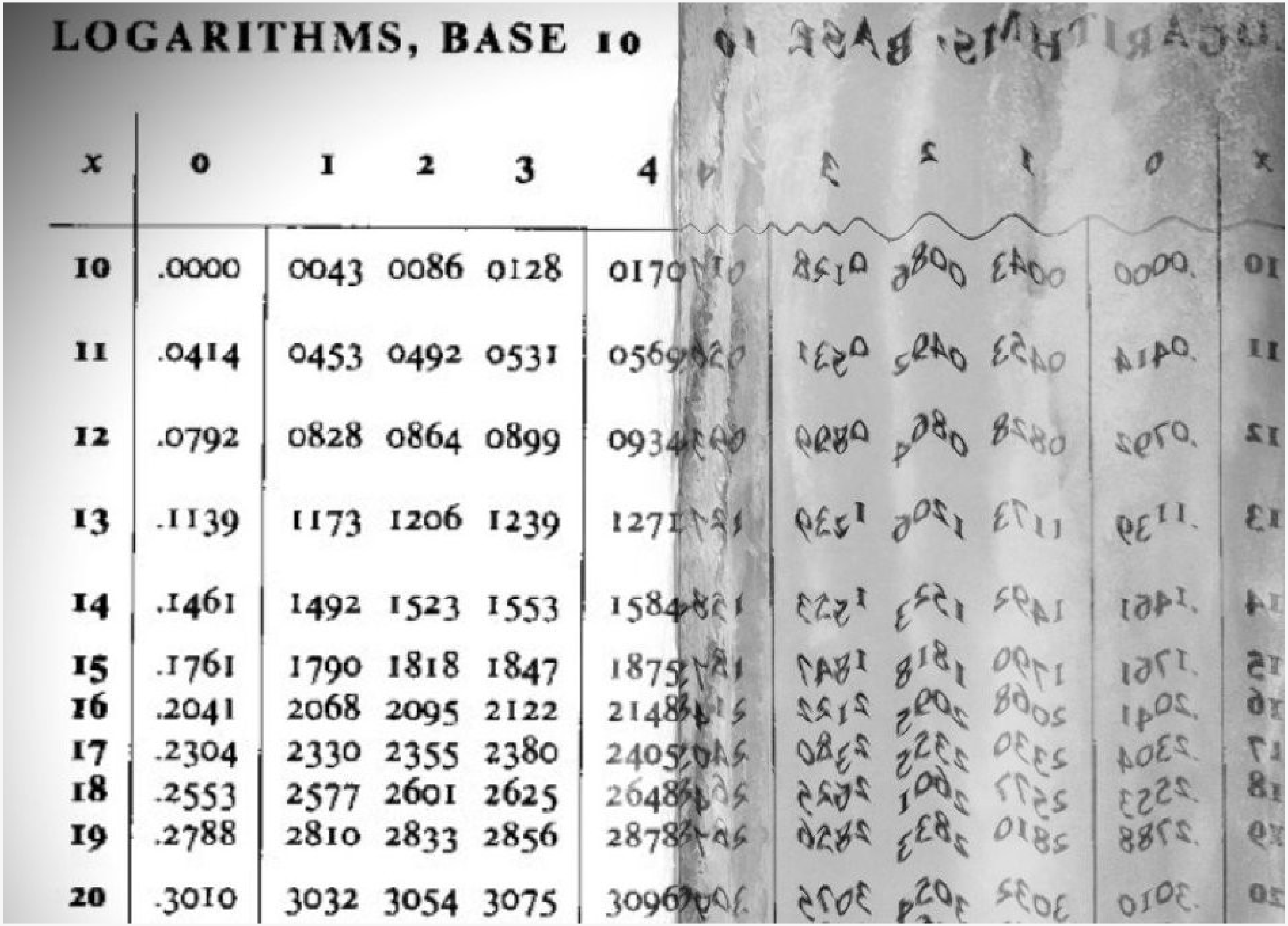


LOGARITHMS, BASE 10

x	0	1	2	3	4
10	.0000	0043	0086	0128	0170
11	.0414	0453	0492	0531	0569
12	.0792	0828	0864	0899	0934
13	.1139	1173	1206	1239	1271
14	.1461	1492	1523	1553	1584
15	.1761	1790	1818	1847	1875
16	.2041	2068	2095	2122	2148
17	.2304	2330	2355	2380	2405
18	.2553	2577	2601	2625	2648
19	.2788	2810	2833	2856	2878
20	.3010	3032	3054	3075	3096

