - 3x = 0$$

$S(2, 1)$  is the intercept of the functions  $f(x)$  and  $g(x)$ .

$$E : \sqrt{2x - 3} = 3x - 5, \quad D(E) = [1.5, \infty), \quad S_E = \{2\}$$

$$\tilde{E} : 9x^2 - 32x + 28 = 0, \quad D(\tilde{E}) = \mathbb{R}, \quad S_{\tilde{E}} = \left\{ \frac{14}{9}, 2 \right\}$$

## Radical equations: graphical solution 6

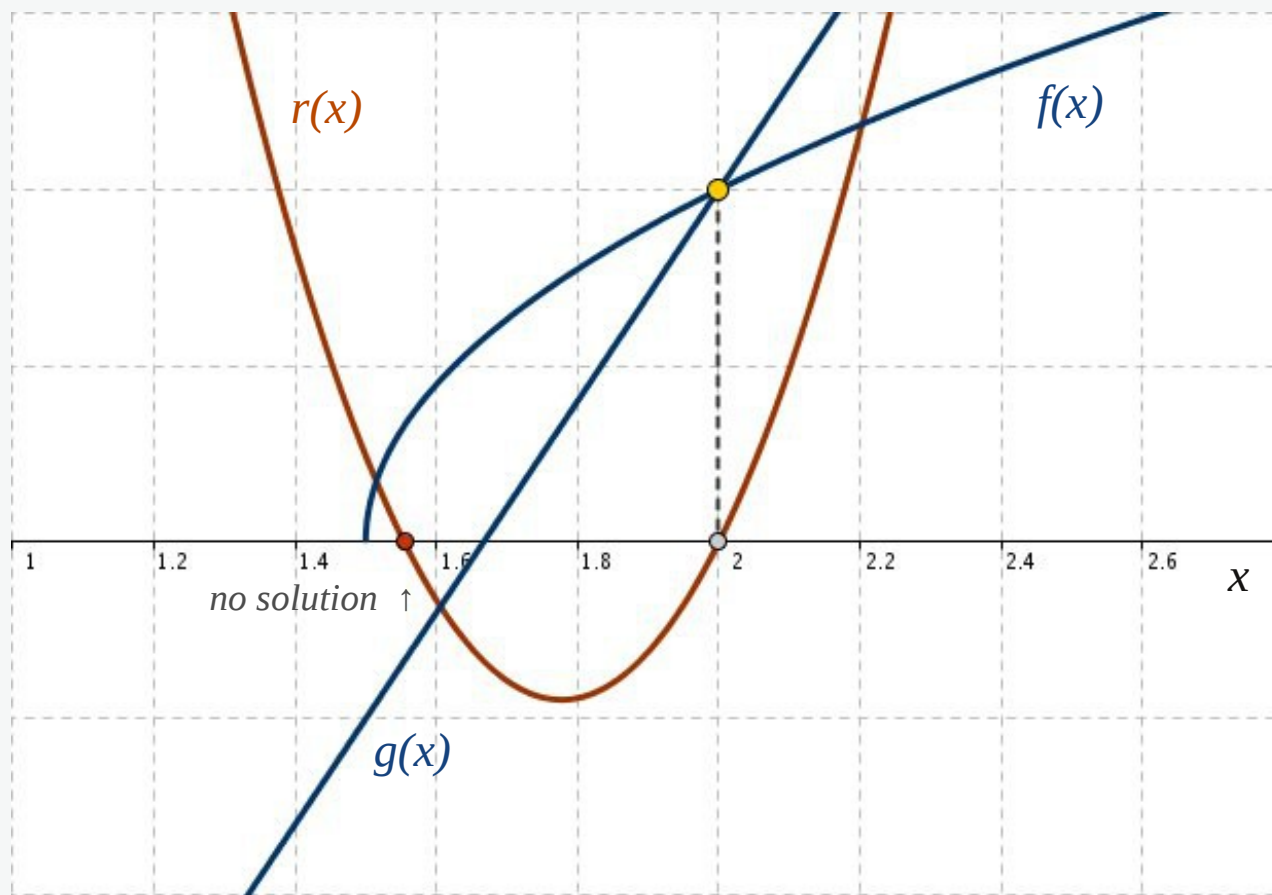


