http://www.ebrew.com/Images_R/rq_wine_glass.jpg

Stories are told of opera singers, who make glasses burst by singing. Indeed glasses may burst, if excited by a specific frequency.

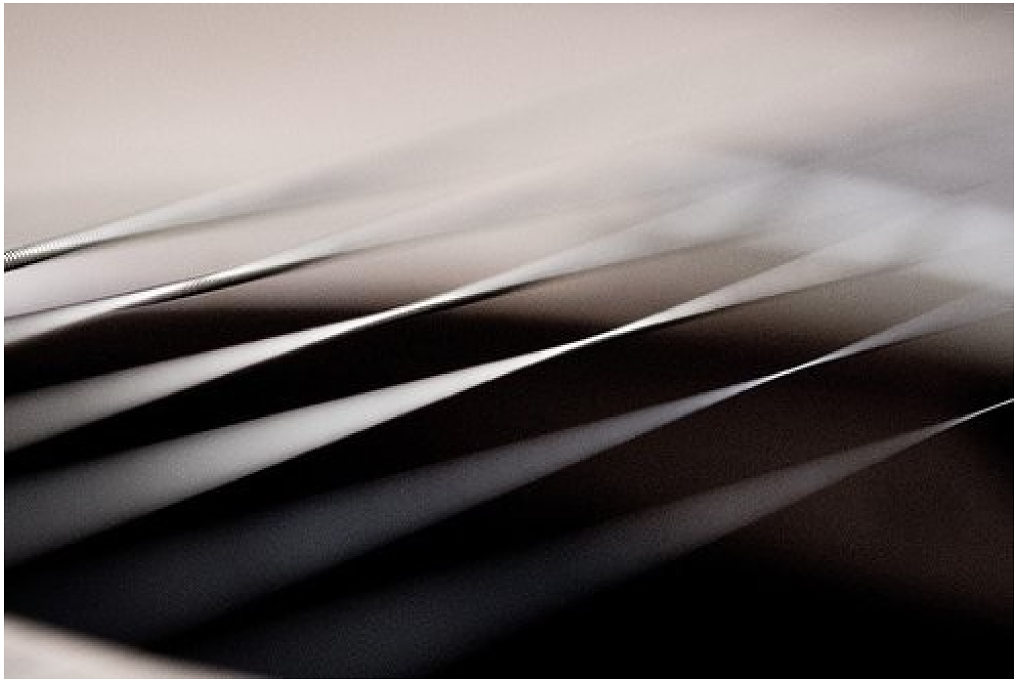
http://www.youtube.com/watch?v=4EnSTLH492U&feature=related

Some videos demonstrating the oscillations of glasses.

- *http://www.youtube.com/watch?v=5VZcY1B8iPM*
- *http://www.youtube.com/watch?v=4EnSTLH492U&feature=related*
- *http://www.youtube.com/watch?v=uVvnw3MfxkI&feature=related*

