

Root functions

Root function $f(x) = \sqrt{x}$

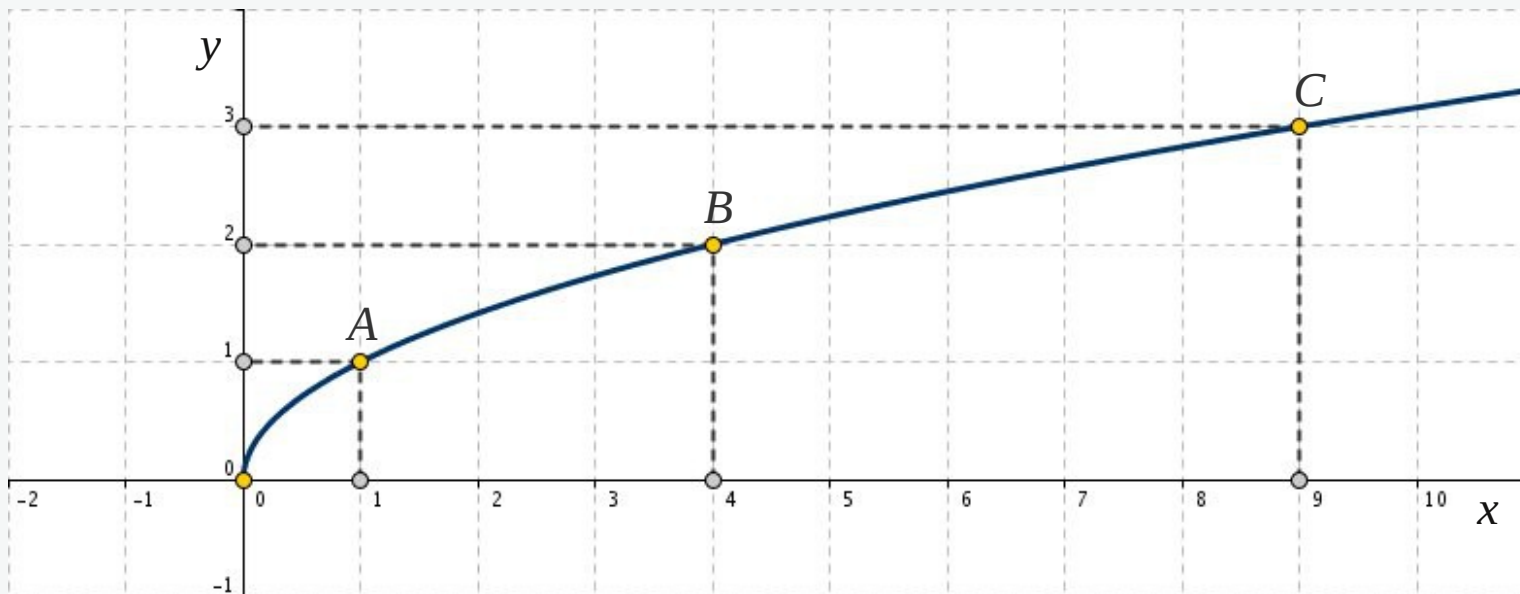


Fig. 1: Root function $f(x) = \sqrt{x}$

The domain (D) and the range (R) of the root function $f(x) = \sqrt{x}$ are

$$D(f) = [0, \infty), \quad R(f) = [0, \infty).$$

Remember, in the field of the real numbers, roots are only defined for positive radicands

