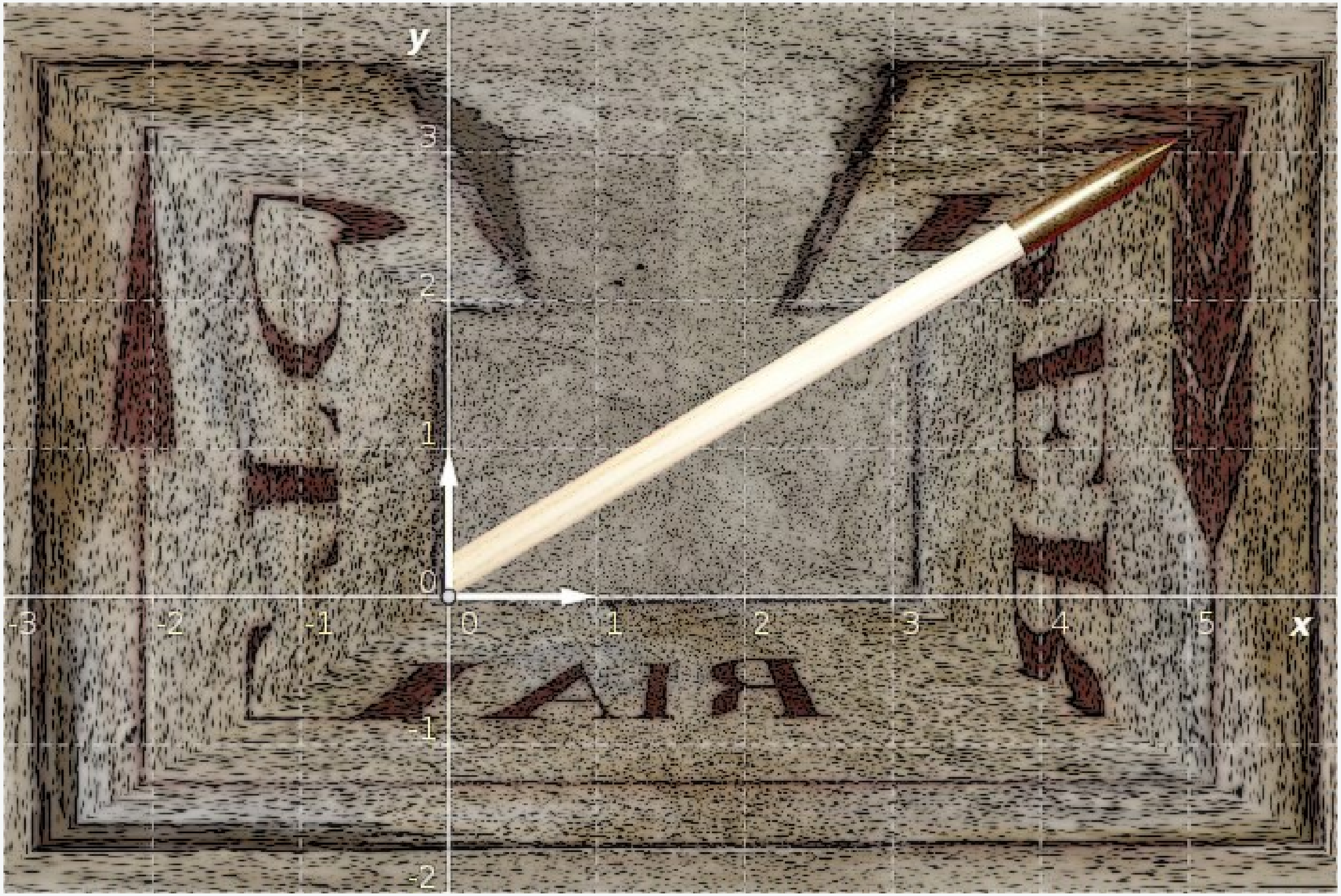
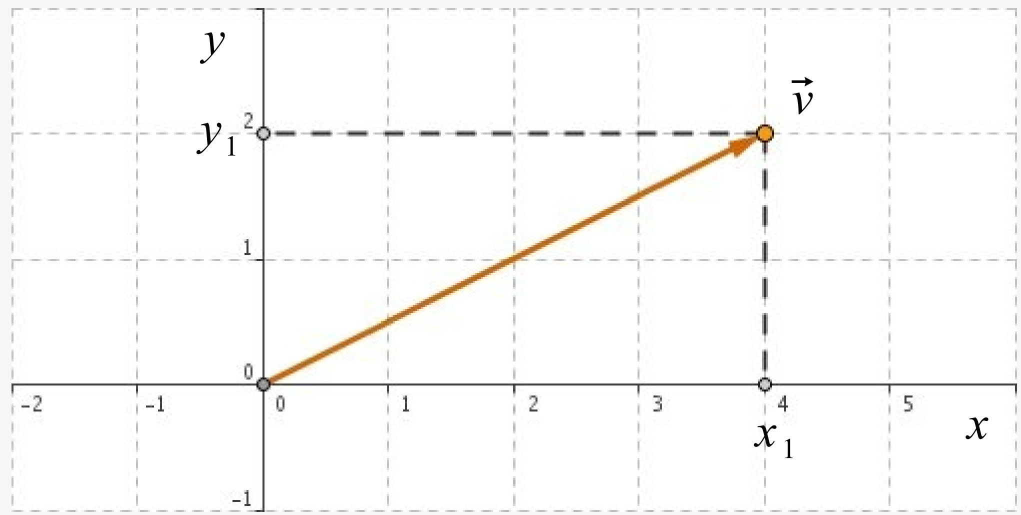