Das Rechnen mit Logarithmen

Spezielle Logarithmen

- Der natürliche Logarithmus ist von besonderer Bedeutung in den Anwendungen: Basiszahl ist die Eulersche Zahl e:

$$\log_e x \equiv \ln x$$

<u>gelesen</u>: *natürlicher Logarithmus von x*

- Der Logarithmus für die Basiszahl $a = 10$, Zehnerlogarithmus, auch Briggscher oder Dekadischer Logarithmus genannt

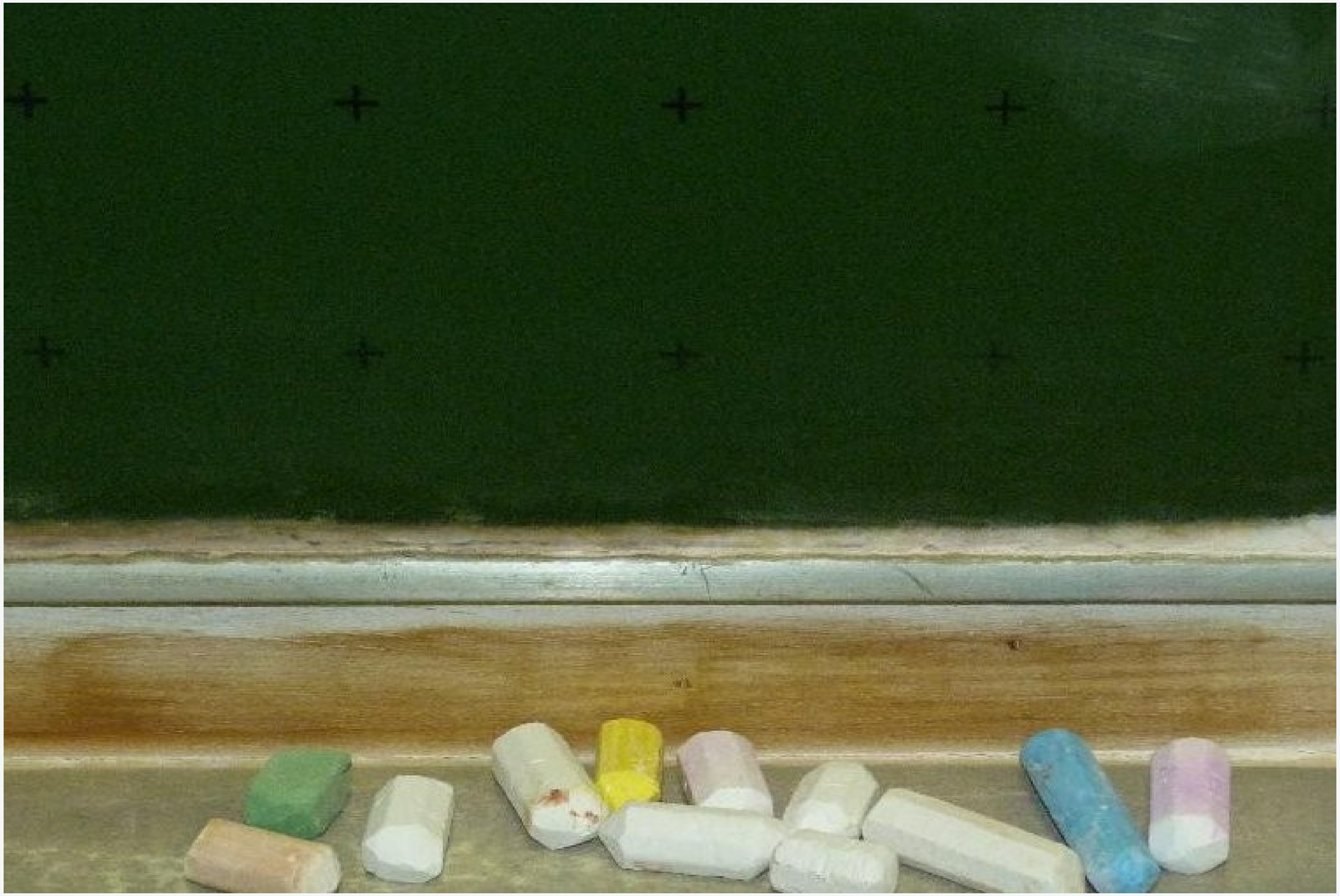
$$\log_{10} x \equiv lg\, x$$

<u>gelesen</u>: *Zehnerlogarithmus von x*

- Der Logarithmus für die Basiszahl $a = 2$, Zweierlogarithmus, auch Binärlogarithmus genannt

$$\log_2 x \equiv lb\, x$$

<u>gelesen</u>: *Zweierlogarithmus von x*



Aufgaben

Aufgabe 1:

Verwandle folgende Potenzgleichungen in Logarithmengleichungen:

$a)\quad 2^5 = 32, \qquad 2^7 = 128, \qquad 2^{-3} = \frac{1}{8}$

$b)\quad 3^3 = 27, \qquad 3^4 = 81, \qquad 3^{-2} = \frac{1}{9}$

$c)\quad 4^0 = 1, \qquad 4^3 = 64, \qquad 4^{-2} = \frac{1}{16}$

Aufgabe 2:

Verwandle folgende Logarithmengleichungen in Potenzgleichungen:

$\log_3 9 = 2, \qquad \log_7 49 = 2, \qquad \log_6 6 = 1, \qquad \log_8 1 = 0, \qquad \log_4 2 = \frac{1}{2}$

a) $2^5 = 32, \quad \log_2 32 = 5, \qquad 2^7 = 128, \quad \log_2 128 = 7,$

$2^{-3} = \frac{1}{8}, \qquad \log_2 \frac{1}{8} = -3,$

b) $3^3 = 27, \quad \log_3 27 = 3, \qquad 3^4 = 81, \quad \log_3 81 = 4,$

$3^{-2} = \frac{1}{9}, \qquad \log_3 \frac{1}{9} = -2,$

c) $4^0 = 1, \quad \log_4 1 = 0, \qquad 4^3 = 64, \quad \log_4 64 = 3,$

$4^{-2} = \frac{1}{16}, \qquad \log_4 \frac{1}{16} = -2,$

$$\log_3 9 = 2, \qquad 3^2 = 9$$

$$\log_7 49 = 2, \qquad 7^2 = 49$$

$$\log_6 6 = 1, \qquad 6^1 = 6$$

$$\log_8 1 = 0, \qquad 8^0 = 1$$

$$\log_4 2 = \frac{1}{2}, \qquad 4^{\frac{1}{2}} = \sqrt{4} = 2$$

Aufgabe 3:

Berechnen Sie die gegebenen Ausdrücke ohne Taschenrechner:

$a)\quad \log_2 32, \quad \log_2 64, \quad \log_2 \frac{1}{16}, \quad \log_2 \frac{1}{128}, \quad \log_2 2^{-4}, \quad \log_2 1$

$b)\quad \log_4 4, \quad \log_4 16, \quad \log_4 \frac{1}{64}, \quad \log_8 64, \quad \log_8 \frac{1}{8}, \quad \log_8 8^{-3}$

$c)\quad \log_6 36, \quad \log_5 125, \quad \log_{16} \frac{1}{16}, \quad \log_7 1, \quad \log_7 \left(\frac{1}{7}\right)^{3}, \quad \log_7 \left(\frac{1}{49}\right)^{2}$

$d)\quad lg\,100, \quad lg\,100000, \quad lg\,\frac{1}{10}, \quad lg\,\frac{1}{1000}, \quad lg\,0.01, \quad lg\,0.0001 .$

Aufgabe 4:

Berechnen Sie x:

$\log_x 25 = 2, \quad \log_x 27 = 3, \quad \log_x \frac{1}{9} = -2, \quad \log_x \frac{1}{9} = -1.$

Logarithmen: Lösung 3

$a)\ \log_2 32 = 5, \quad \log_2 64 = 6, \quad \log_2 \frac{1}{16} = -4,$

$\log_2 \frac{1}{128} = -7, \quad \log_2 2^{-4} = -4, \quad \log_2 1 = 0,$

$b)\ \log_4 4 = 1, \quad \log_4 16 = 2, \quad \log_4 \frac{1}{64} = -3,$

$\log_8 64 = 2, \quad \log_8 \frac{1}{8} = -1, \quad \log_8 8^{-3} = -3,$

$c)\ \log_6 36 = 2, \quad \log_5 125 = 3, \quad \log_{16} \frac{1}{16} = -1,$

$\log_7 1 = 0, \quad \log_7 \left(\frac{1}{7}\right)^3 = -3, \quad \log_7 \left(\frac{1}{49}\right)^2 = -4,$

$d)\ lg\ 100 = 2, \quad lg\ 100000 = 5, \quad lg \frac{1}{10} = -1,$

$lg \frac{1}{1000} = -3, \quad lg\ 0.01 = -2, \quad lg\ 0.0001 = -4$

Logarithmen: Lösung 4

$$\log_x 25 = 2, \qquad x^2 = 25, \qquad x = 5,$$

$$\log_x 27 = 3, \qquad x^3 = 27, \qquad x = 3,$$

$$\log_x \frac{1}{9} = -2, \qquad x^{-2} = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}, \qquad x = 3$$

$$\log_x \frac{1}{9} = -1, \qquad x^{-1} = \frac{1}{9} = 9^{-1}, \qquad x = 9$$

Berechnen Sie:

Aufgabe 5:

$a)$ $\log_2 32 + \log_2 \frac{1}{8}$, $\log_4 64 + \log_4 \frac{1}{16}$, $\lg 10000 - \lg \frac{1}{100} + \lg \frac{1}{10}$,

$b)$ $\log_2 16 + \lg \frac{1}{1000}$, $\log_5 25 + \log_3 \frac{1}{9}$, $\log_6 1 + \log_7 1 - \log_8 1$,

$c)$ $\log_4 \frac{1}{4} - \log_5 \frac{1}{5}$, $\left(\log_4 \frac{1}{4}\right)^2 + \left(\log_9 \frac{1}{9}\right)^2$, $\log_7 49 + 2 \log_6 \frac{1}{6}$.

Aufgabe 6: $\log_8 16$, $\log_{32} 64$, $\log_9 27$, $\log_{16} 64$.

Aufgabe 7:

$\log_3 \left(\frac{1}{3^4}\right)$, $\log_6 \left(\frac{1}{6}\right)^8$, $\log_7 \left(\frac{1}{7}\right)^{-2}$, $\log_7 \frac{1}{49}$, $\lg \frac{1}{100}$, $\log_{0.1} 100$.

Aufgabe 8: $\log_{\sqrt{2}} 2$, $\log_{\sqrt{7}} 7$, $\log_{\sqrt{2}} 16$, $\log_{\sqrt{3}} 9$, $\log_{\sqrt{7}} (7\sqrt{7})$

a) $\log_2 32 + \log_2 \frac{1}{8} = 5 - 3 = 2, \qquad \log_4 64 + \log_4 \frac{1}{16} = 3 - 2 = 1,$

$$\lg 10000 - \lg \frac{1}{100} + \lg \frac{1}{10} = 4 - (-2) - 1 = 5,$$

b) $\log_2 16 + \lg \frac{1}{1000} = 4 - 3 = 1, \qquad \log_5 25 + \log_3 \frac{1}{9} = 2 - 2 = 0,$

$$\log_6 1 + \log_7 1 - \log_8 1 = 0 + 0 - 0 = 0,$$

c) $\log_4 \frac{1}{4} - \log_5 \frac{1}{5} = -1 - (-1) = 0,$

$$\left(\log_4 \frac{1}{4}\right)^2 + \left(\log_9 \frac{1}{9}\right)^2 = (-1)^2 + (-1)^2 = 2,$$

$$\log_7 49 + 2 \log_6 \frac{1}{6} = 2 + 2 \cdot (-1) = 0.$$

Lösung 6:

$$\log_8 16 = \tfrac{4}{3}, \quad \log_{32} 64 = \tfrac{6}{5}, \quad \log_9 27 = \tfrac{3}{2}, \quad \log_{16} 64 = \tfrac{3}{2}.$$

Lösung 7:

$$\log_3\left(\frac{1}{3^4}\right) = \log_3(3^{-4}) = -4, \qquad \log_6\left(\frac{1}{6}\right)^8 = \log_6(6^{-8}) = -8,$$

$$\log_7\left(\frac{1}{7}\right)^{-2} = \log_7(7^{-1})^{-2} = 2, \qquad \log_7 \frac{1}{49} = \log_7(7^{-2}) = -2,$$

$$\lg \frac{1}{100} = -2, \qquad \log_{0.1} 100 = \log_{0.1}(10)^2 = \log_{0.1}(0.1^{-1})^2 = -2.$$

$$\log_{\sqrt{2}} 2 = \log_{\sqrt{2}} (\sqrt{2})^2 = 2, \qquad \log_{\sqrt{7}} 7 = 2,$$

$$\log_{\sqrt{2}} 16 = \log_{\sqrt{2}} (2^4) = \log_{\sqrt{2}} \left((\sqrt{2})^2\right)^4 = \log_{\sqrt{2}} (\sqrt{2})^8 = 8,$$

$$\log_{\sqrt{3}} 9 = \log_{\sqrt{3}} \left(\sqrt{3}\right)^4 = 4,$$

$$\log_{\sqrt{7}} (7\sqrt{7}) = \log_{\sqrt{7}} \left((\sqrt{7})^2\sqrt{7}\right) = \log_{\sqrt{7}} \left((\sqrt{7})^3\right) = 3,$$