$$A = (1, 1), \quad B = (4, 2), \quad C = (9, 3)$$

Root functions

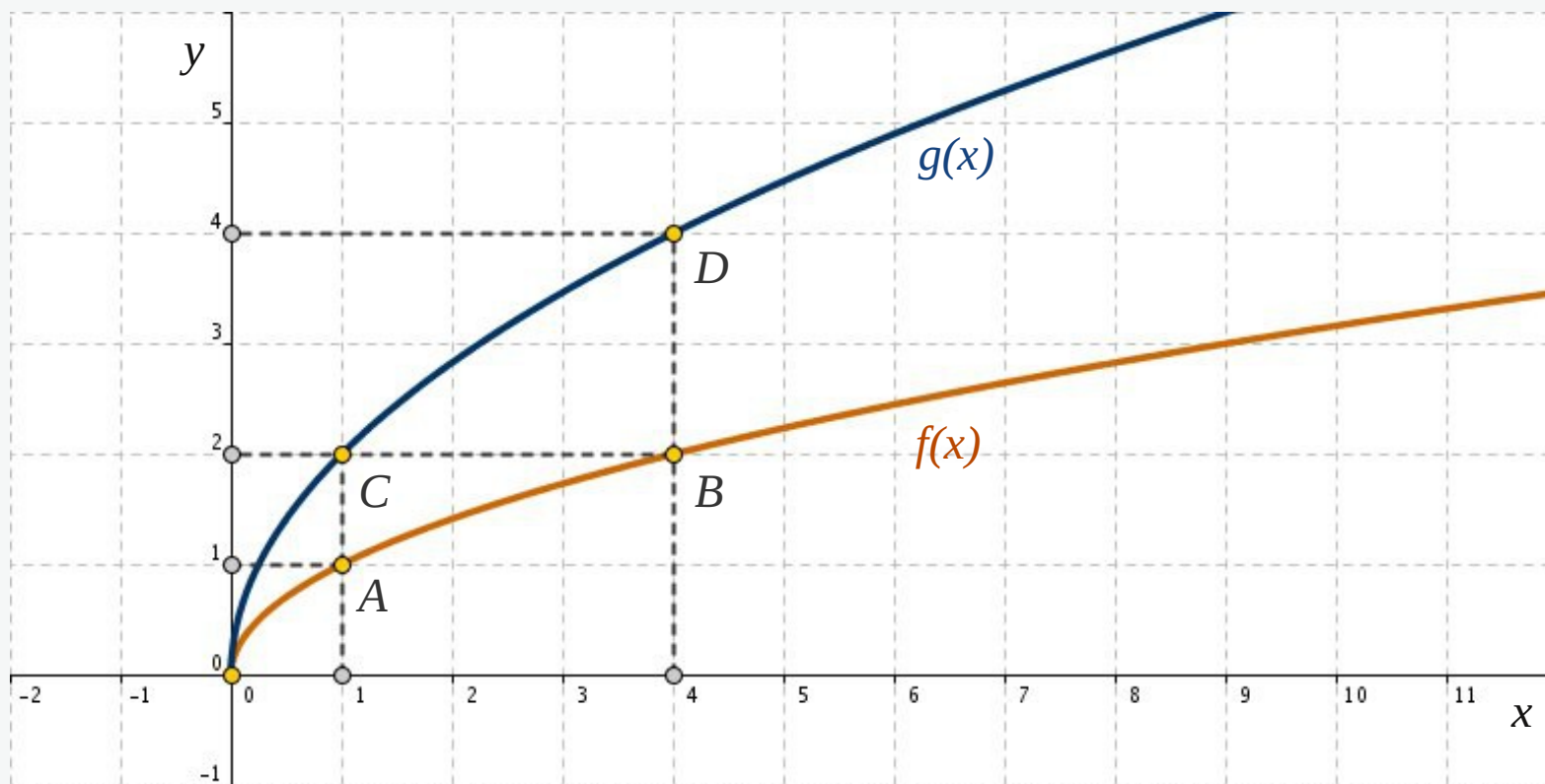


Fig. 2: Root functions $f(x)$ and $g(x)$

$$f(x) = \sqrt{x}, \quad g(x) = 2\sqrt{x}$$