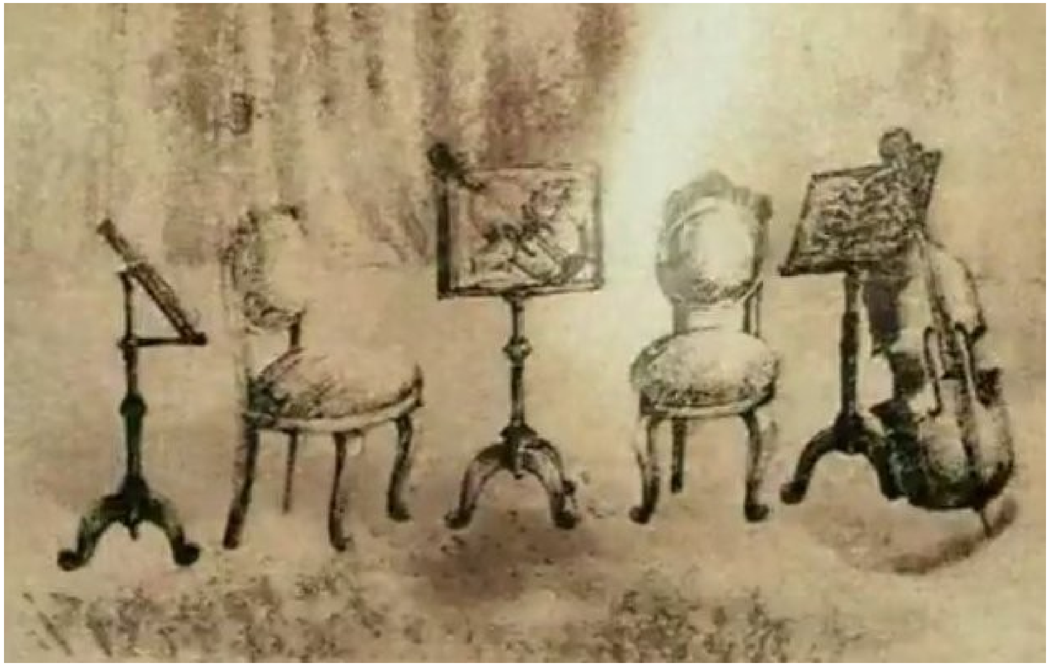
http://marqspusta.com/newz/wp-content/uploads/2009/10/springswing1.jpg

How does a child manage to rock without help? By proper swinging of legs and body.



http://www.flickr.com/photos/jonblock/448581594/

The sound of musical instruments is due to oscillations. The tune depends on the frequency.



http://www.youtube.com/watch?v=vORsKyopHyM

It is well known that music essentially consists of vibrations or oscillations. But if we look more closely, we notice that some kind of wave is always involved: the sine wave.