Vectors

The figure below shows a Cartesian coordinate system. The points in the plane can be identified as elements of $\mathbb{R}^2$. We attribute two coordinates to the point v on the plane and write

$$\vec{v} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \in \mathbb{R}^2$$



Thus the set of all column vectors represents the whole set of points

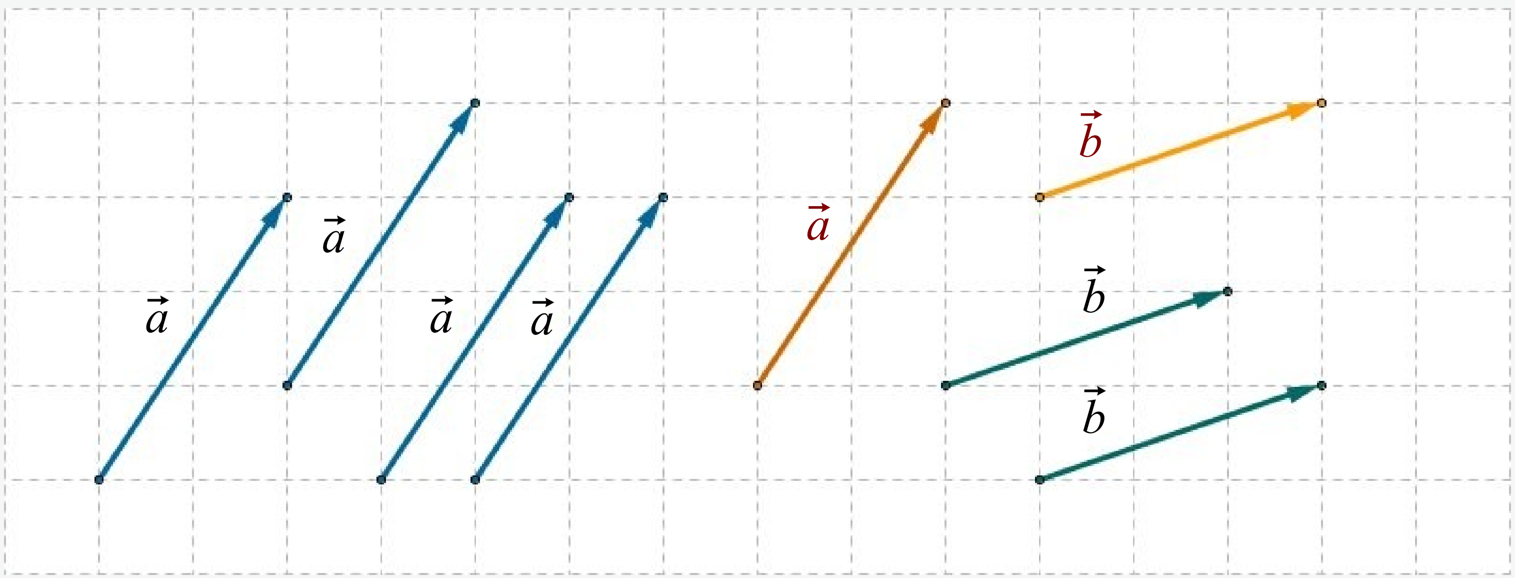
$$\mathbb{R}^2 = \left\{ \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \;\middle|\; x_1, \ y_1 \in \mathbb{R} \right\}$$