Fig. S-6b: Functions  $f(x)$ ,  $g(x)$  and  $r(x)$

$$f(x) = \sqrt{2x - 3}, \quad g(x) = 3x - 5, \quad r(x) = 9x^2 - 32x + 28$$

## Radical equations: graphical solution 7

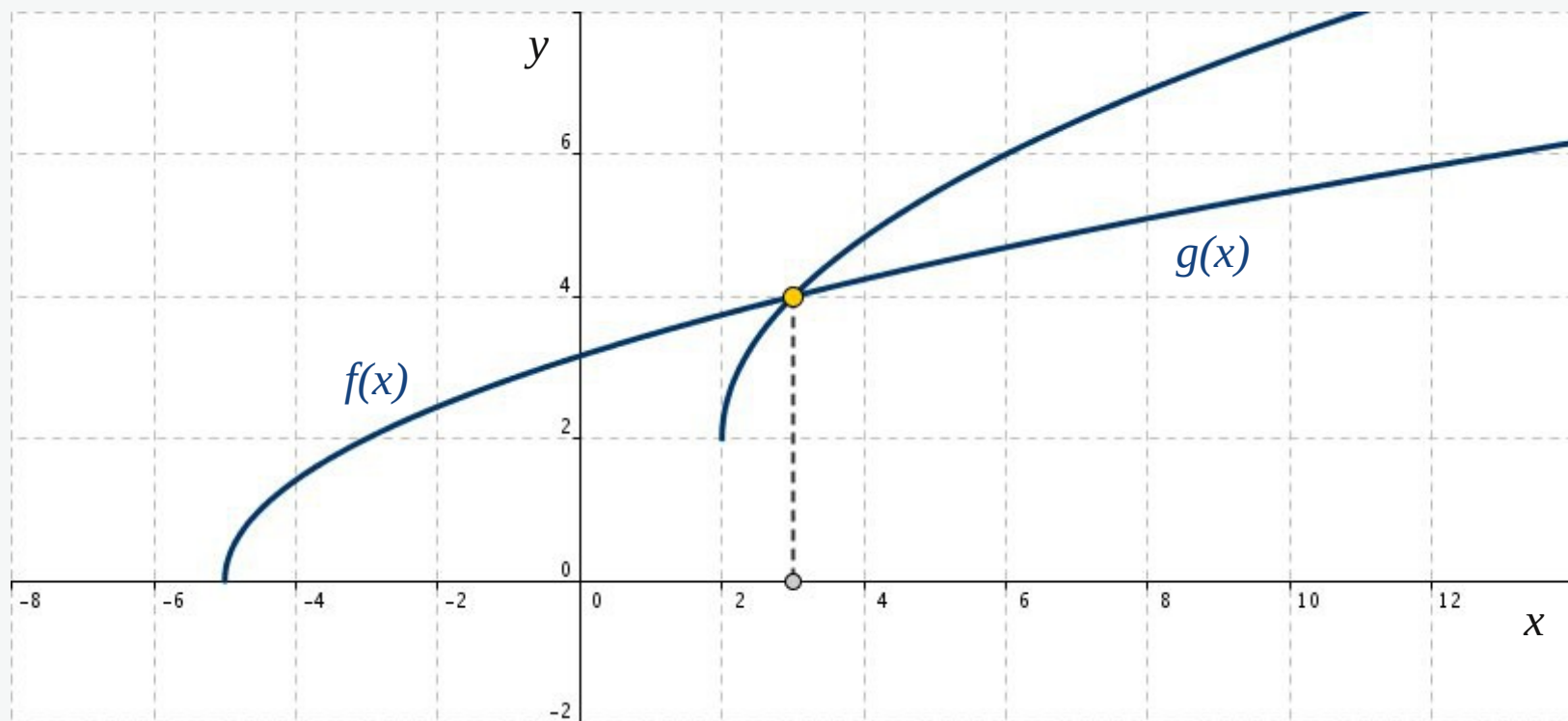


Fig. S-7: Functions  $f(x)$  and  $g(x)$

$$f(x) = \sqrt{2x + 10}, \quad g(x) = 2 + \sqrt{4x - 8}$$

$S(3, 4)$  is the intersection of the functions  $f(x)$  and  $g(x)$ .