Sine Wave

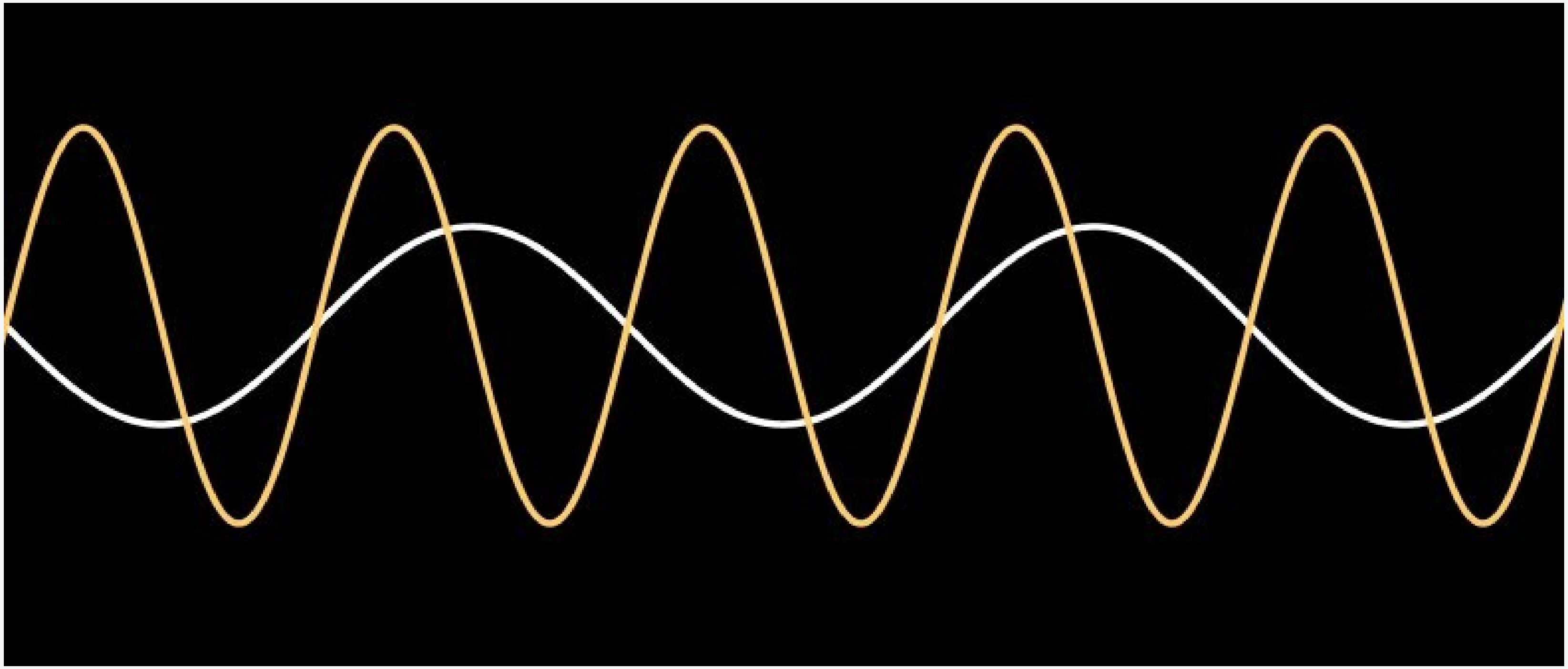


Fig. 1: Sine wave

But originally the sine function was not related to waves, its significance was in geometry.

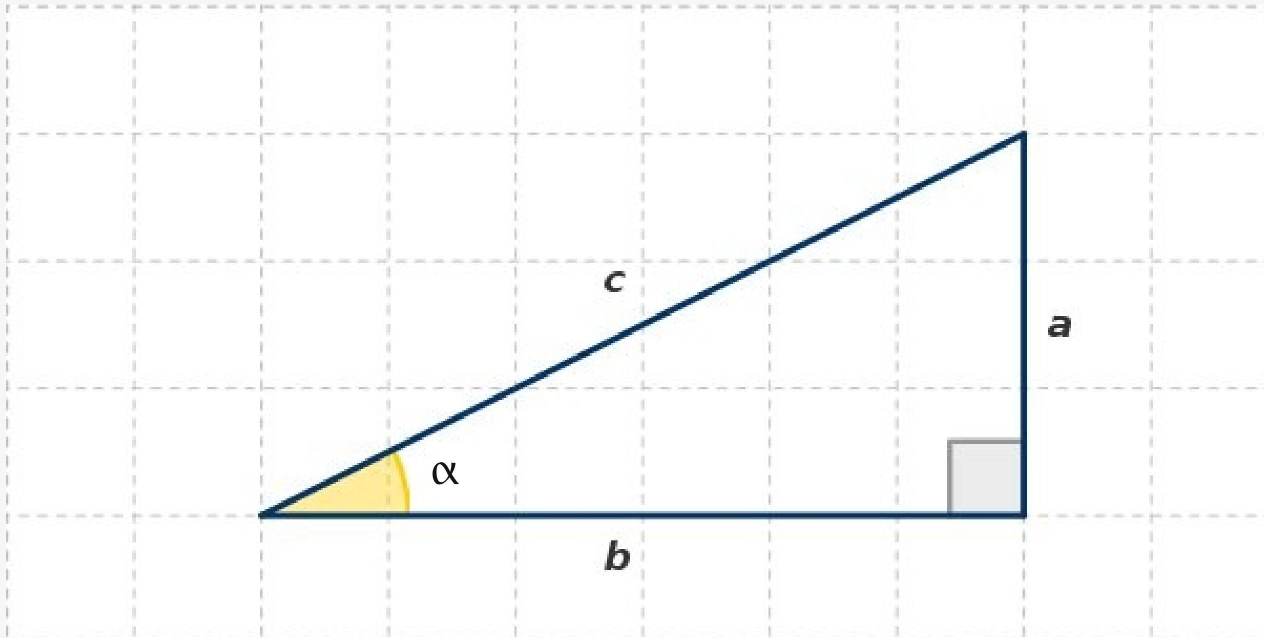


Fig. 2-1: A right triangle.