$$A = (1, 1), \quad B = (4, 2), \quad C = (1, 2), \quad D = (4, 4)$$

Root functions

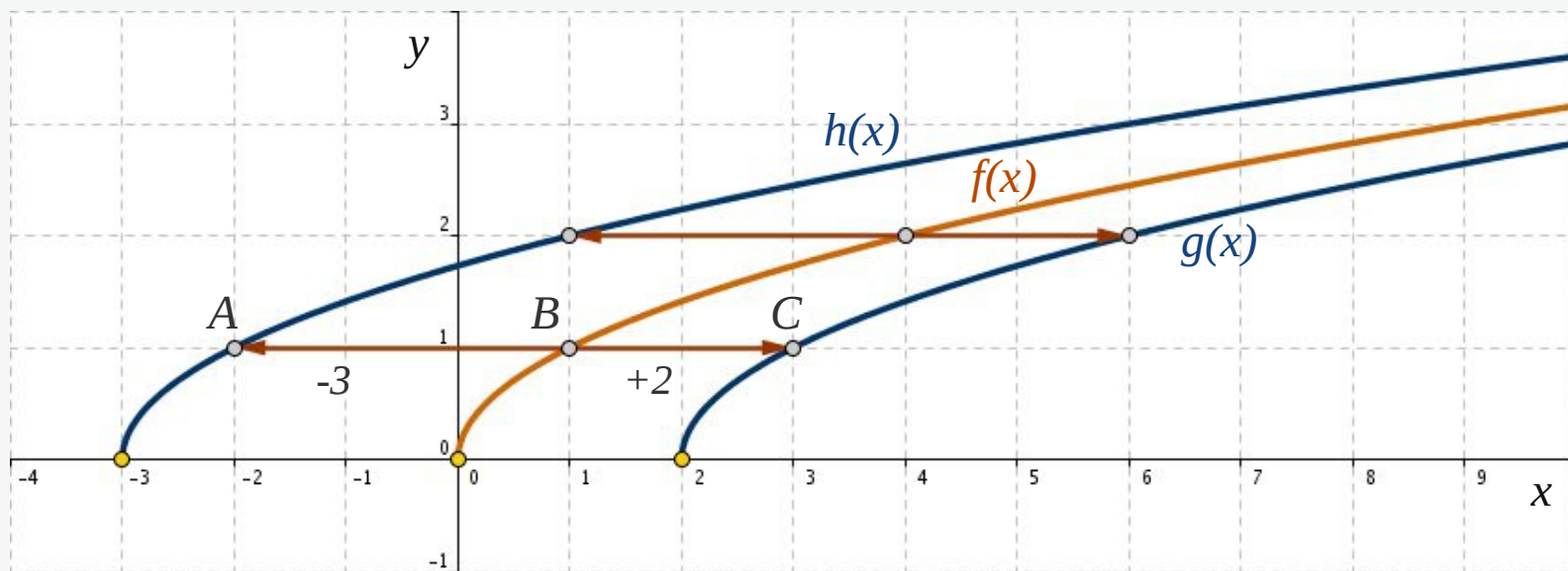


Fig. 3: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{x}, \quad g(x) = \sqrt{x-2}, \quad h(x) = \sqrt{x+3}$$