Definition 1:

A vector is the set of all arrows with a given length and direction.
A single arrow out of this set is a representative of the vector.

Definition 2:

Vectors are oriented objects.



important! length and direction of the vector

not important! where it is

Magnitude (also length) of a vector is a measure of the length of the arrow:

$$|\vec{a}| = a \geqslant 0$$

A vector is uniquely defined by:

1. length and direction *or*
2. initial point and end point

Special Vectors:

zero vector : each vector of length zero $|\vec{0}| = 0\,, \quad \vec{a} + \vec{0} = \vec{a}$

unit vector : each vector of length one $|\vec{u}| = 1$

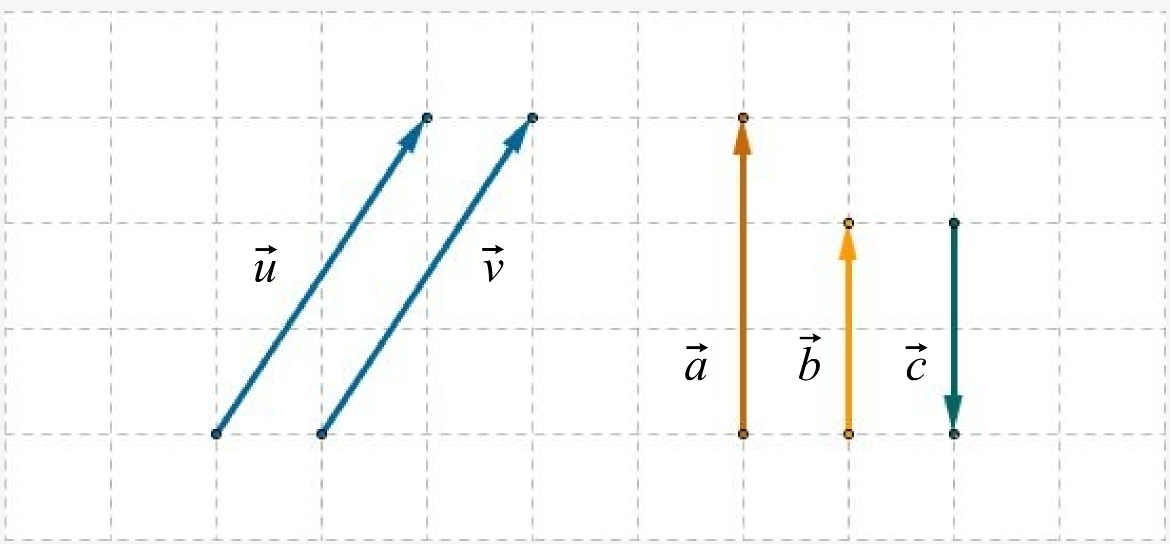
Two vectors are equal, if they have same length and same direction:

$$\vec{u} = \vec{v}\,.$$

$\vec{u} \uparrow\uparrow \vec{v}\,,\quad \vec{a} \uparrow\uparrow \vec{b}$ – parallel vectors