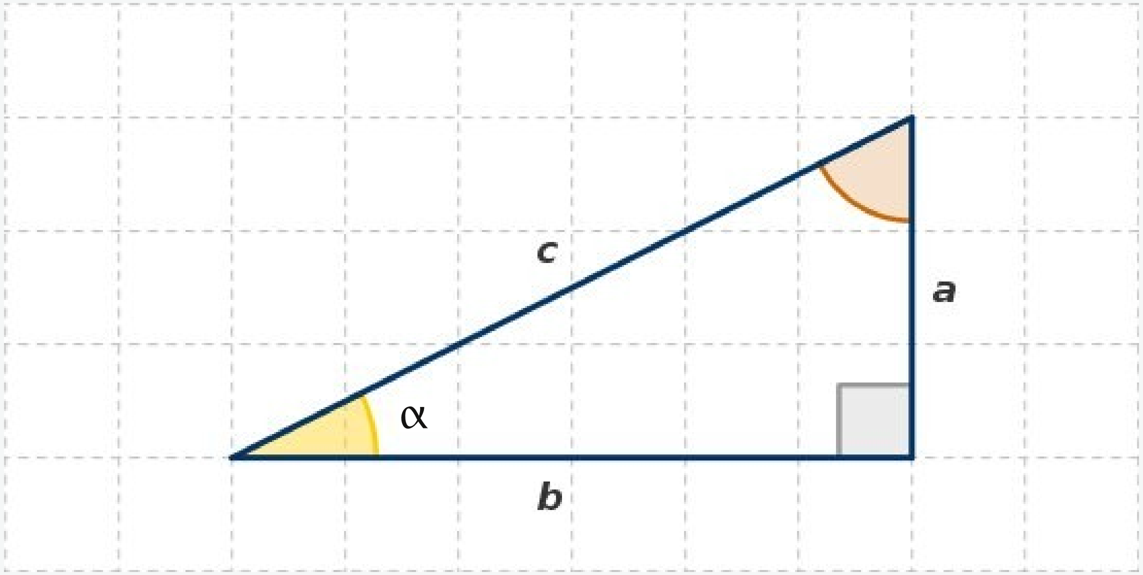
We consider a right triangle (Fig. 2-1). The angle α is one of the two smaller angles. The sine of α (written $\sin\alpha$) is the ratio of the length of the opposite side a to the length of the hypotenuse c. Correspondingly, the cosine of α is the ratio of the length of the adjacent side b to the length of the hypoteuse c.

Trigonometric functions

$$\sin\alpha = \frac{a}{c}$$

$$\cos\alpha = \frac{b}{c}$$

$$\tan\alpha = \frac{a}{b} = \frac{a/c}{b/c} = \frac{\sin\alpha}{\cos\alpha}$$



$$\cot\alpha = \frac{b}{a} = \frac{b/c}{a/c} = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$$

a is the opposite side (opposite cathetus),
b is the adjacent side (adjacent cathetus),
c is the hypotenuse.