$$A = (-2, 1), \quad B = (1, 1), \quad C = (3, 1)$$

$$f(x) = \sqrt{x} \xrightarrow{\overrightarrow{BC}} g(x) = \sqrt{x-2}$$

$$f(x) = \sqrt{x} \xrightarrow{\overrightarrow{BA}} h(x) = \sqrt{x+3}$$

Root functions

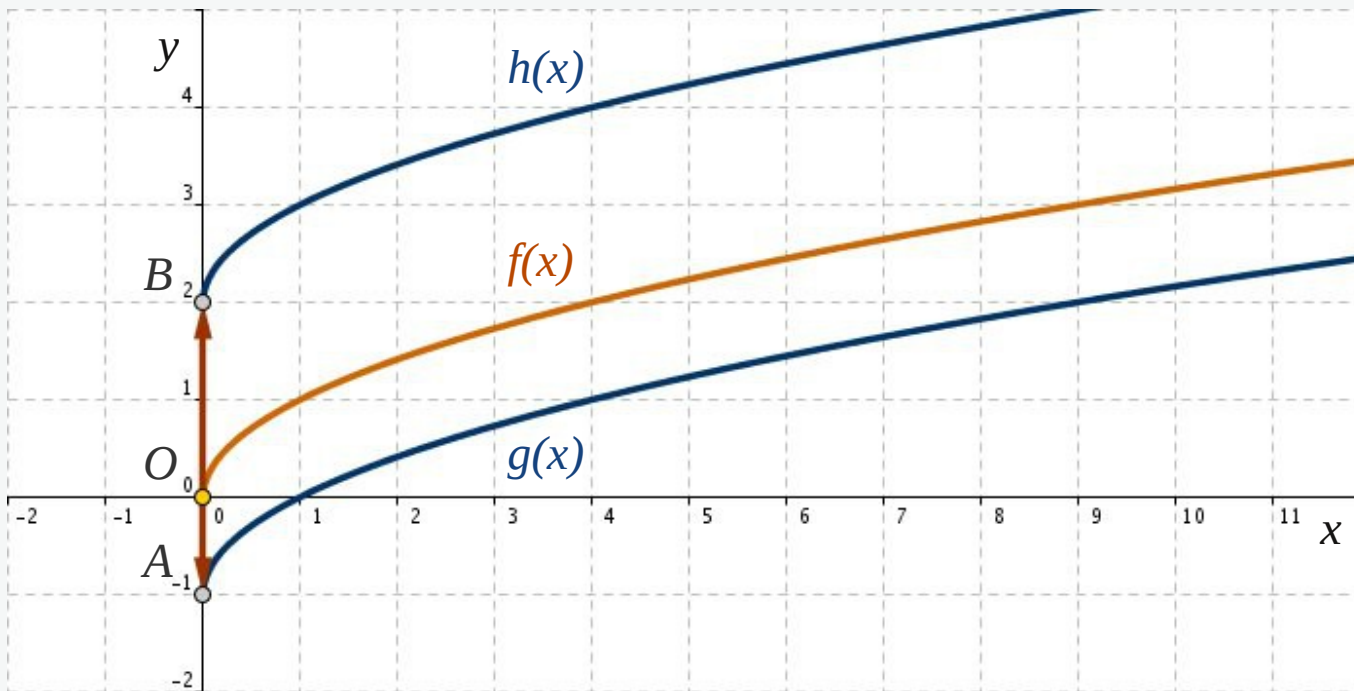


Fig. 4: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{x}, \quad g(x) = \sqrt{x} - 1, \quad h(x) = \sqrt{x} + 2$$

$$f(x) = \sqrt{x} \xrightarrow{\vec{OA}} g(x) = \sqrt{x} - 1$$

$$f(x) = \sqrt{x} \xrightarrow{\vec{OB}} h(x) = \sqrt{x} + 2$$

Root functions

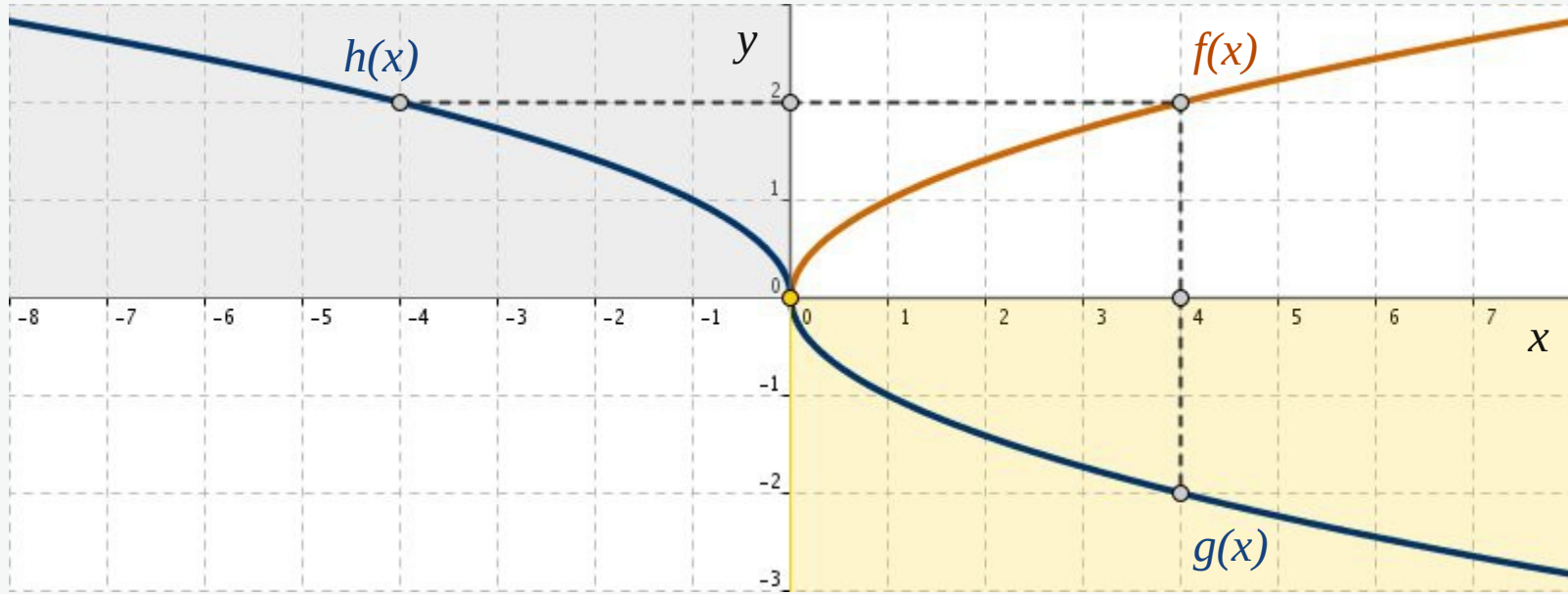


Fig. 5: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{x}, \quad g(x) = -\sqrt{x}, \quad h(x) = \sqrt{-x}$$