$\vec{a} \uparrow\downarrow \vec{c}\,,\quad \vec{b} \uparrow\downarrow \vec{c}$ – antiparallel vectors

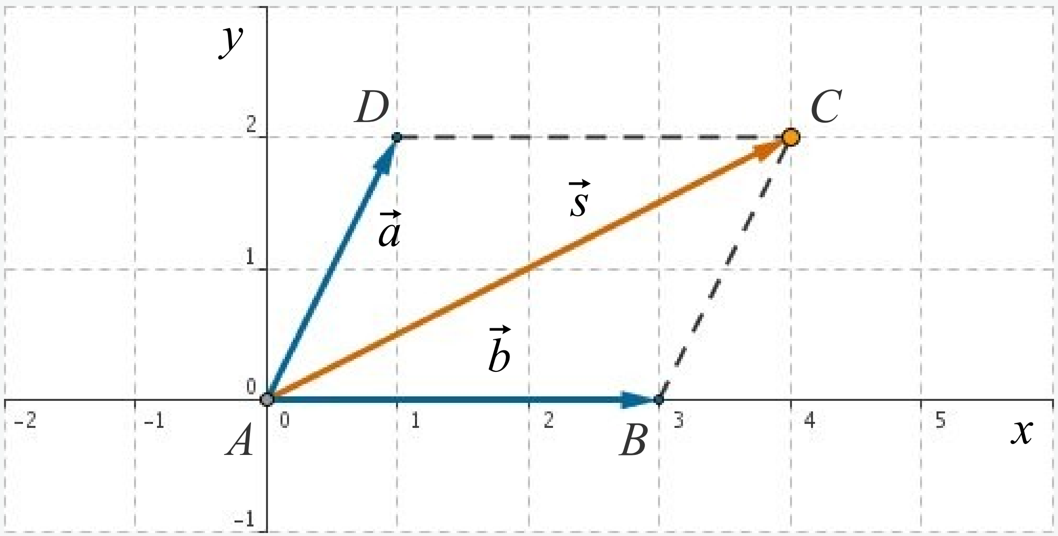
$\vec{b} = -\vec{c}$ – $\vec{b}$ and $\vec{c}$ have same length and opposite direction



Two vectors add to produce the oriented diagonal of a parallelogram

$$\vec{s} = \vec{a} + \vec{b}$$

Parallelogram rule



$$\overrightarrow{AB} = \overrightarrow{DC}\,, \quad \overrightarrow{BC} = \overrightarrow{AD}$$

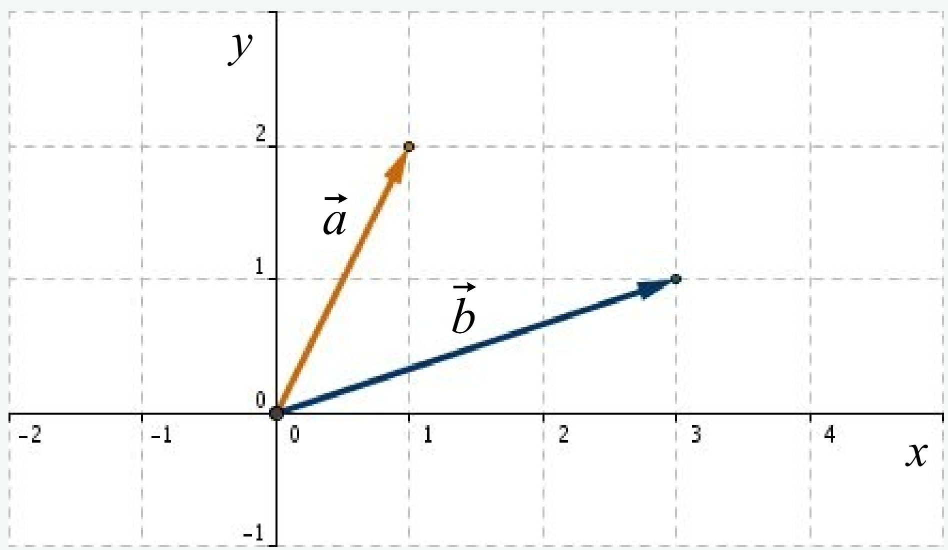
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC} = \vec{s}$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