Erste Rechenregel

$$\log_b (x \cdot y) = \log_b x + \log_b y$$

Der Logarithmus eines Produkts ist gleich der Summe der Logarithmen der beiden Faktoren

$$b, \ x, \ y > 0$$

Zweite Rechenregel

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

Der Logarithmus eines Quotienten ist gleich der Differenz der Logarithmen von Zähler und Nenner

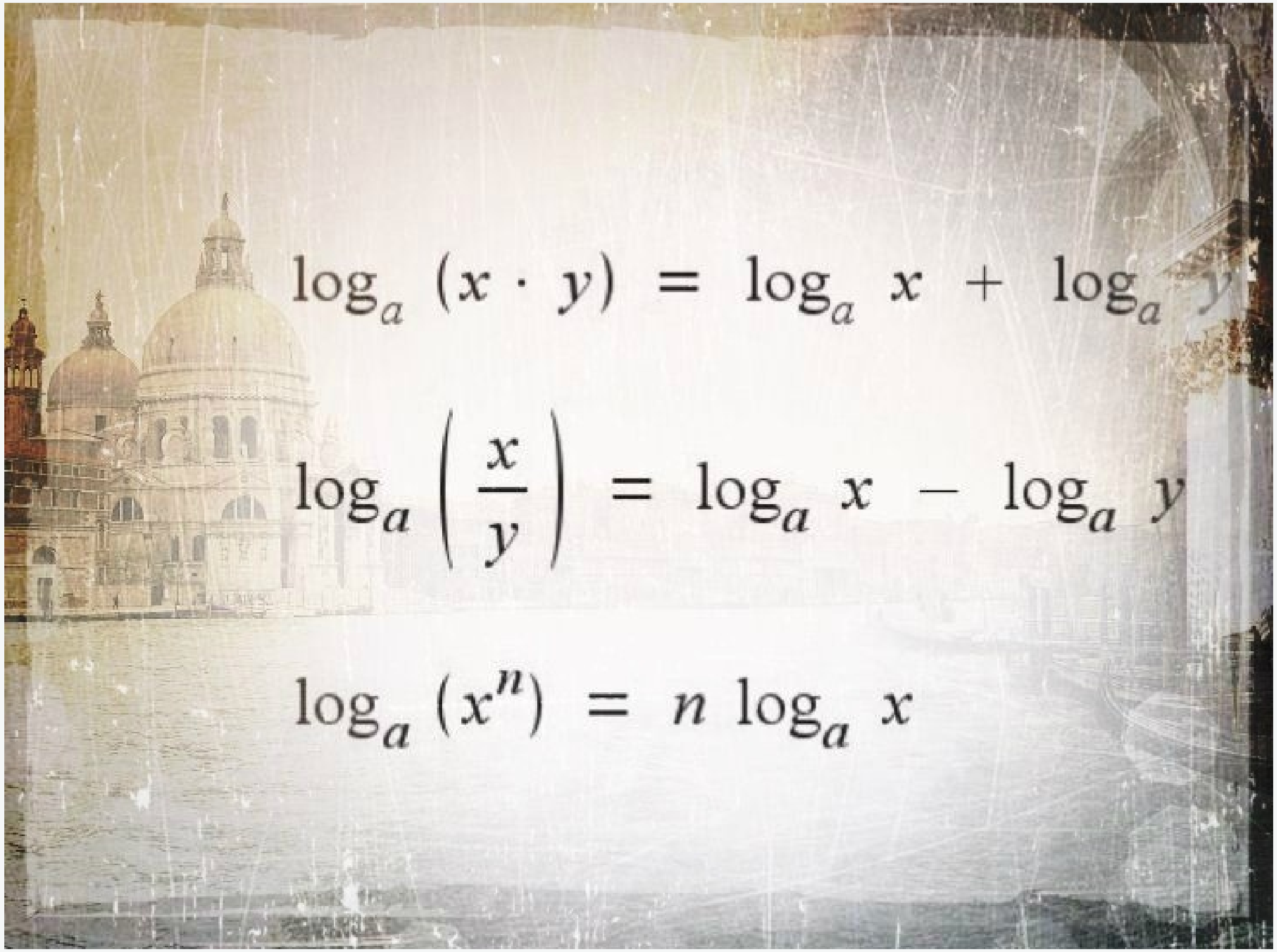
$$b\,,\; x\,,\; y > 0$$

Dritte Rechenregel

$$\log_b (x^n) = n \log_b x$$

Der Logarithmus einer Potenz ist gleich dem Produkt aus dem Exponenten und dem Logarithmus der Basis.

$$b\,,\ x > 0$$



Rechenregeln für Logarithmen

Aufgabe 9: Berechnen Sie die gegebenen Ausdrücke:

$a)$ $\log_{12} 4 + \log_{12} 3, \qquad \log_{14} 2 + \log_{14} 7, \qquad \log_{33} 3 + \log_{33} 11,$

$b)$ $lg\, 2 + lg\, 5000, \qquad \log_5 75 - \log_5 3, \qquad lg\, 300 - lg\, 3,$

$c)$ $\log_{\sqrt{2}} 4 - \log_{\sqrt{2}} 2\sqrt{2}, \qquad \log_{\sqrt{3}} 6 - \log_{\sqrt{3}} 2\sqrt{3}.$

a) $\log_{12} 4 + \log_{12} 3 = \log_{12} (4 \cdot 3) = \log_{12} 12 = 1,$

$\log_{14} 2 + \log_{14} 7 = \log_{14} (2 \cdot 7) = \log_{14} 14 = 1,$

$\log_{33} 3 + \log_{33} 11 = \log_{33} (3 \cdot 11) = \log_{33} 33 = 1,$

b) $lg\, 2 + lg\, 5000 = lg\, 10\,000 = lg\, 10^4 = 4\, lg\, 10 = 4,$

$$\log_5 75 - \log_5 3 = \log_5 \frac{75}{3} = \log_5 25 = 2 \log_5 5 = 2,$$

$lg\, 300 - lg\, 3 = lg\, 100 = 2.$

c) $$\log_{\sqrt{2}} 4 - \log_{\sqrt{2}} 2\sqrt{2} = \log_{\sqrt{2}} \left(\frac{4}{2\sqrt{2}} \right) = \log_{\sqrt{2}} \sqrt{2} = 1,$$

$$\log_{\sqrt{3}} 6 - \log_{\sqrt{3}} 2\sqrt{3} = \log_{\sqrt{3}} \sqrt{3} = 1.$$

Die gegebenen Terme sind mit Hilfe der Rechengesetze für Logarithmen (so weit wie möglich) additiv zu zerlegen:

a) $\log_3 (5\,x)\,, \quad \log_3 (3\,x)\,, \quad \log_2 (4\,x)$

b) $\log_3 \frac{1}{3}\,, \quad \log_3 \frac{1}{9}\,, \quad 2\,\log_3 \frac{1}{27}$

c) $\log (a\,b)\,, \quad \log (a\,b\,c)\,, \quad \log (a\,b\,c\,d)$

d) $\log \frac{a}{b}\,, \quad \log \frac{a\,c}{b}\,, \quad \log \frac{a\,b}{c\,d}$

e) $\log a^2\,b\,, \quad \log a^3\,b^2\,, \quad \log a^5\,b^3\,c$

f) $\log \frac{a^2\,c}{b}\,, \quad \log \frac{a\,c}{b\,d^3}\,, \quad \log \frac{\sqrt{a}\,b}{c^4}$

a) $\log_3(5x) = \log_3 5 + \log_3 x\,, \qquad \log_3(3x) = \log_3 3 + \log_3 x = 1 + \log_3 x$

$\log_2(4x) = \log_2 4 + \log_2 x = \log_2 2^2 + \log_2 x = 2 + \log_2 x$

b) $\log_3 \frac{1}{3} = \log_3 1 - \log_3 3 = -1, \qquad \log_3 \frac{1}{3} = \log_3 3^{-1} = -\log_3 3 = -1,$

$\log_3 \frac{1}{9} = \log_3 1 - \log_3 9 = 0 - \log_3 3^2 = -2\log_3 3 = -2$

$2\,\log_3 \frac{1}{27} = 2(\log_3 1 - \log_3 27) = 2(\log_3 1 - \log_3 3^3) = -2\cdot 3\,\log_3 3 = -6$

c) $\log(a\,b) = \log a + \log b\,, \qquad \log(a\,b\,c) = \log a + \log b + \log c$

$\log(a\,b\,c\,d) = \log a + \log b + \log c + \log d$

d) $\log \frac{a}{b} = \log a - \log b\,, \qquad \log \frac{a\,c}{b} = \log a + \log c - \log b$

$\log \frac{a\,b}{c\,d} = \log a + \log b - \log c - \log d$

e) $\log a^2 b = \log a^2 + \log b = 2 \log a + \log b$

$\log a^3 b^2 = \log a^3 + \log b^2 = 3 \log a + 2 \log b$

$\log a^5 b^3 c = \log a^5 + \log b^3 + \log c = 5 \log a + 3 \log b + \log c$

f) $\log \frac{a^2 c}{b} = \log a^2 + \log c - \log b = 2 \log a + \log c - \log b$

$$\log \frac{a c}{b d^3} = \log (a c) - \log (b d^3) = \log a + \log c - \log b - \log d^3 =$$

$$= \log a + \log c - \log b - 3 \log d$$

$$\log \frac{\sqrt{a}\, b}{c^4} = \log (\sqrt{a}\, b) - \log (c^4) = \log a^{1/2} + \log b - 4 \log c =$$

$$= \frac{1}{2} \log a + \log b - 4 \log c$$

Fassen Sie die folgenden Logarithmen zusammen:

$a)\ \log_u a + 2\log_u b$

$b)\ \log_u a + \log_u b - \log_u c$

$c)\ \log_u a + \frac{1}{2}\log_u b + 2\log_u c$

$d)\ \log_u a + \frac{1}{2}\log_u b - 3\log_u c$

$e)\ \log_u a + \frac{1}{2}\log_u b + \frac{1}{4}\log_u c$

a) $\log_u a + 2\log_u b = \log_u a + \log_u b^2 = \log_u (a b^2)$

b) $\log_u a + \log_u b - \log_u c = \log_u \left(\frac{a b}{c}\right)$

c) $\log_u a + \frac{1}{2}\log_u b + 2\log_u c = \log_u a + \log_u \sqrt{b} + \log_u c^2 = \log_u (a\sqrt{b}\, c^2)$

d) $\log_u a + \frac{1}{2}\log_u b - 3\log_u c = \log_u a + \log_u \sqrt{b} + \log_u c^{-3} =$

$$= \log_u (a\sqrt{b}\, c^{-3}) = \log_u \frac{a\sqrt{b}}{c^3}$$

e) $\log_u a + \frac{1}{2}\log_u b + \frac{1}{4}\log_u c = \log_u a + \log_u b^{\frac{1}{2}} + \log_u c^{\frac{1}{4}} =$

$$= \log_u a + \log_u b^{\frac{1}{2}} + \log_u c^{\frac{1}{4}} = \log_u \left(a\, b^{\frac{1}{2}} c^{\frac{1}{4}}\right) = \log_u \left(a\sqrt{b\sqrt{c}}\,\right)$$

Die gegebenen Terme sind mit Hilfe der Rechengesetze für Logarithmen (so weit wie möglich) additiv zu zerlegen:

Aufgabe 12:

a) $\log_2(4a), \quad \log_3(27b), \quad \log_2\left(\frac{c}{8}\right), \quad \log_3\left(\frac{9\sqrt{3}}{b}\right), \quad \log_4\left(\frac{8}{a}\right).$

b) $\log_b(xyz), \quad \log_b(xy^2), \quad \log_b(x^2yz^3), \quad \log_b\left(\frac{xy}{z}\right), \quad \log_b\left(\frac{xy^2}{z^3u}\right).$

c) $\log_b(\sqrt{x}\,y), \quad \log_b(\sqrt{xy}), \quad \log_b(\sqrt{x}\,\sqrt[3]{y}), \quad \log_b\left(\frac{\sqrt[5]{xy}}{z}\right), \quad \log_b\left(\frac{\sqrt[4]{x}}{\sqrt{yz}}\right).$

Aufgabe 13:

$$\log_b\left(\sqrt[3]{\frac{xy}{z}}\right), \quad \log_b\left(\sqrt[5]{\frac{12x^3}{y^2}}\right), \quad \log_b\left(\sqrt[4]{\sqrt{x}\,y^2}\right), \quad \log_b\left(\sqrt{\frac{xy^2}{\sqrt[3]{z}}}\right).$$

a) $\log_2(4a) = 2 + \log_2 a, \quad \log_3(27b) = 3 + \log_3 b, \quad \log_2\left(\frac{c}{8}\right) = \log_2 c - 3,$

$$\log_3\left(\frac{9\sqrt{3}}{b}\right) = \log_3(3^2 3^{\frac{1}{2}}) - \log_3 b = \log_3\left(3^{\frac{5}{2}}\right) - \log_3 b = \frac{5}{2} - \log_3 b,$$

$$\log_4\left(\frac{8}{a}\right) = \log_4(4 \cdot 2) - \log_4 a = \log_4(4 \cdot 4^{\frac{1}{2}}) - \log_4 a = \frac{3}{2} - \log_4 a.$$

b) $\log_b(xyz) = \log_b x + \log_b y + \log_b z, \qquad \log_b(xy^2) = \log_b x + 2\log_b y,$

$$\log_b(x^2yz^3) = 2\log_b x + \log_b y + 3\log_b z, \qquad \log_b\left(\frac{xy}{z}\right) = \log_b x +$$