$$f(x) = \sqrt{x} \quad \xrightarrow{\text{x-Achse}} \quad g(x) = -\sqrt{x}$$

$$f(x) = \sqrt{x} \quad \xrightarrow{\text{y-Achse}} \quad h(x) = \sqrt{-x}$$

Root functions

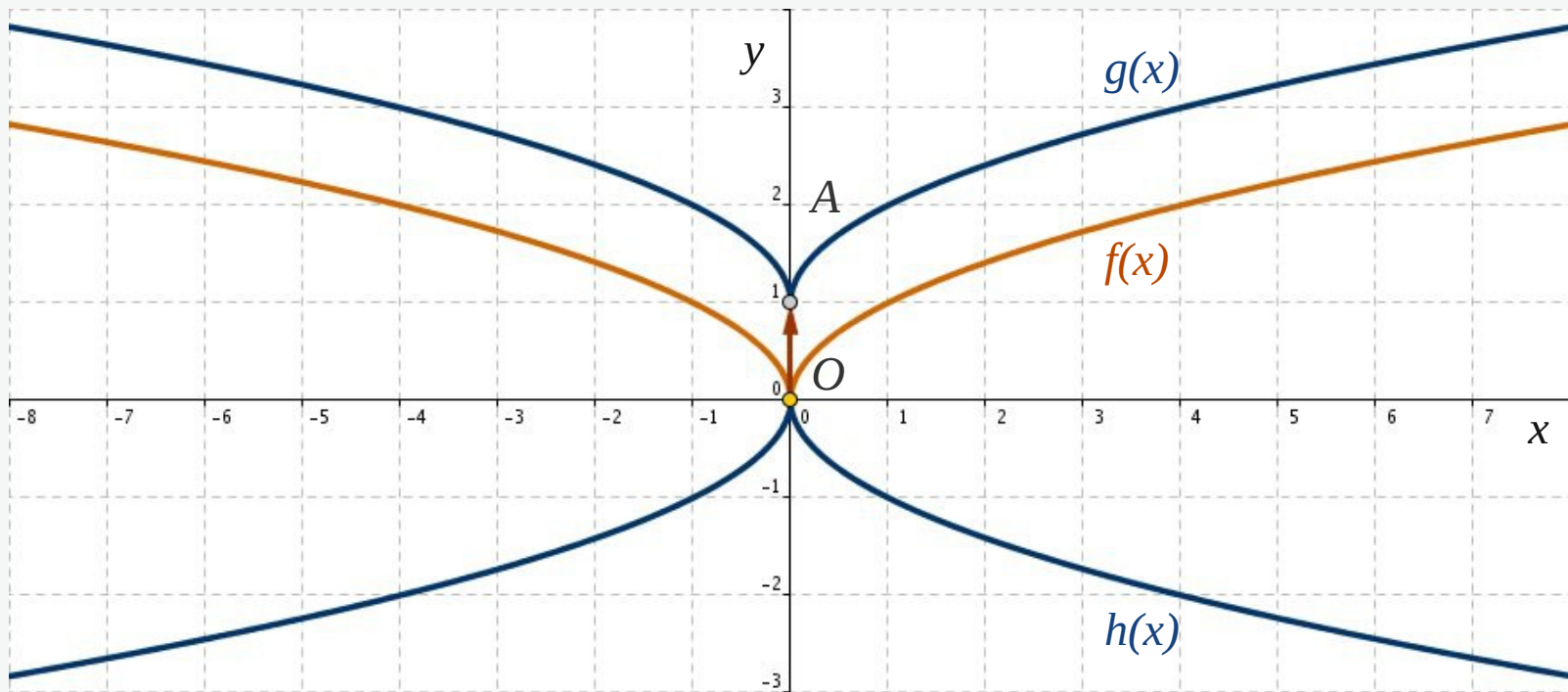


Fig. 6: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{|x|}, \quad g(x) = \sqrt{|x|} + 1, \quad h(x) = -\sqrt{|x|}$$