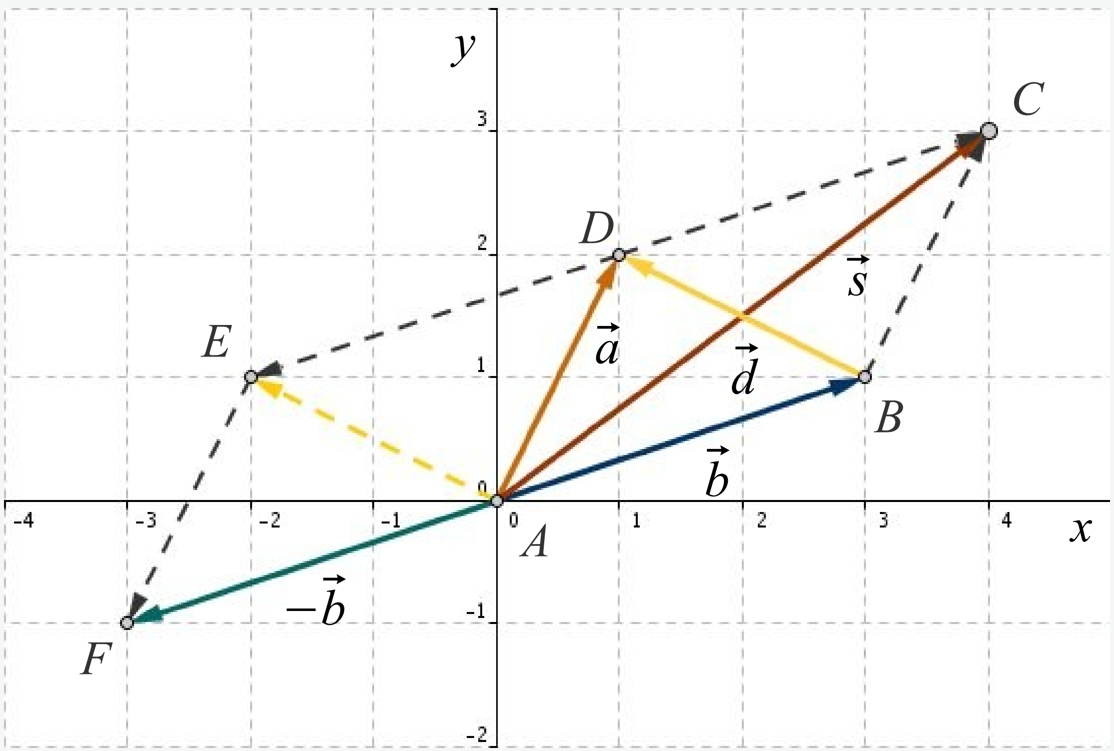
Vector addition and subtraction: Example 1

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{a} + \vec{b} = \begin{pmatrix} 1+3 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad \vec{a} - \vec{b} = \begin{pmatrix} 1-3 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