## Radical equations: graphical solution 7

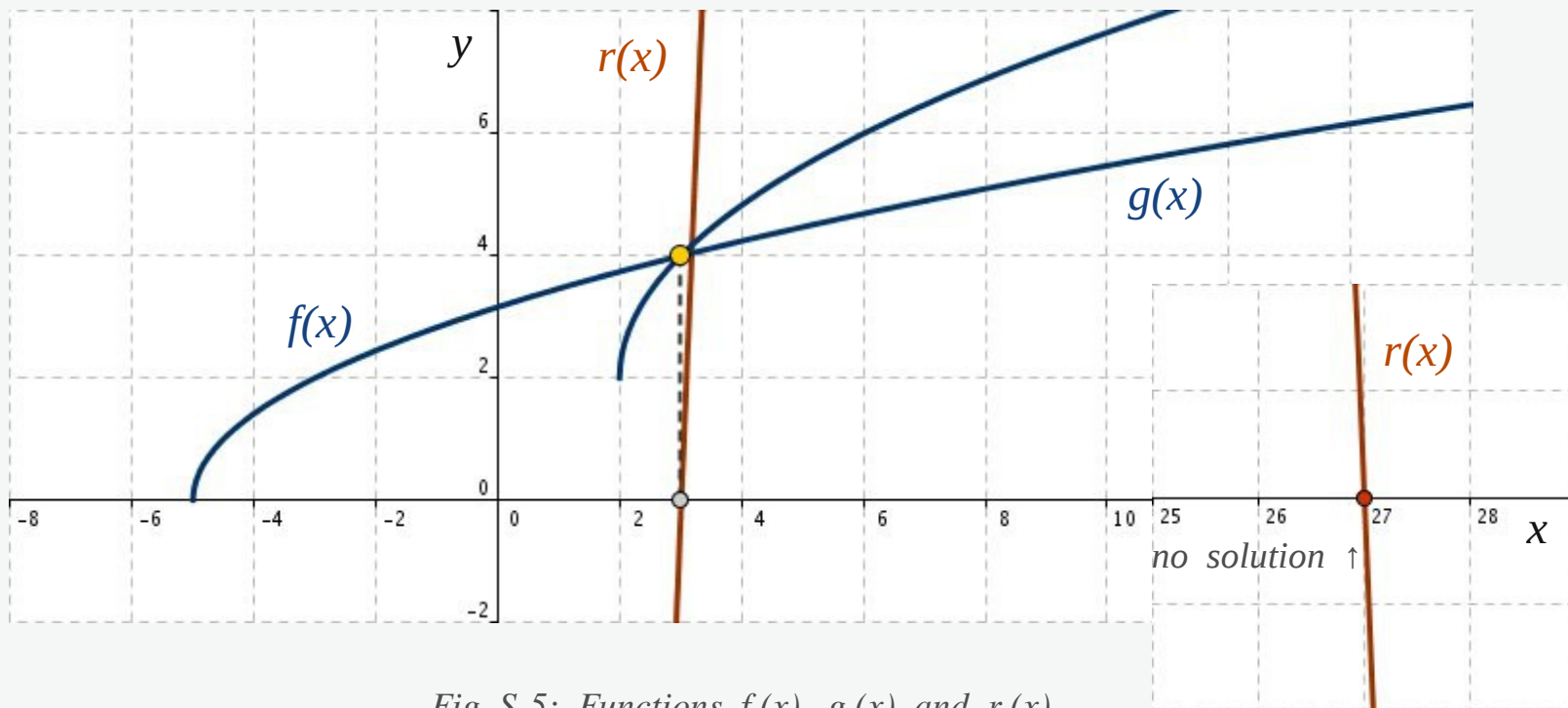


Fig. S-5: Functions  $f(x)$ ,  $g(x)$  and  $r(x)$

$$f(x) = \sqrt{2x + 10}, \quad g(x) = 2 + \sqrt{4x - 8}$$

$$r(x) = -x^2 + 30x - 81$$