$$f(x) = \sqrt{|x|} \xrightarrow[\vec{OA}]{x\text{-Achse}} h(x) = -\sqrt{|x|}$$

$$f(x) = \sqrt{|x|} \xrightarrow{\vec{OA}} g(x) = \sqrt{|x|} + 1$$

Root functions

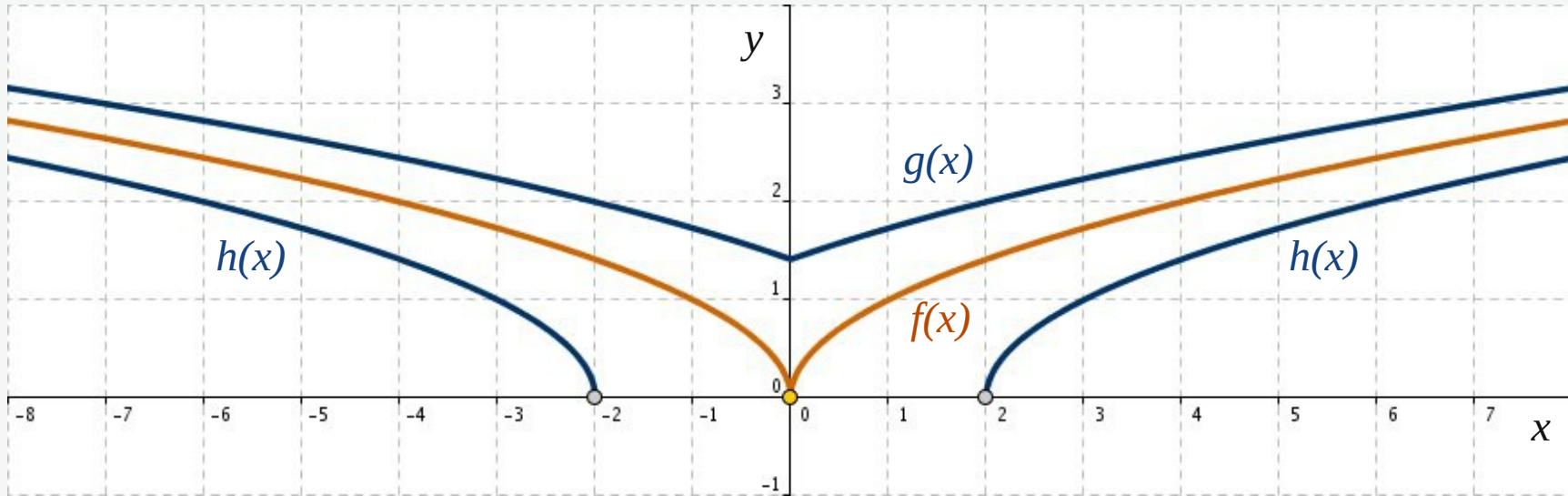


Fig. 7: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{|x|}, \quad g(x) = \sqrt{|x| + 2}, \quad h(x) = \sqrt{|x| - 2}$$

$$D(f(x)) = D(g(x)) = \mathbb{R}, \quad D(h(x)) = (-\infty, -2] \cup [2, \infty)$$

Root functions: exercises 1,2

Exercise 1:

Describe how the graph of the function $f(x)$ is transformed to that of $g(x)$

$$a) f(x) = \sqrt{x}, \quad g(x) = \sqrt{x+3} + 1$$

$$b) f(x) = \sqrt{x}, \quad g(x) = 1.5\sqrt{x+3}$$

Exercise 2:

Plot the following functions

$$f(x) = \sqrt{2-x}, \quad g(x) = \sqrt{x+2} + 1$$

Root functions: solution 1a

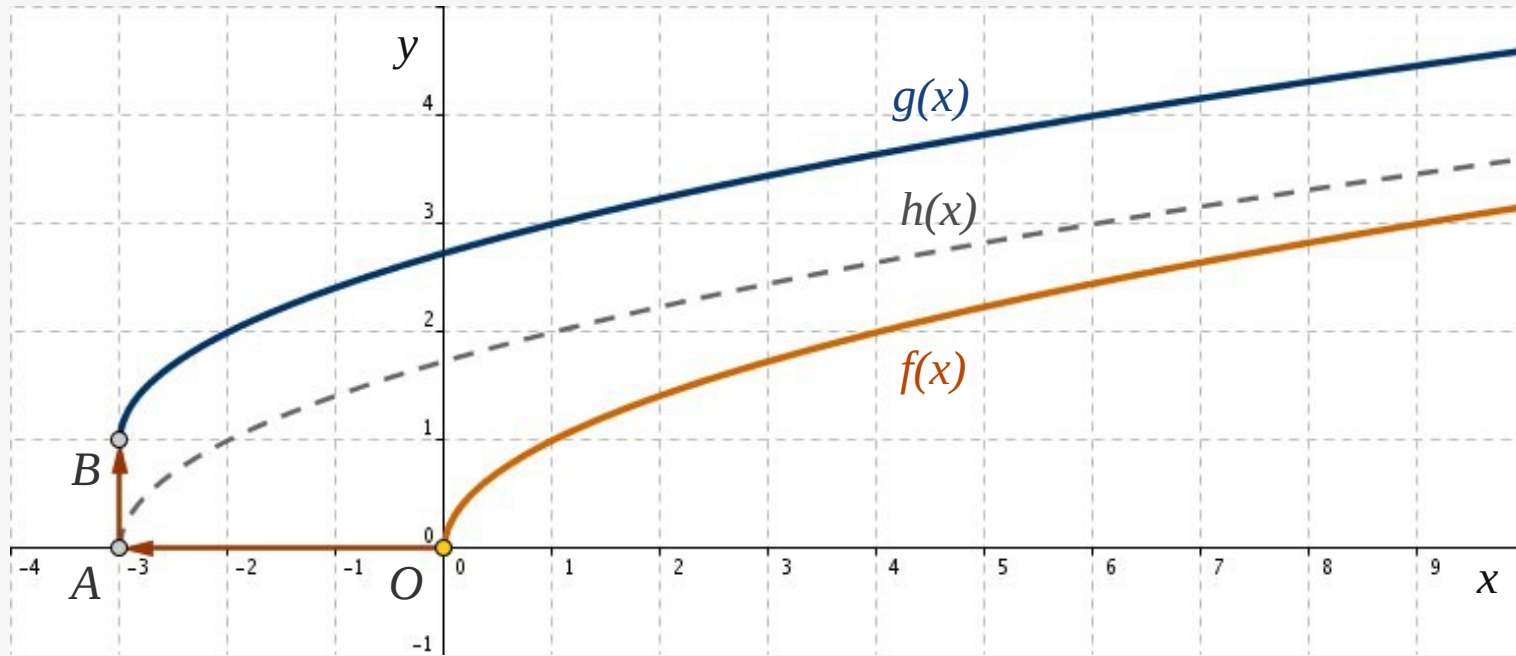


Fig. 8-1: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{x} \quad \begin{matrix} \vec{OA} \\ \overline{} \\ \rightarrow \end{matrix} \quad h(x) = \sqrt{x+3} \quad \begin{matrix} \vec{AB} \\ \overline{} \\ \rightarrow \end{matrix} \quad g(x) = \sqrt{x+3} + 1$$