Right triangle

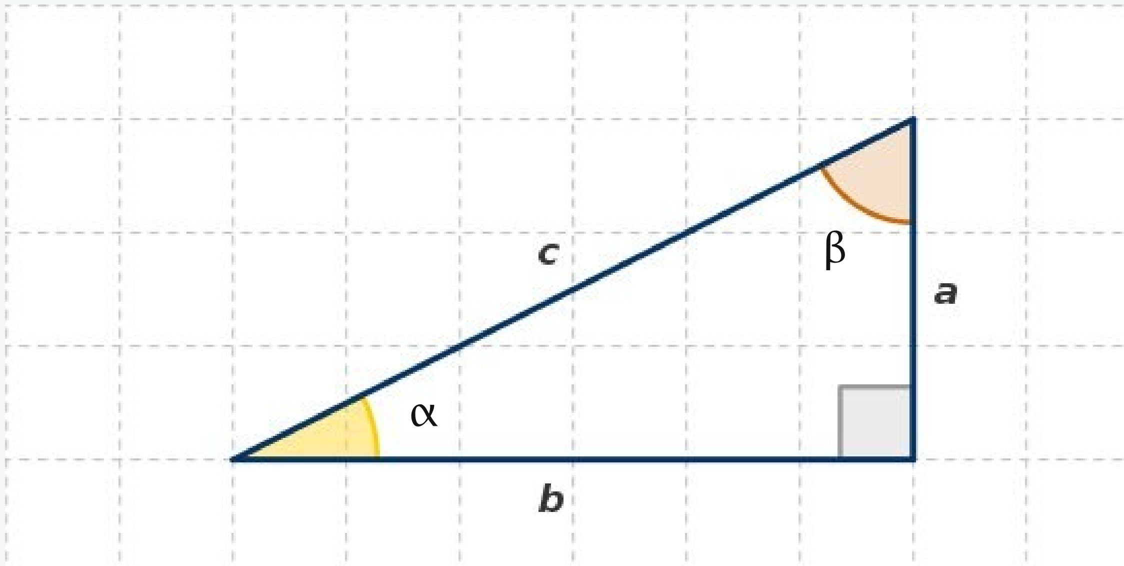


Fig. 2-2: Right triangle.

$$\sin\beta = \frac{b}{c}, \qquad \cos\beta = \frac{a}{c}$$

$$\tan\beta = \frac{b}{a} = \frac{b/c}{a/c} = \frac{\sin\beta}{\cos\beta}$$

$$\cot\beta = \frac{a}{b} = \frac{a/c}{b/c} = \frac{\cos\beta}{\sin\beta} = \frac{1}{\tan\beta}$$

Important Relations

It is easy to deduce from the above definitions of the trigonometric functions the following important relations for acute angles:

$$\sin^2\alpha + \cos^2\alpha = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