## Radical equations: graphical solution 8

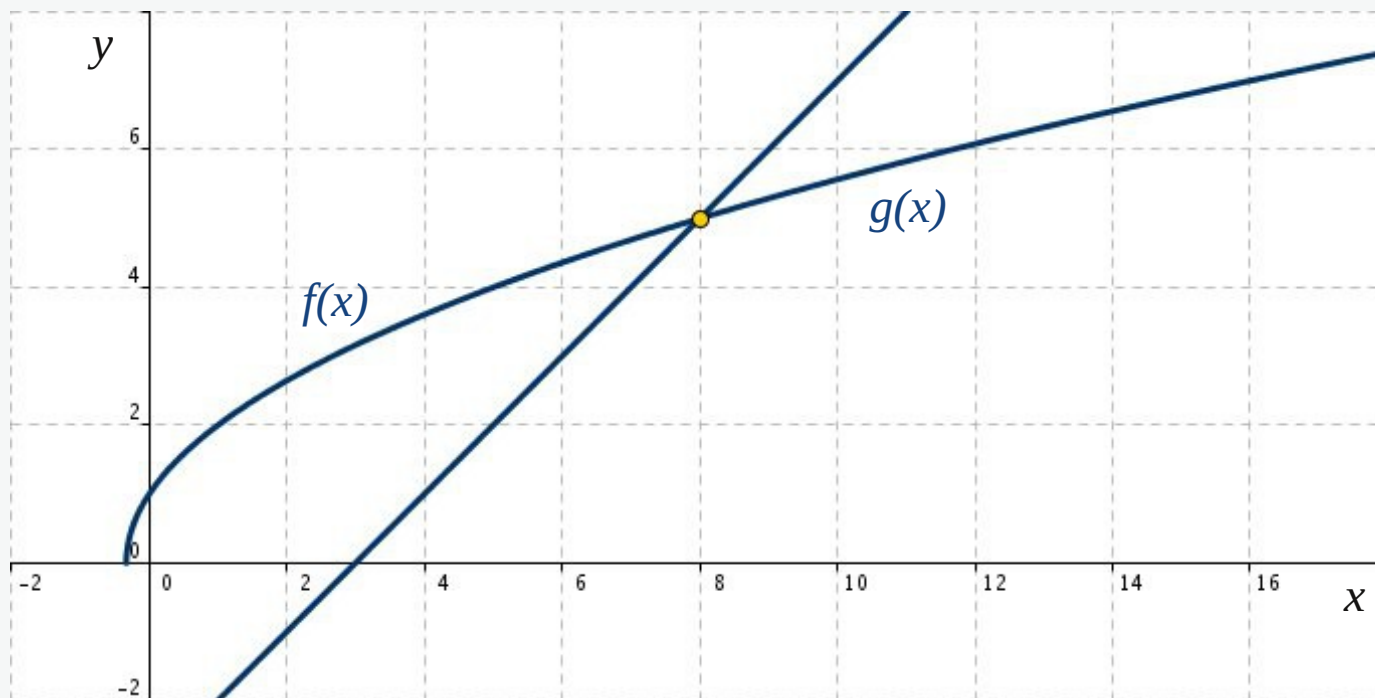


Fig. S-8: Functions  $f(x)$  and  $g(x)$

$$f(x) = \sqrt{3x + 1}, \quad g(x) = x - 3$$

$S(8, 5)$  is the intercept of the functions  $f(x)$  and  $g(x)$ .