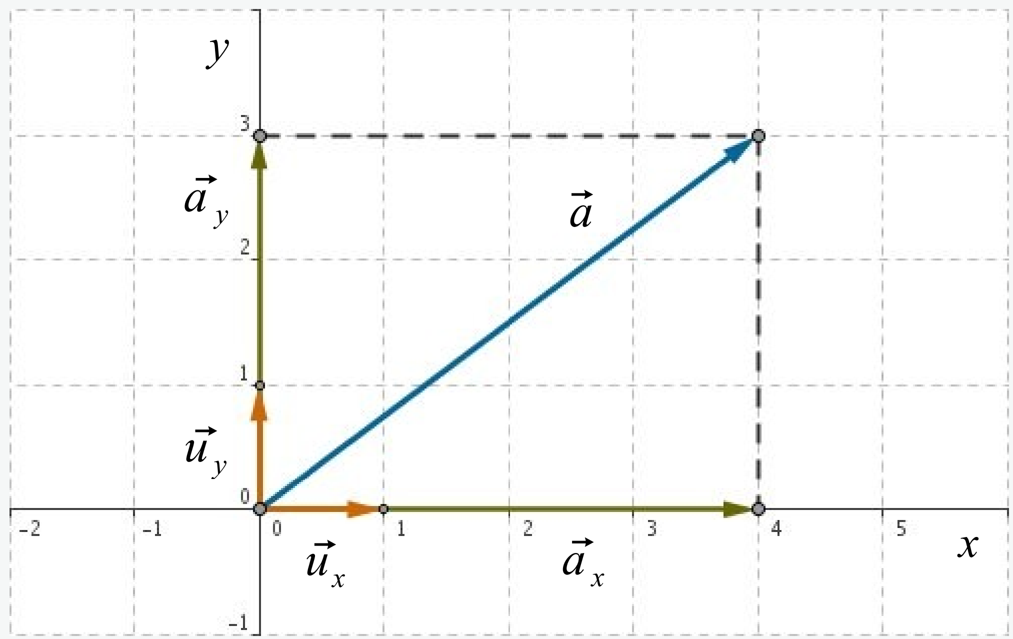


$$\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \qquad \vec{a} + \vec{b} = \begin{pmatrix} 1+3 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \qquad \vec{a} - \vec{b} = \begin{pmatrix} 1-3 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$



$$\vec{a} + \vec{b} = \overrightarrow{AD} + \overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC} = \vec{s}$$

$$\vec{a} - \vec{b} = \overrightarrow{AD} - \overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE} = \overrightarrow{BD} = \vec{d}$$

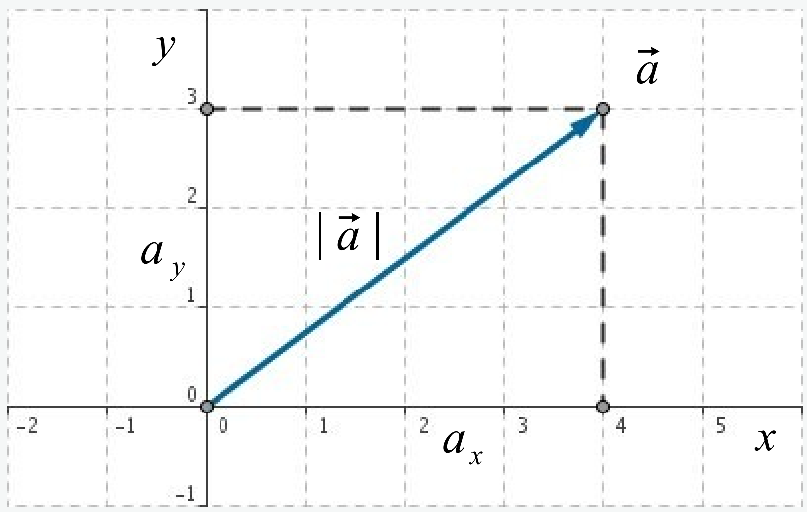


$$\vec{a} = \vec{a}_x + \vec{a}_y = a_x \vec{u}_x + a_y \vec{u}_y = \begin{pmatrix} a_x \\ a_y \end{pmatrix} \quad \leftarrow \text{ column vector}$$

$$\vec{u}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{u}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad – \text{ basis vectors}$$

$\vec{a}_x , \ \vec{a}_y$ – vector components

$a_x , \ a_y$ – vector coordinates



$$\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}, \qquad |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

Example: Position vector of point $P = (4, 3)$.

$$\vec{r}(P) = \overrightarrow{OP} = 4\,\vec{u}_x + 3\,\vec{u}_y = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$|\vec{r}(P)| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$