Pythagorean theorem: $a^2 + b^2 = c^2$

$$\frac{a}{c} = \sin\alpha = \cos\beta = \cos(90^\circ - \alpha)$$

$$\frac{b}{c} = \cos\alpha = \sin\beta = \sin(90^\circ - \alpha)$$

Right triangle

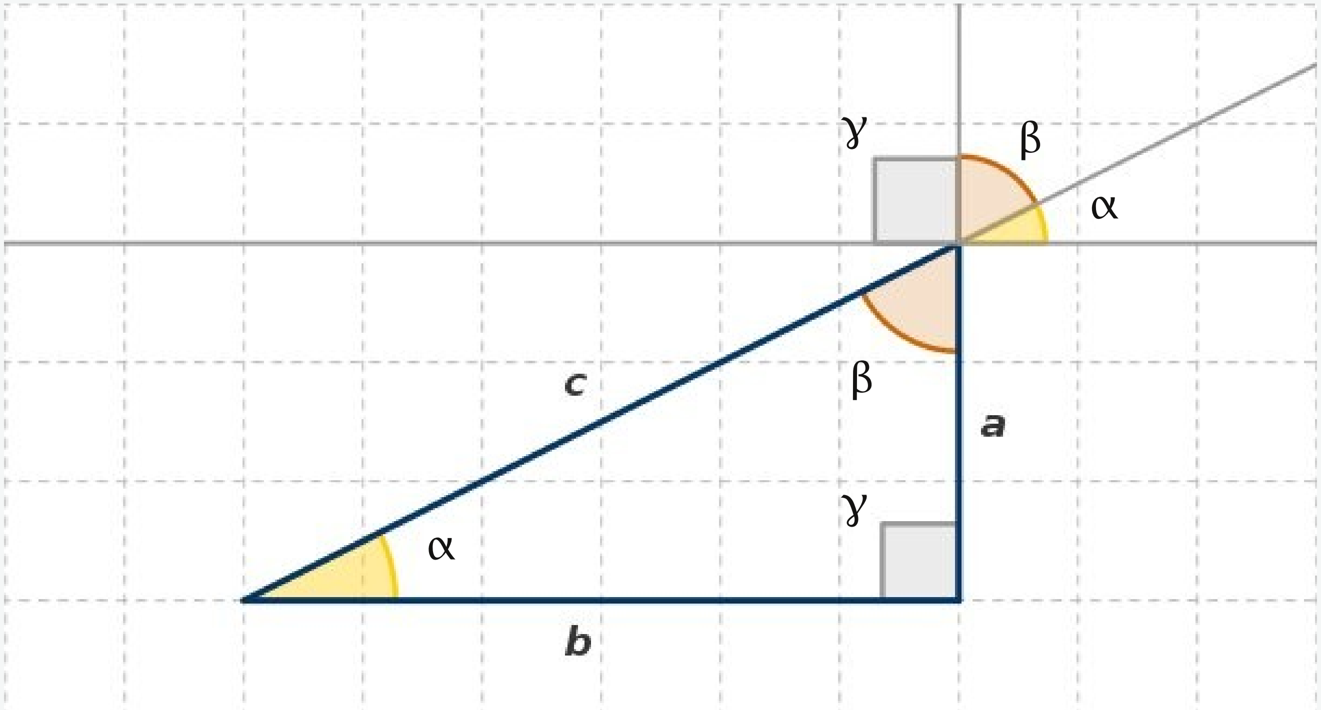


Fig. 2-3: Angles of right triangle

$$\gamma = 90^\circ, \qquad \alpha + \beta = 90^\circ \quad \Rightarrow$$

$$\alpha = 90^\circ - \beta, \qquad \beta = 90^\circ - \alpha$$

0, 45 and 90 degree angles

1) $\alpha = 0^\circ, \quad a = 0$

$$\sin(0^\circ) = \cos(90^\circ - 0^\circ) = \cos(90^\circ) = \tan(0^\circ) = 0$$

2) $\alpha = 90^\circ, \quad a = c \quad \Rightarrow \quad \frac{a}{c} = 1, \quad \frac{a}{b} \to \infty$

$$\sin(90^\circ) = \cos(90^\circ - 90^\circ) = \cos(0^\circ) = 1, \quad \cot(90^\circ) = 0$$

$\tan\alpha$ is undefined

3) $\alpha = 45^\circ, \quad \alpha = \beta, \quad a = b \quad \Rightarrow \quad \frac{a}{b} = 1$

$$c^2 = a^2 + b^2 = a^2 + a^2 = 2a^2, \quad c = \sqrt{2}\,a$$

$$\frac{a}{c} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan(45^\circ) = \cot(45^\circ) = \frac{a}{a} = 1$$