Root functions: solution 1b

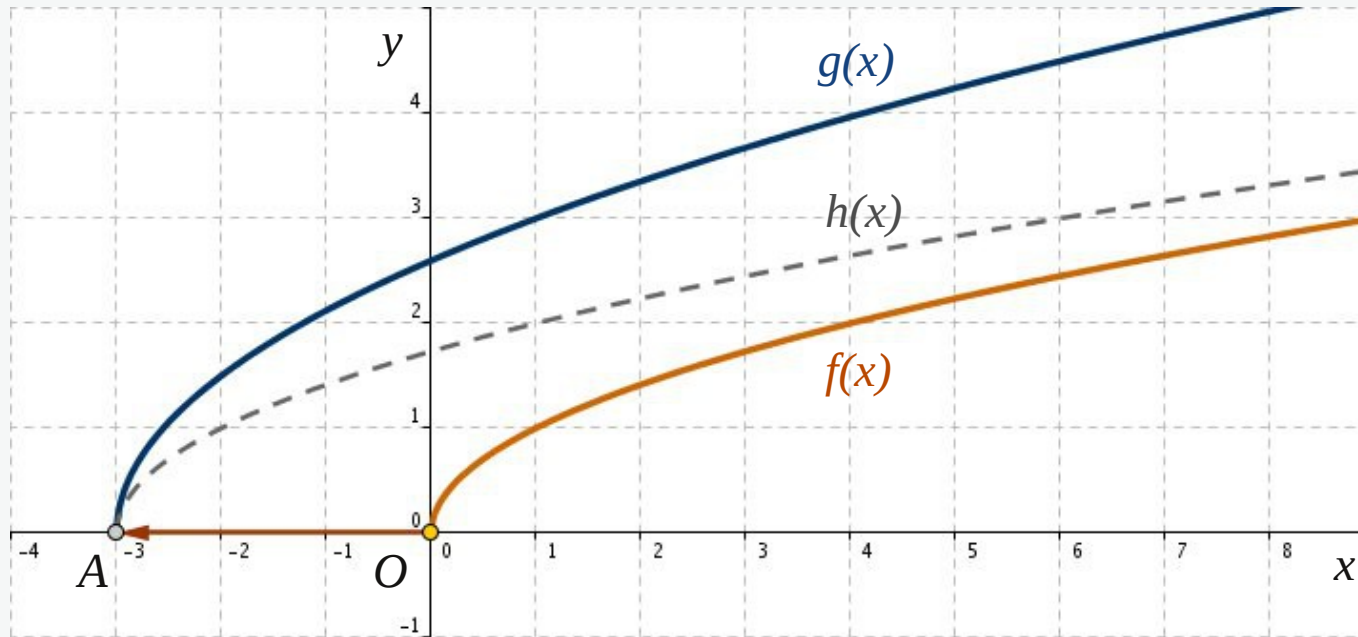


Fig. 8-2: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{x} \quad \begin{matrix} \vec{OA} \\ \rightarrow \end{matrix} \quad h(x) = \sqrt{x + 3} \quad \begin{matrix} \times 1.5 \\ \rightarrow \end{matrix} \quad g(x) = 1.5\sqrt{x + 3}$$

Root functions: solution 2

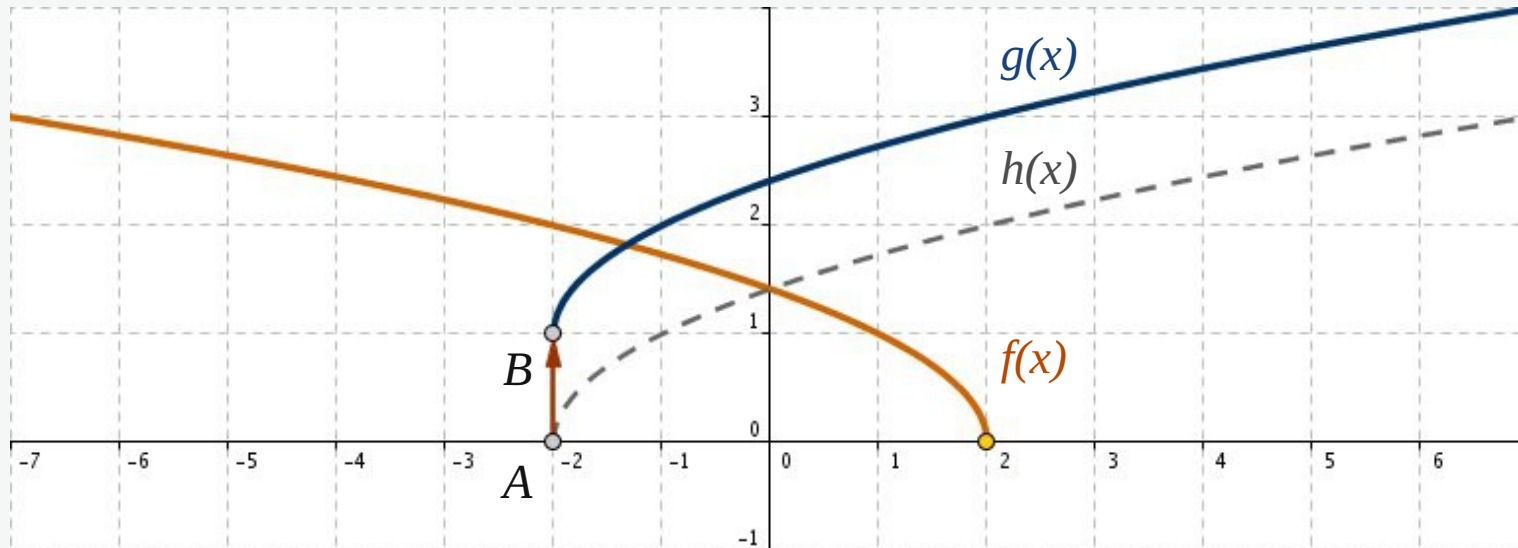


Fig. 9: Root functions $f(x)$, $g(x)$ and $h(x)$

$$f(x) = \sqrt{2 - x}$$

$$h(x) = \sqrt{x + 2} \quad \xrightarrow{\overline{AB}} \quad g(x) = \sqrt{x + 2} + 1$$