## Radical equations: graphical solution 9

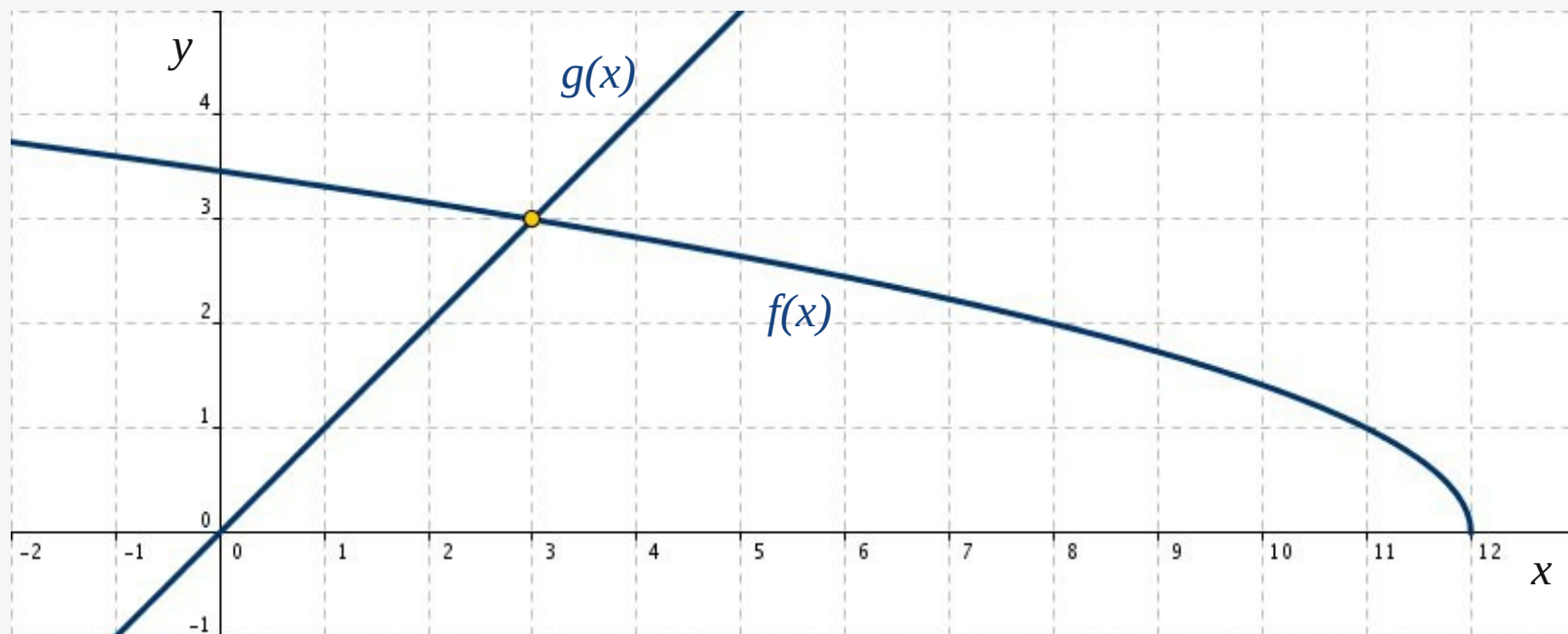


Fig. S-9: Functions  $f(x)$  and  $g(x)$

$$f(x) = \sqrt{-x + 12}, \quad g(x) = x$$

$S(3, 3)$  is the intercept of the functions  $f(x)$  and  $g(x)$ .