$$+ \log_b y - \log_b z, \qquad \log_b\left(\frac{xy^2}{z^3u}\right) = \log_b x + 2\log_b y - 3\log_b z - \log_b u.$$

c) $\log_b(\sqrt{x}\,y) = \frac{1}{2}\log_b x + \log_b y, \qquad \log_b(\sqrt{xy}) = \frac{1}{2}(\log_b x + \log_b y),$

$$\log_b(\sqrt{x}\,\sqrt[3]{y}) = \frac{1}{2}\log_b x + \frac{1}{3}\log_b y, \qquad \log_b\left(\frac{\sqrt[5]{xy}}{z}\right) = \frac{1}{5}(\log_b x +$$

$$+ \log_b y) - \log_b z, \qquad \log_b\left(\frac{\sqrt[4]{x}}{\sqrt{yz}}\right) = \frac{1}{4}\log_b x - \frac{1}{2}(\log_b y + \log_b z).$$

Logarithmen: *Lösung 13*

$$\log_b\left(\sqrt[3]{\frac{xy}{z}}\right)=\log_b\left(\frac{xy}{z}\right)^{\frac{1}{3}}=\frac{1}{3}\left(\log_b x+\log_b y-\log_b z\right),$$

$$\log_b\left(\sqrt[5]{\frac{12x^3}{y^2}}\right)=\log_b\left(\frac{12x^3}{y^2}\right)^{\frac{1}{5}}=\frac{1}{5}\left(\log_b 12+3\log_b x-2\log_b y\right),$$

$$\log_b\left(\sqrt[4]{\sqrt{x}\,y^2}\right)=\frac{1}{4}\log_b\left(\sqrt{x}\,y^2\right)=\frac{1}{4}\left(\frac{1}{2}\log_b x+2\log_b y\right),$$

$$\log_b\left(\sqrt{\frac{xy^2}{\sqrt[3]{z}}}\right)=\frac{1}{2}\log_b\left(\frac{xy^2}{\sqrt[3]{z}}\right)=\frac{1}{2}\left(\log_b x+2\log_b y-\frac{1}{3}\log_b z\right).$$

Die gegebenen Terme sind mit Hilfe der Rechengesetze für Logarithmen (so weit wie möglich) additiv zu zerlegen:

Aufgabe 14:

$$\log_b(x+y), \quad \log_b(x-y), \quad \log_b(x^2+y^2), \quad \log_b(x^2-y^2), \quad \log_b(x^4+y^4),$$
$$\log_b(x^4-y^4), \quad \log_b\left(\frac{x^2+y^2}{x^2-y^2}\right), \quad \log_b(x^2-4y^2), \quad \log_b(9x^2-25y^2).$$

Aufgabe 15:

$$\log_b\left(\sqrt{1-x^2}\right), \qquad \log_b\left(\frac{1}{\sqrt{1-x^2}}\right), \qquad \log_b\left(\frac{1}{\sqrt{x}\,(a^2-x^2)}\right).$$

Lösung 14:

$$\log_b(x^2 - y^2) = \log_b\left((x+y)(x-y)\right) = \log_b(x+y) + \log_b(x-y),$$

$$\log_b(x^4 - y^4) = \log_b\left((x^2+y^2)(x^2-y^2)\right) = \log_b\left((x^2+y^2)(x+y)(x-y)\right) =$$

$$= \log_b(x^2+y^2) + \log_b(x+y) + \log_b(x-y),$$

$$\log_b\left(\frac{x^2+y^2}{x^2-y^2}\right) = \log_b\left(\frac{x^2+y^2}{(x+y)(x-y)}\right) = \log_b(x^2+y^2) - \log_b(x+y) -$$

$$-\log_b(x-y), \quad \log_b(x^2 - 4y^2) = \log_b(x-2y) + \log_b(x+2y),$$

$$\log_b(9x^2 - 25y^2) = \log_b(3x-5y) + \log_b(3x+5y).$$

Lösung 15:

$$\log_b\left(\sqrt{1-x^2}\right) = \frac{1}{2}\log_b\left(1-x^2\right) = \frac{1}{2}\left(\log_b(1-x) + \log_b(1+x)\right),$$

$$\log_b\left(\frac{1}{\sqrt{1-x^2}}\right) = -\frac{1}{2}\log_b\left(1-x^2\right) = -\frac{1}{2}\left(\log_b(1-x) + \log_b(1+x)\right),$$

$$\log_b\left(\frac{1}{\sqrt{x}\,(a^2-x^2)}\right) = -\log_b\left(\sqrt{x}\,(a^2-x^2)\right) = -\frac{1}{2}\log_b x - \log_b(a-x) -$$

$$-\log_b(a+x).$$