30 and 60 degree angles

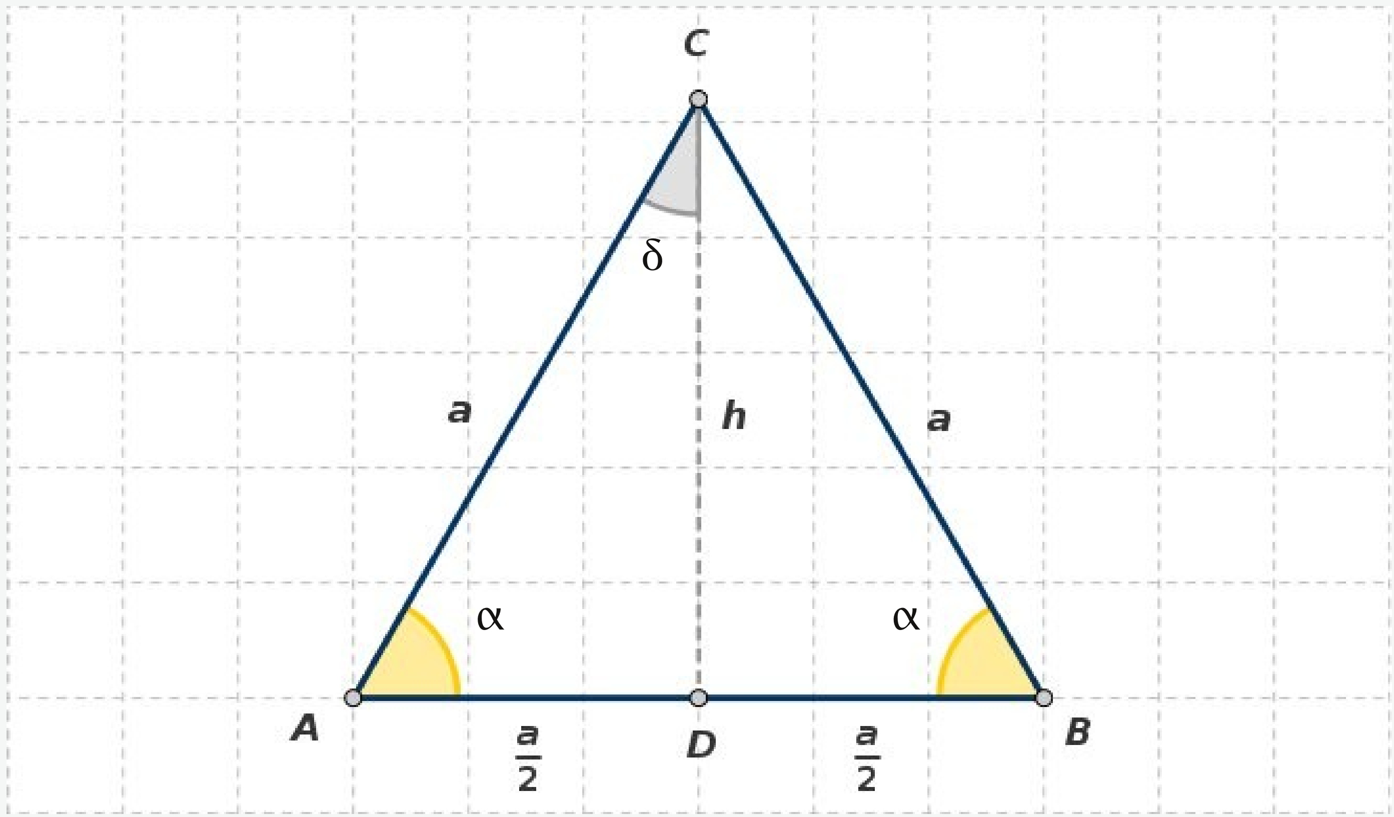


Fig. 2-4: Construction of 30 and 60 degree angles

Angles of 30^o and 60^o in a right triangle are obtained by drawing the height of an equilateral triangle. For sub-triangle *ADC* holds:

$$| AD | = \frac{a}{2}, \qquad \alpha = 60^\circ, \qquad \delta = 30^\circ$$

30 and 60 degree angles

The height h of the equilateral triangle:

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{a}{2}\sqrt{3}$$

$$\sin\alpha = \sin(60^\circ) = \cos(30^\circ) = \frac{h}{a} = \frac{\sqrt{3}}{2}$$

$$\tan\alpha = \tan(60^\circ) = \cot(30^\circ) = \frac{h}{\frac{a}{2}} = \sqrt{3}$$

$$\cos\alpha = \cos(60^\circ) = \sin(30^\circ) = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$

$$\tan\delta = \tan(30^\circ) = \cot(60^\circ) = \frac{1}{\sqrt{3}}$$